Intrinsic topological order in materials with strong spin-orbit coupling

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October 4, 2016

Réunion thématique "Matériaux et Phases topologiques"
Lyon, 3-4 Octobre

Acknowledgements

- N. Regnault (ENS, CNRS)
- B.A. Bernevig (Princeton University)
- F. Pollmann (Max Planck Institute for the physics of complex systems)
- Y.-C. He (Max Planck Institute for the physics of complex systems $\rightarrow$ Harvard)
- R. Moessner (Max Planck Institute for the physics of complex systems)

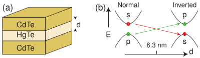
Motivations

Spin-orbit coupling

Essential ingredient of topological insulators

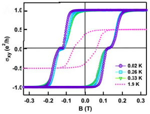
Inhibitor

2D time-reversal symmetric TI



Hg(Cd)Te 2007

Chern insulator



Cr thin films 2015

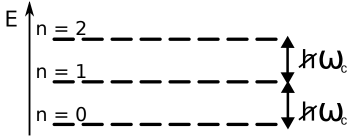
- Rashba spin-orbit coupling in TRS topological insulators
- Dzyaloshinskii-Moriya interaction in frustrated antiferromagnets

Strong interactions: FQH-like phases?

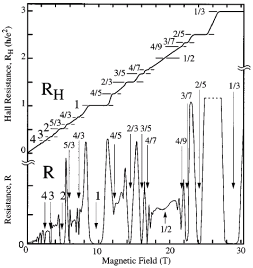
How are the strongly interacting phases affected by spin-orbit perturbations?

Electrons in a magnetic field in 2D: the FQHE

Single-particle picture:
Landau levels (LL)



filling fraction $\nu = N/N_\phi$



Quantized Hall conductance: $\sigma_{xy} = \nu e^2/h$ (topological invariant)

$\nu = n$

Integer Quantum Hall Effect

interactions can be neglected

single-particle problem

$\nu < 1$

Fractional quantum Hall effect

interactions are crucial

N-body problem

Numerical methods are necessary (ED, DMRG)

The Laughlin state

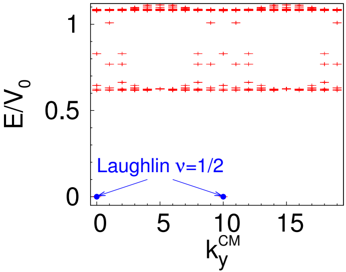
First observed FQH fractions: $\nu = 1/3$

$\rightarrow$ Series of Laughlin fractions: $\nu = 1/m$, fermionic but also bosonic

Let us focus on the $\nu = 1/2$ case

- Bosonic state
- Model Hamiltonian: $H_{2body} = P_{LLL} V_0 \delta(r_i - r_j) P_{LLL}$
- Excitations have fractional charge $e/2$ and fractional statistics (semions)

Exact diagonalization with periodic boundary conditions
$N = 10, \ N_\phi = 20$

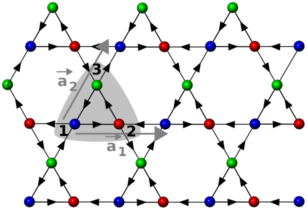


Exact twofold degeneracy on the torus

Signature of intrinsic topological order

Quantized conductance without a magnetic field

Kagome lattice model with NN hopping and spin-orbit coupling



$$H_0 = -t \sum_{<i,j>,\sigma=\uparrow\downarrow} c^\dagger_{i\sigma} c_{j\sigma} + i\lambda \sum_{<i,j>,\sigma,\sigma'} (\mathbf{E_{ij}} \times \mathbf{r_{ij}}) \cdot \sigma_{\sigma,\sigma'} c^\dagger_{i\sigma} c_{j\sigma'}$$

$$\sigma = \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{pmatrix}$$ Pauli matrices

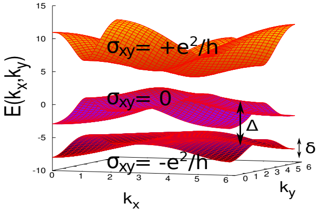
- $(\mathbf{E_{ij}} \times \mathbf{r_{ij}}) \perp$ kagome plane $\rightarrow c^\dagger_{i\uparrow} c_{j\uparrow} + c^\dagger_{i\downarrow} c_{j\downarrow}$
- $(\mathbf{E_{ij}} \times \mathbf{r_{ij}}) \not\perp$ kagome plane $\rightarrow$ interspin terms $c^\dagger_{i\uparrow} c_{j\downarrow} + c^\dagger_{i\downarrow} c_{j\uparrow}$

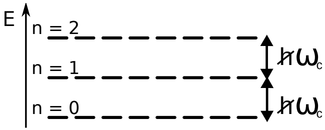
Quantized conductance without a magnetic field

$$H_0 = (-t + i\lambda') \sum_{<i,j>} c^\dagger_{i\uparrow} c_{j\uparrow} + (-t - i\lambda') \sum_{<i,j>} c^\dagger_{i\downarrow} c_{j\downarrow}$$

Spin-polarized model: $H_0 = (-t + i\lambda') \sum_{<i,j>} c^\dagger_{i\uparrow} c_{j\uparrow}$

(time-reversal symmetry can be broken by coupling to a ferromagnet)





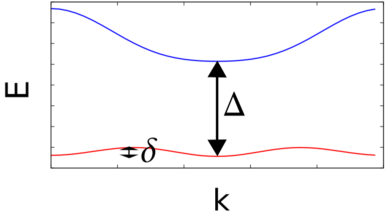
- complex hopping →
- band structure →

- Aharonov-Bohm phase
- Landau levels

Engineering a fractional Chern insulator

Goal: obtain a Chern band energetically similar to the LLL

A strong dispersion could destroy a potential CI fractional state



- V interaction scale: $\delta \ll V \ll \Delta$
- flat band limit: continuously deform the lowest band $\delta \to 0$
- lowest (occupied) band projection: $\Delta \to \infty$ ($\simeq$ LLL projection)

FCI

- $N_x \times N_y$ unit cells
- periodic boundary conditions
- N bosons, $\nu = \frac{N}{N_x N_y} = 1/2$
- Hubbard interaction : $\rho_i \rho_i$: projection into the lowest band

FQH

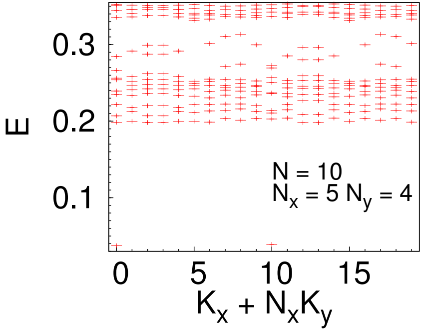
N_ϕ flux quanta

torus

$\nu = \frac{N}{N_\phi} = 1/2$

$\delta(\mathbf{r}_i - \mathbf{r_j})$
LLL projection

The Laughlin state on the kagome lattice Chern insulator



$$K_x = \sum_i k_{xi} \bmod N_x$$

$$K_y = \sum_i k_{yi} \bmod N_y$$

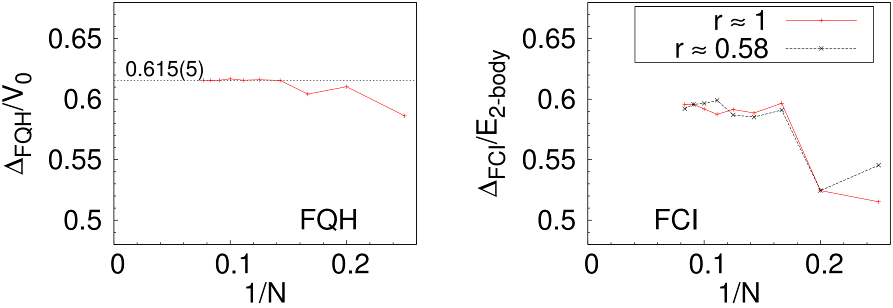
Exact diagonalization

Twofold quasidegenerate ground state

- Even in the ideal conditions, Laughlin phase not always realized
- $V \gg \Delta$ can work in some cases

Gap extrapolation

Relevance of thermodynamic limit extrapolation from small systems?



CR, T. Neupert, Z. Papic, N. Regnault, PRB 2014

Very little finite size effect

$$\Delta_{FQH}/V_0 = 0.615(5) \simeq \Delta_{FCI}/E_{2-\mathrm{body}} = 0.60(3)$$

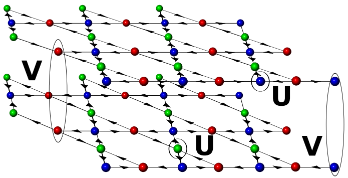
Time-reversal symmetric topological insulator

Original spinful TRS Hamiltonian

$$H_0 = -t \sum_{<i,j>,\sigma=\uparrow\downarrow} c^\dagger_{i\sigma} c_{j\sigma} + i\lambda \sum_{<i,j>,\sigma,\sigma'} (\mathbf{E_{ij}} \times \mathbf{r_{ij}}) \cdot \sigma_{\sigma,\sigma'} c^\dagger_{i\sigma} c_{j\sigma'}$$

- $\uparrow, \downarrow$ pseudospin index for bosons
- $\mathbb{Z}_2$ topological invariant:
  - 1 if only $H_0^{\uparrow\uparrow} + H_0^{\downarrow\downarrow}$ terms
  - can be driven to trivial phase for large strength of $H_0^{\uparrow\downarrow}$

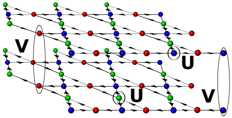
Quantitative statement in the presence of strong interactions?



$$H = H_0^{\uparrow\uparrow} + H_0^{\downarrow\downarrow} + H_0^{\uparrow\downarrow} + \frac{U}{2} (: \rho_{i\uparrow}\rho_{i\uparrow} : + : \rho_{i\downarrow}\rho_{i\downarrow} :) + V : \rho_{i\uparrow}\rho_{i\downarrow} :$$

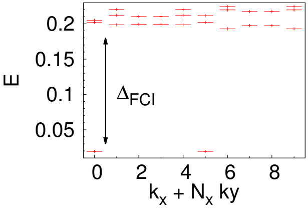
From FCI to FTI ($H_0^{\uparrow\downarrow} = 0$)

Expected phase diagram of the $\nu = 1/2$ FTI?



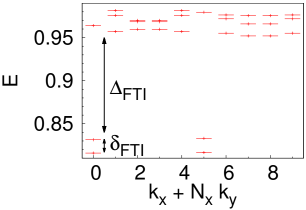
- $V = 0$: $\mathrm{Laughlin} \otimes \overline{\mathrm{Laughlin}}$
- Large V/U: fully polarized system
  $S_z \equiv \frac{N^{\uparrow} - N^{\downarrow}}{2} = N/2$
- U/V only parameter

FCI : $\mathrm{Laughlin{:}}\ \mathrm{deg.} = 2$



$N = 5,\ N_x = 5,\ N_y = 2$

FTI : $\mathrm{Laughlin} \otimes \overline{\mathrm{Laughlin}}{:}\ \mathrm{deg.} = 2 \times 2$

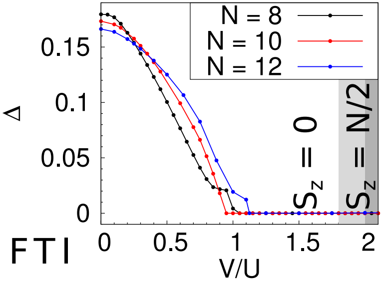


$N = 10,\ N_x = 5,\ N_y = 2,\ V = 0.5U$

Stability of the $\nu = 1/2$ FTI: V coupling ($H_0^{\uparrow\downarrow} = 0$)

Bulk signature:

- Energy spectrum: ground state fourfold almost degeneracy + gap



→ stable up to $V/U \simeq 1$

→ fully polarized at $V/U \simeq 2$

CR, B.A. Bernevig, N. Regnault PRB 2014

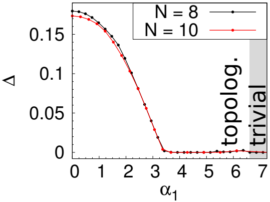
Stability of the $\nu = 1/2$ FTI: $H_0^{\uparrow\downarrow} \neq 0$

$$H_R(\mathbf{k}) = \begin{pmatrix} h_{\mathrm{CI}}(\mathbf{k}) & R \\ R^t & h^*_{\mathrm{CI}}(-\mathbf{k}) \end{pmatrix}$$

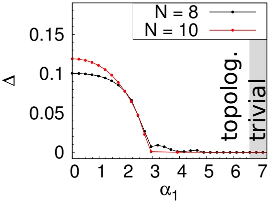
$R = -R^t \in \mathbb{R}$, constant, breaks inversion symmetry

$$R = \alpha_1 R_1 + \underbrace{\alpha_2 R_2 + \alpha_3 R_3}_{\text{breaks } C_3 \text{ sym.}}$$

- $\alpha_2 = \alpha_3 = 0 \rightarrow$ preserves C_3

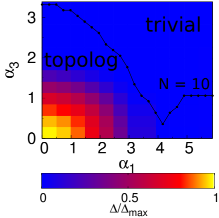


No interlayer interaction



$V = 0.5U$

- $\alpha_3 \neq 0 \rightarrow$ breaks C_3

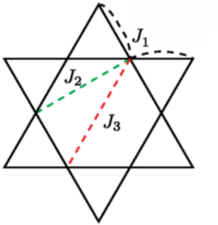


No interlayer inter.

- **FTI stability zone has significant overlap with one-body topological region**

CR, B.A. Bernevig, N. Regnault PRB 2014

Spontaneous time-reversal symmetry breaking: the kagome chiral spin liquid



$$H = J_1 \sum_{<i,j>} \mathbf{S_i S_j} + J_2 \sum_{\ll i,j \gg} S_i^z S_j^z + J_3 \sum_{\lll i,j \ggg} S_i^z S_j^z$$

$J_1 = J_2 = J_3$: DMRG studies have found ground state properties

S. S. Gong, W. Zhu, and D. N. Sheng, Sci. Rep. 2014

S.S Gong, W. Zhu, L. Balents, D.N. Sheng, PRB 2015

Y.-C. He, Y. Chen, PRL 2015

- no lattice symmetry breaking
- spontaneous time-reversal symmetry breaking
- akin to $\nu = 1/2$ Laughlin state (Kalmeyer-Laughlin)

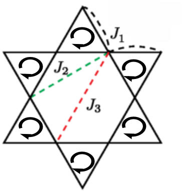
Chiral spin liquid

Breakdown of the CSL with Dzyaloshinskii-Moriya interaction

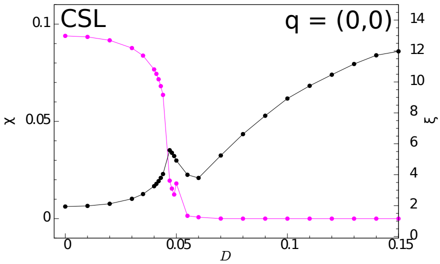
Stability with respect to spin-orbit coupling?

Dzyaloshinskii-Moriya interaction: $H_D = D \sum_{<i,j>} \mathbf{e_z} \cdot (\mathbf{S_i} \times \mathbf{S_j})$

Herbertsmithite: $D/J_1 \simeq 0.05 - 0.1$



Scalar spin chirality
$\chi = \sum_{i,j,k \in \Delta} \mathbf{S_i} \cdot (\mathbf{S_j} \times \mathbf{S_k})$
(measures time-reversal symmetry breaking)



DMRG on an infinitely long cylinder of 4 unit cells of circumference

CR et. al, in preparation

Conclusion

- FQH-like phases at $B = 0$:
  - from the interplay of spin-orbit coupling and interactions
  - from interactions alone
- Stability of TRS fractional topological insulators
  - with coupling interaction
  - with Rashba coupling
- Rare-earth triangular antiferromagnet? (anisotropic spin-spin interactions due to spin-orbit, DM forbidden by symmetry)