Laboratoire de Physique Théorique

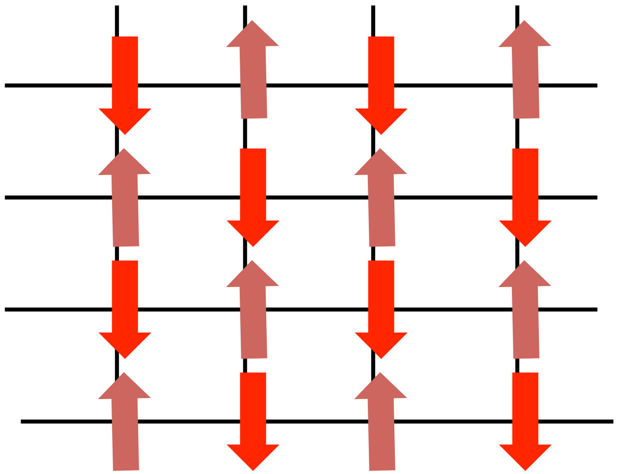
Pierre Pujol
Laboratoire de Physique Théorique
LCAR - LCPQ - LPCNO - LPT
Université Paul Sabatier, Toulouse

Topological states in quantum antiferromagnets

Thanks to I. Makhfudz, S. Takayoshi and A. Tanaka

Quantum AF systems : GS zoology

Non frustrated AF:

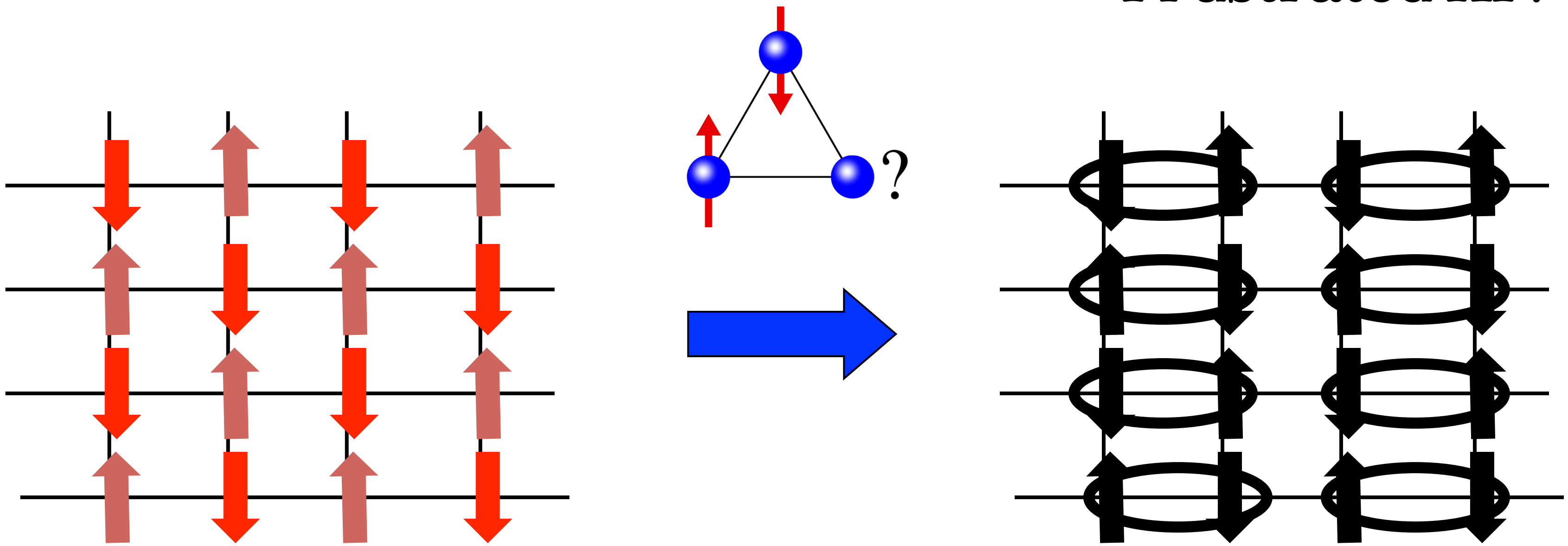


(anti-ferro) magnetic order

Quantum AF systems : GS zoology

Non frustrated AF:

Frustrated AF:

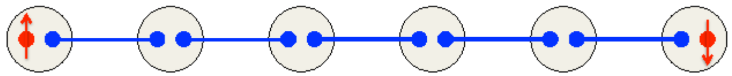


(anti-ferro) magnetic order

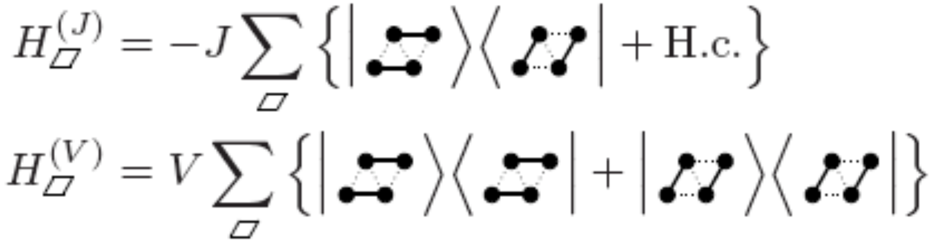
Non magnetic order

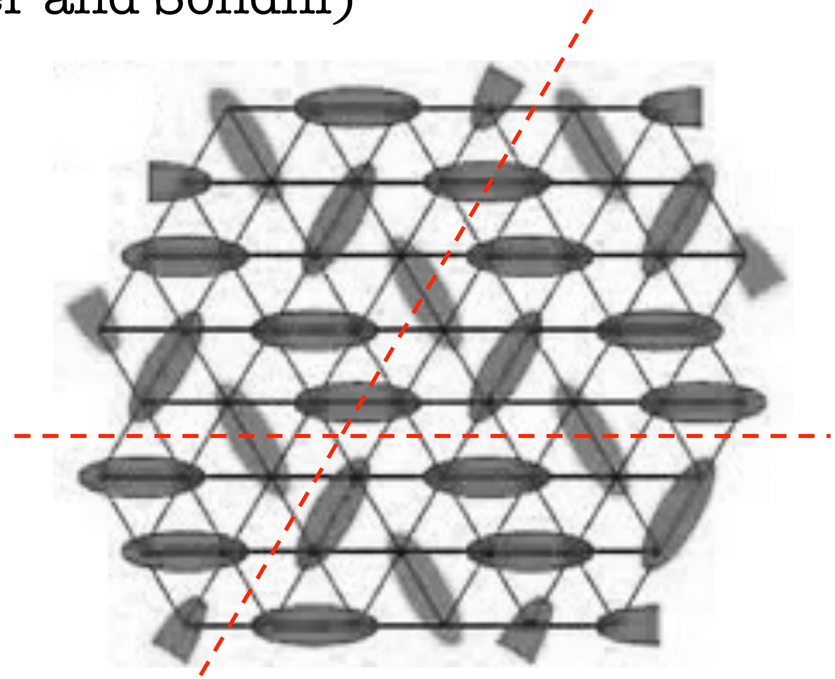
Quantum AF systems : GS zoology

GS of the AKLT type (SPT)



Z_2 spin liquid : The Rokhsar-Kivelson model in the triangular lattice (Moessner and Sondhi)





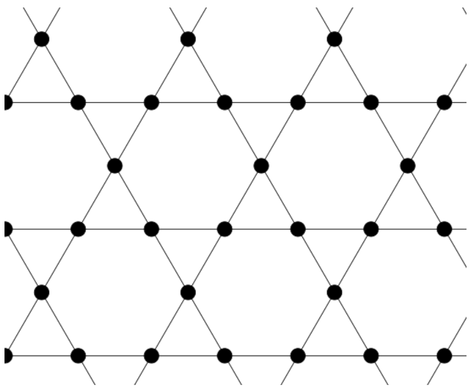
Chiral spin liquids

➡ Looking for a QHE state in magnetic degrees of freedom (Kalmeyer and. Laughlin, Wen, Wilczek, and Zee, Yang, Warman, and Girvin.....)

Chiral spin liquids

➡ Looking for a QHE state in magnetic degrees of freedom (Kalmeyer and. Laughlin, Wen, Wilczek, and Zee, Yang, Warman, and Girvin.....)

➡ The kagome lattice with explicitely broken TRI is a good candidate (ex. Fradkin et al., Moessner et al.):



$$H = J \sum_{\langle i,j \rangle} \{ S_i^x S_j^x + S_i^y S_j^y + \lambda S_i^z S_j^z \} - h_{\rm ext} \sum_i S_i^z,$$

$$H_{\rm ch} = h \sum_{\triangle} \chi_{ijk}(\triangle) = h \sum_{\triangle} \boldsymbol{S}_i \cdot (\boldsymbol{S}_j \times \boldsymbol{S}_k),$$

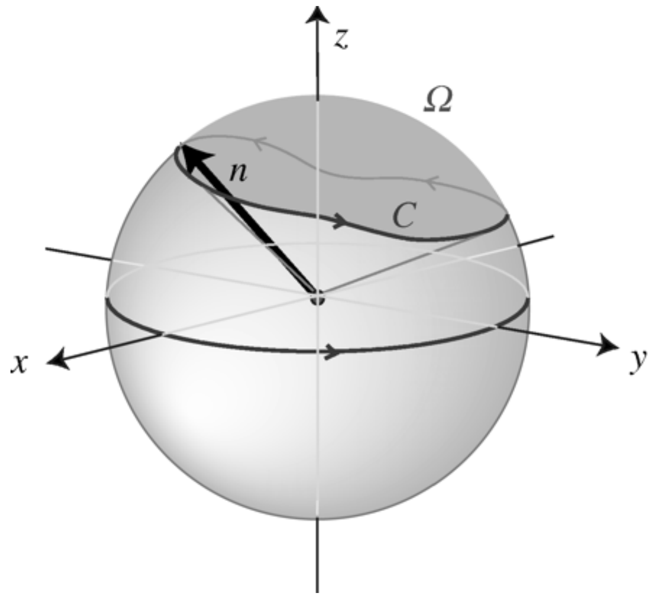
The path integral approach

- Write down a path integral for spins (Haldane)

- A particular contribution to the action, the Berry phase term

$$Z = \int D\mathbf{n}\, e^{-S_E(\mathbf{n})}$$

$$S_E = -\langle \mathbf{n}|H|\mathbf{n}\rangle + is\omega(\mathbf{n})$$



The path integral approach

Haldane's NLSM for spin chains:

$$\mathcal{S}_{\text{eff}}[\boldsymbol{n}(\tau,x)] = \frac{1}{2g}\int d\tau dx \{(\partial_\tau \boldsymbol{n})^2 + (\partial_x \boldsymbol{n})^2\} \; + 2\pi i S Q_{\tau x}$$

$$Q_{\tau x} = \frac{1}{4\pi}\int d\tau dx \boldsymbol{n}\cdot\partial_\tau \boldsymbol{n} \times \partial_x \boldsymbol{n} \in \mathbb{Z}.$$

The path integral approach

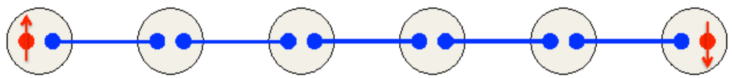
Planar and CP^1 representation:

$$\boldsymbol{n}^{\mathrm{pl}} \equiv (\cos\phi, \sin\phi, 0) \qquad Q_{\mathrm{v}} = \frac{1}{2\pi}\int d\tau dx(\partial_\tau\partial_x - \partial_x\partial_\tau)\phi \in \mathbb{Z}.$$

$$a_\mu = \partial_\mu\phi/2, \qquad \mathcal{S}_\Theta = i\frac{\Theta}{2\pi}\int d\tau dx(\partial_\tau a_x - \partial_x a_\tau) \quad (\Theta = 2\pi S)$$

➡ And for an open spin S chain, integrate to get a boundary topological term (Ng, 1994)
Example : spin 1 AKLT chain

$$\mathcal{S}_{\mathrm{edge}} = \pm iS\int d\tau a_\tau$$



The path integral approach

2-D case (square lattice), monopoles play a role (Haldane):

$$\boldsymbol{n} = z^\dagger \frac{\boldsymbol{\sigma}}{2} z$$
$$a_\mu = i z^\dagger \partial_\mu z$$

$$\mathcal{S}_{\rm eff}^{\rm 2d} = \frac{1}{2K}\int d\tau d^2\boldsymbol{r} (\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 + i\frac{\pi S}{2} Q_{\rm mon}^{\rm tot}$$
$$= \int d\tau d^2\boldsymbol{r} \left\{ \frac{1}{2K}(\epsilon_{\mu\nu\lambda}\partial_\nu a_\lambda)^2 + i\frac{S}{4}\epsilon_{\mu\nu\lambda}\partial_\mu\partial_\nu a_\lambda \right\}$$

➡ Only even-integer spins admit non-degenerate GS

Work donne in colaboration with S. Takayoshi and A. Tanaka, arXiv:1609.01316

The path integral approach

- And for a system with boundary, integrate again to get a boundary topological term

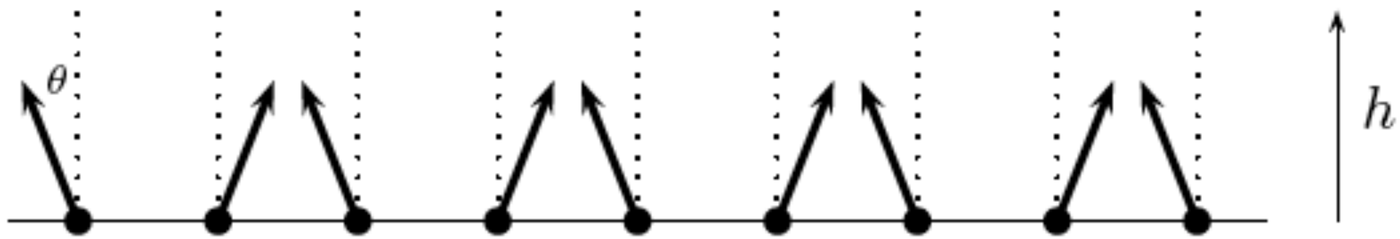
$$\mathcal{S}_{y\text{-edge}} = \pm i\frac{S}{4}\int d\tau dx(\partial_\tau a_x - \partial_x a_\tau) = \pm i\frac{\pi S}{2}Q_{\tau x}$$

0 (mod 4) vs 2 (mod 4) : Only the second is SPT

The path integral approach

- Adapt the path integral approach to the presence of a magnetic field (Tanaka, Totsuka, Hu)

- The starting point is a quasi-classical configuration with non-zero neat magnetization



$$\vec{S}_j = (S\sin\theta_j \ \cos\phi_j, S\sin\theta_j \ \sin\phi_j, S\cos\theta_j)$$

The path integral approach

Integrate out short ranged fields to get the effective action for the Goldstone field :

$$S = \int dx d\tau \left\{ \frac{K_\tau}{2} (\partial_\tau \phi)^2 + \frac{K_x}{2} (\partial_x \phi)^2 + i \left(\frac{S-m}{a} \right) (\partial_\tau \phi) \right\}$$

Berry phase term

Coupling to (Chern) Charge degrees of freedom

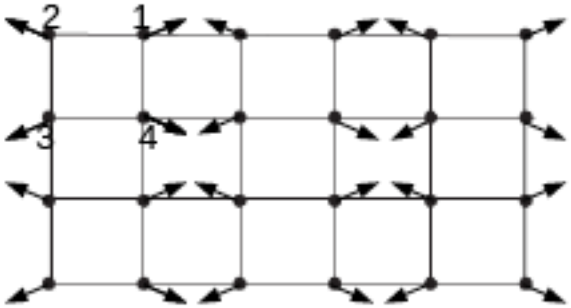
→ Then, extend the study to the presence of moving holes

$$H_{hopp} = -t \sum_j \langle \Omega_j | \Omega_{j+1} \rangle \, \psi^\dagger_{j+1} \psi_j + \text{h.c.}$$

$$\begin{aligned}\mathcal{S}_{eff} = \int dx d\tau \Big\{ & \frac{1}{2} K_x (\partial_x \phi)^2 + \frac{1}{2} K_t (\partial_\tau \phi)^2 \\ & + i \left(1 - \frac{\delta}{2S}\right) \left(\frac{S-m}{a}\right) (\partial_\tau \phi) \Big\} \\ & + i g_1 \int dx d\tau \left\{ \bar{\Psi}(x) \sigma_3 \left(\partial_x + i \frac{S(S-m)}{m} \partial_x \phi \right) \Psi(x) \right\} \\ & - \int dx d\tau \left\{ \bar{\Psi}(x) \mathbb{I} \left(\partial_\tau - i g_2 (\partial_\tau \phi) \right) \Psi(x) \right\},\end{aligned}$$

Coupling to (Chern) Charge degrees of freedom

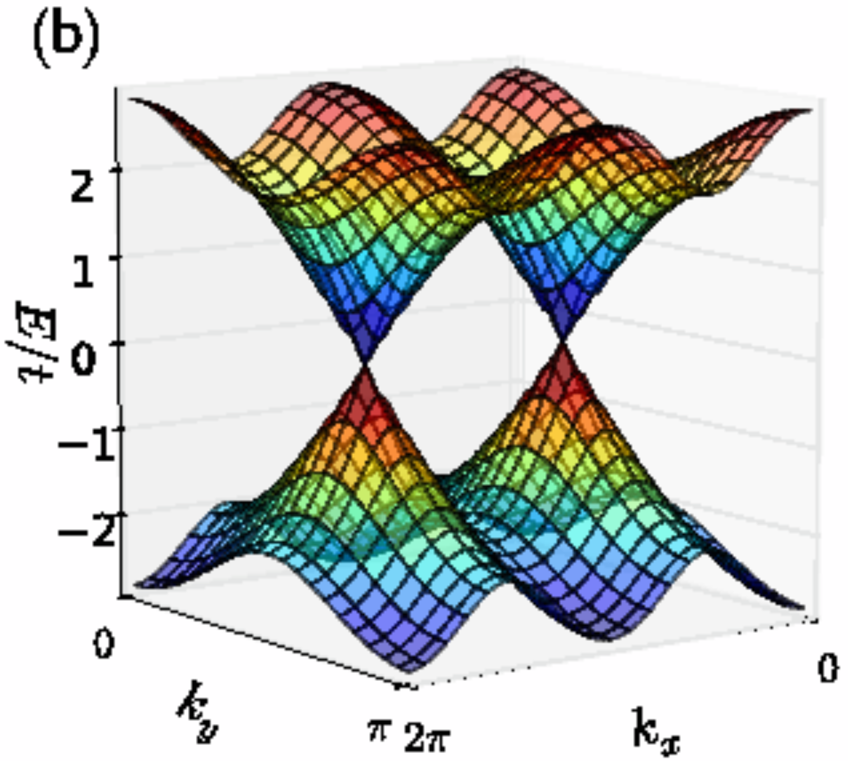
$$H = J\sum_{ij}\mathbf{S}_i\cdot\mathbf{S}_j + D\sum_i (S_i^z)^2 + \sum_{ij}\mathbf{D}_{DM}^{ij}\cdot(\mathbf{S}_i\times\mathbf{S}_j)$$



Doped electrons feel an effective flux of ± π per plaquette

Work donne in colaboration with I. Makhfudz,
*Phys. Rev. **B** 92, pp. 144507, 2015.*

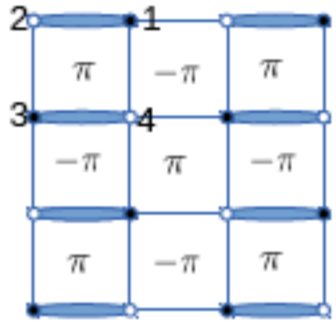
Coupling to (Chern) Charge degrees of freedom



➔ Dirac-like dispersion relation at half filling for the electrons

Coupling to (Chern) Charge degrees of freedom

Add dimerization to gap the charge degrees of freedom



➡ Effective action for the charge in the con

$$S_{\overline{\psi},\psi} = \int d^2x \int d\tau \overline{\psi}[\gamma^\mu(-i\partial_\mu - eA_\mu) + m_\psi]\psi$$

Coupling to (Chern) Charge degrees of freedom

Integrate out these “harmless” degrees of freedom and get an effective action for the spin sector

$$S_\phi = \int d^2x \int d\tau \frac{K_\tau}{2}(\partial_\tau \phi)^2 + \frac{K_r}{2}(\nabla\phi)^2 + i\left(\frac{S}{a^2}\right)\partial_\tau\phi$$

$$S_{CS}[A] = i\frac{N_f}{2}\frac{e^2}{4\pi}\frac{m_\psi}{|m_\psi|}\int d^3x\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda \qquad A_\mu = \partial_\mu\phi$$

$$\mathcal{L}_{CS} = i\frac{\kappa}{2\pi}\epsilon^{\mu\nu\lambda}\partial_\mu\phi\partial_\nu\partial_\lambda\phi$$

Coupling to (Chern) Charge degrees of freedom

Effective action for the spin sector : Dual vortex theory

$$L[J_{\mathcal{V}}] = \int_k J^{\mu}_{\mathcal{V}}(k) \frac{1}{k^2} \left(\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) - \kappa \pi \epsilon_{\mu\nu\alpha} k^{\alpha} \right) J^{\nu}_{\mathcal{V}}(-k)$$

$$J^{\lambda}_{\mathcal{V}} = (1/(2\pi)) \epsilon^{\lambda\mu\nu} \partial_\mu \partial_\nu \phi$$

New contribution to the Berry phase term :

$$\mathcal{N}_{\text{linking}} = \frac{1}{4\pi} \oint_{\gamma_1} \oint_{\gamma_2} \frac{(\mathbf{r}_1 - \mathbf{r}_2) \cdot (d\mathbf{r}_1 \times d\mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|^3}$$

$$e^{i 4\pi \kappa q_1 q_2 + i q_1 q_2 E_{\text{Coulomb}}}$$

Coupling to (Chern) Charge degrees of freedom

➡ The partition function of an anyon gaz

$$\Theta = 2\kappa\pi = \pi e^2/2$$

here

$$e = -\frac{1}{2\sqrt{2}}\left[1 - \frac{J}{\sqrt{J^2 + D_{DM}^2}}\right]^{\frac{1}{2}}$$

upper bound $\Theta = \pi/16$

➡ Protection against the spin gap

UNIVERSITÉ TOULOUSE III PAUL SABATIER Université de Toulouse

Laboratoire de Physique Théorique

LPT LABORATOIRE DE PHYSIQUE THÉORIQUE TOULOUSE

Perspectives: in search of a chiral plateau state

➡ Find a microscopic model where the vortex condensation is possible

➡ This would realize the equivalent of a QHE state in the spin sector

➡ This scenario is expected to reproduce when coupling to a generic Chern insulator

Thank you for your attention!