# Multi-terminal Josephson junctions as topological matter

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with

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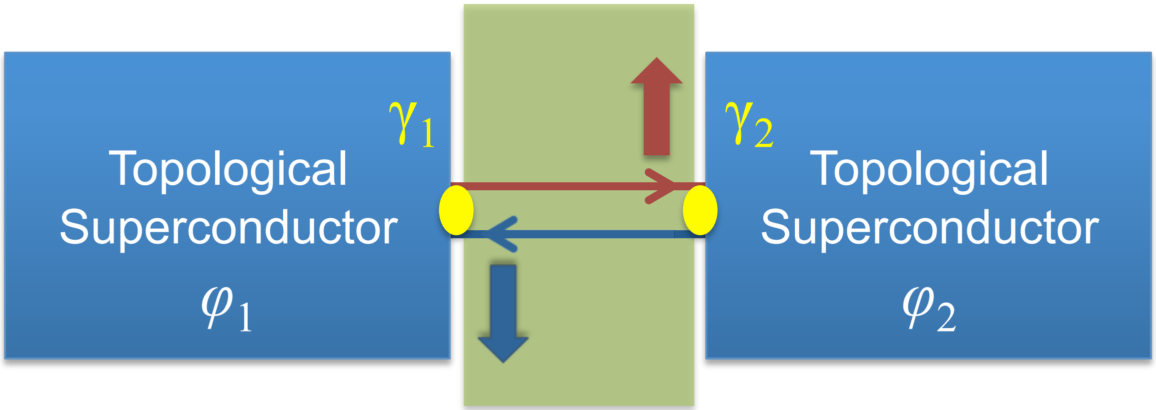
**TopoLyon 2016:** "Matériaux et phases topologiques"

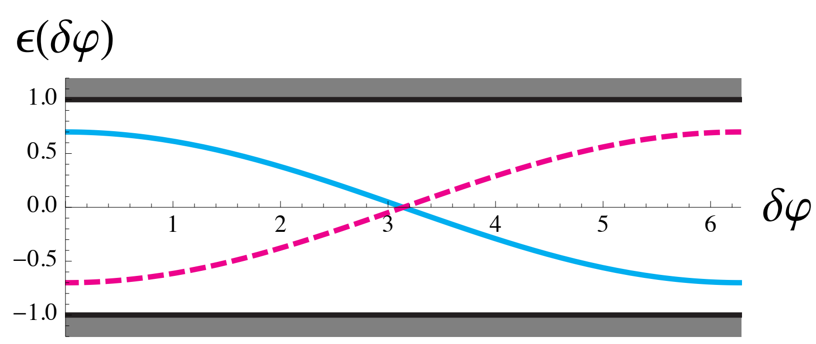
October 2, 2016

# Motivation

Josephson junctions as probes of topology:




2 Majorana bound states form
1 Andreev bound state (ABS)




→ Josephson effect yields signatures of topological superconductivity

Kitaev (2003), Fu & Kane (2009)

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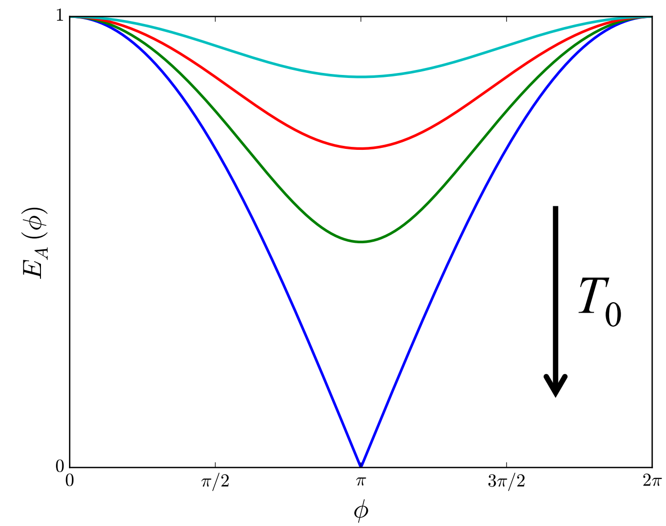
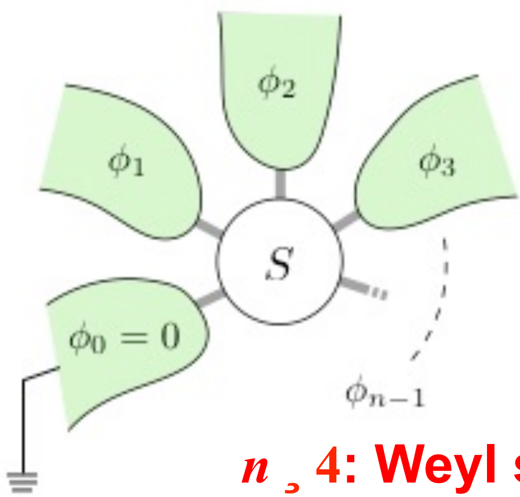
# Motivation

Josephson junctions as topological materials ?




conventional s-wave superconductors:
Andreev bound state spectrum …

- two-terminal junction:
  only accidental zero-energy states
- multi-terminal junction ?




$n$ terminals
$\rightarrow$ $n - 1$ independent phases

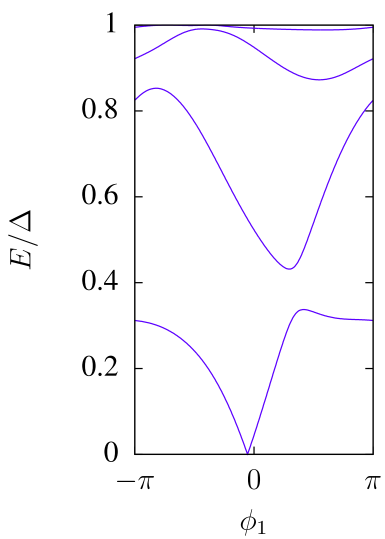
**$n$ ¸ 4: Weyl singularities !**

# Main result

topologically-protected

Weyl singularities in the ABS spectrum

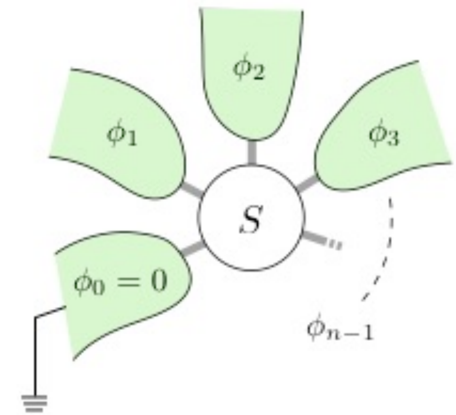
of junctions with $n \geq 4$ terminals




superconducting phases

= quasi-momenta

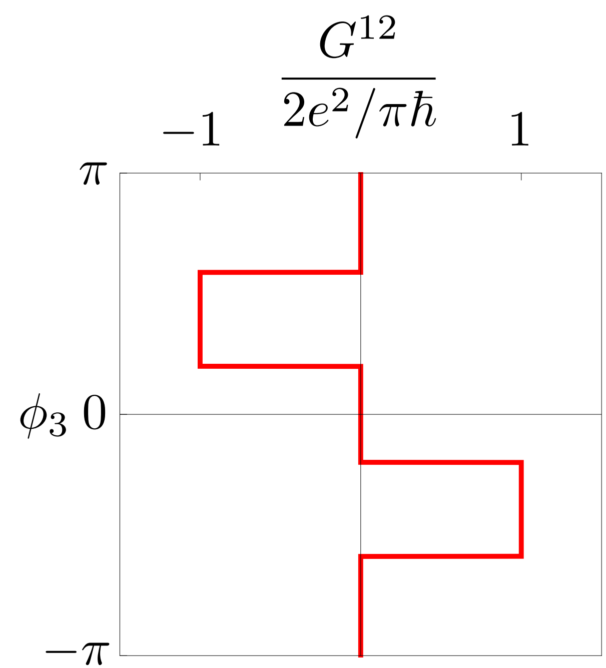
$\rightarrow$ $n$-1 dimensional "bandstructure"




manifestations:

quantized transconductance

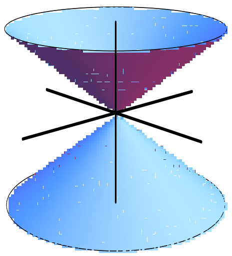
between 2 voltage-biased terminals



# Outline

- Weyl singularities
- Andreev bound state (ABS) spectrum of multi-terminal junctions
- Quantized transconductance
- Beyond the adiabatic regime
- Conclusion

# Weyl singularities



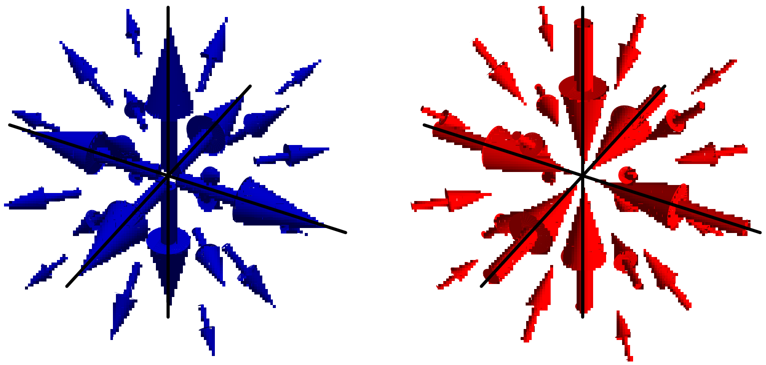
- topologically protected zero-energy states

3D Weyl Hamiltonian: $H_W = \sum_{i,j=x,y,z} v_{ij} k_i \sigma_j$

Weyl points carry a topological charge:

- Weyl points are monopoles of Berry curvature

$$\chi = \frac{1}{2\pi} \oint d\mathbf{S}(\mathbf{k}) \cdot \mathbf{B}(\mathbf{k})$$
$$= \operatorname{sign} \det[\{v_{ij}\}]$$

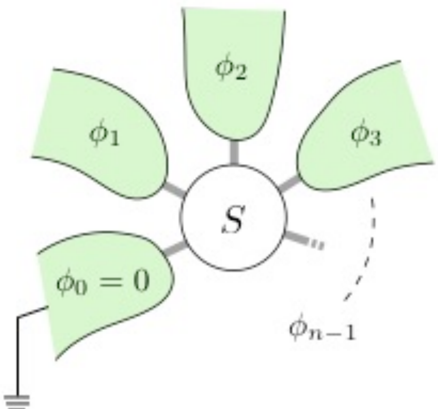


- Weyl points only appear in pairs
  of positive and negative charge
  $\rightarrow$ total charge = 0

Nielsen & Ninomiya (1981)

Weyl semimetals have been discovered recently (TaAs …)

# ABS spectrum




- scattering region described by scattering matrix $S$
  in the space of $N = \sum_\alpha N_\alpha$ channels

- time-reversal symmetry: $S = S^T$

- ABS spectrum determined through $\det\left[1 - e^{-2i\chi} A(\hat{\phi})\right] = 0$

  Beenakker (1991)

  with $\chi = \arccos(E/\Delta)$ & $A(\hat{\phi}) = S\, e^{i\hat{\phi}} S^* e^{-i\hat{\phi}}$

- eigenvalues $e^{\pm i a_k}$
  corresponding to energies §$E_k$ with $E_k = \Delta \cos(a_k/2)$

- zero-energy state at $\Phi^{(0)}$ : doubly degenerate eigenvalue $-1$

# Weyl-Hamiltonian

- in the vicinity of the zero-energy solution at $\Phi^{(0)}$ :

  effective low-energy Weyl Hamiltonian

  in the subspace of the 2 orthogonal eigenstates:

  $H_W = \sum_{\alpha,\, i} M_{\alpha i}\, \delta\phi_\alpha\, \tau_i$

→ 3 independent phases are needed to tune the energy to $E = 0$

→ Weyl singularities exist in junctions with $n \geq 4$ terminals

- topological charge of the Weyl point in a 3D subspace:

  $\chi = \text{sign}\ \det\,[\{M_{\alpha i}\}]$

- time-reversal symmetry: Weyl point at $\Phi^{(0)}$

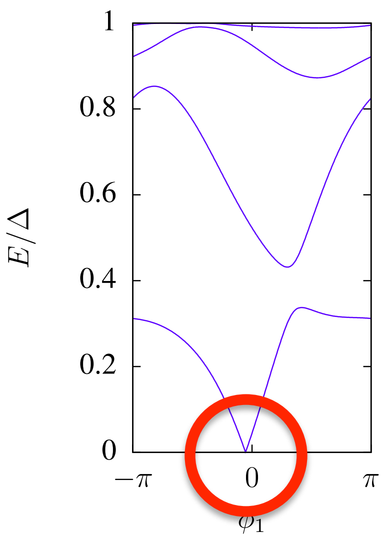
  → Weyl point with the same topological charge at $-\Phi^{(0)}$

- Weyl points come in multiples of 4

# ABS spectrum

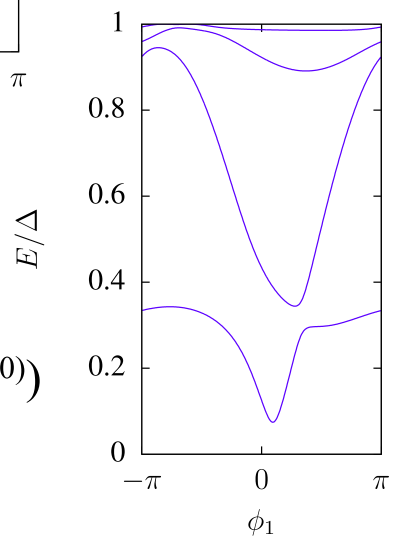
example:

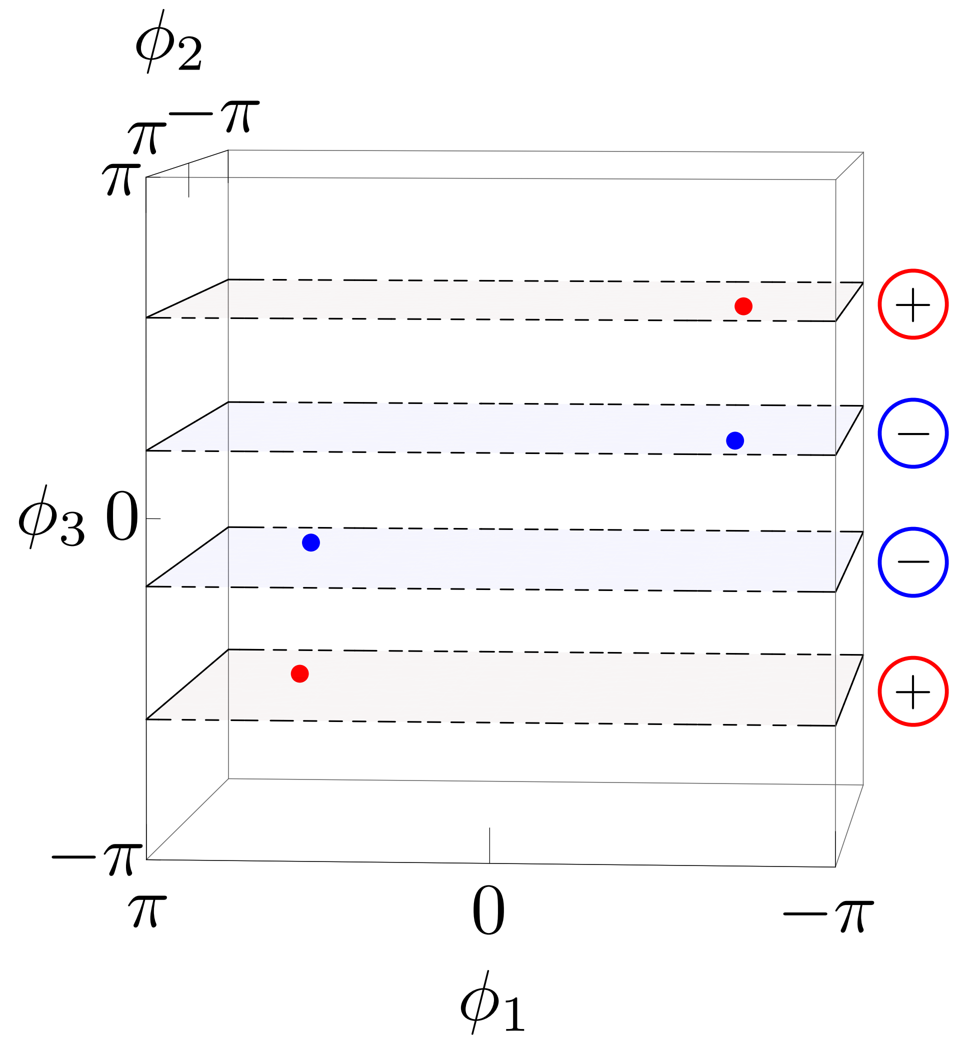
4-terminal junction




$(\phi_2,\phi_3) = (\phi_2^{(0)},\phi_3^{(0)})$

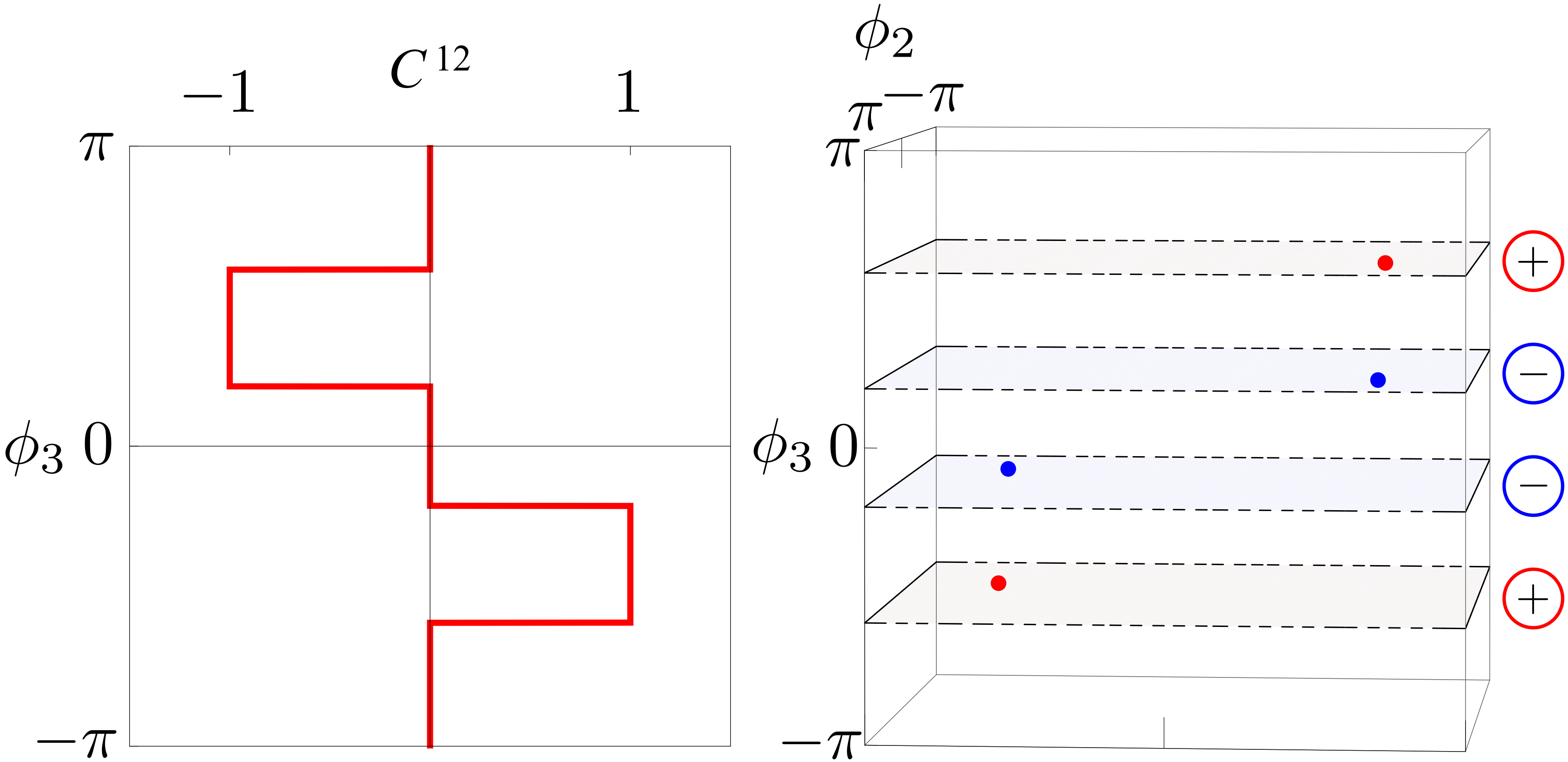
$(\phi_2,\phi_3) \neq (\phi_2^{(0)},\phi_3^{(0)})$



# ABS spectrum

Chern number:




where $C^{12} = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\phi_1 \, d\phi_2 \; B^{12}$

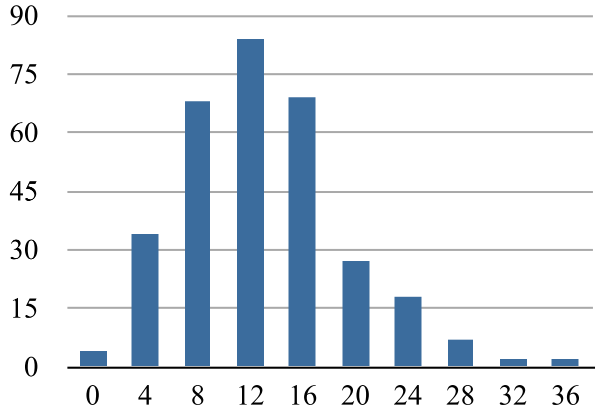
# 4-terminal junctions: Occurence of Weyl points

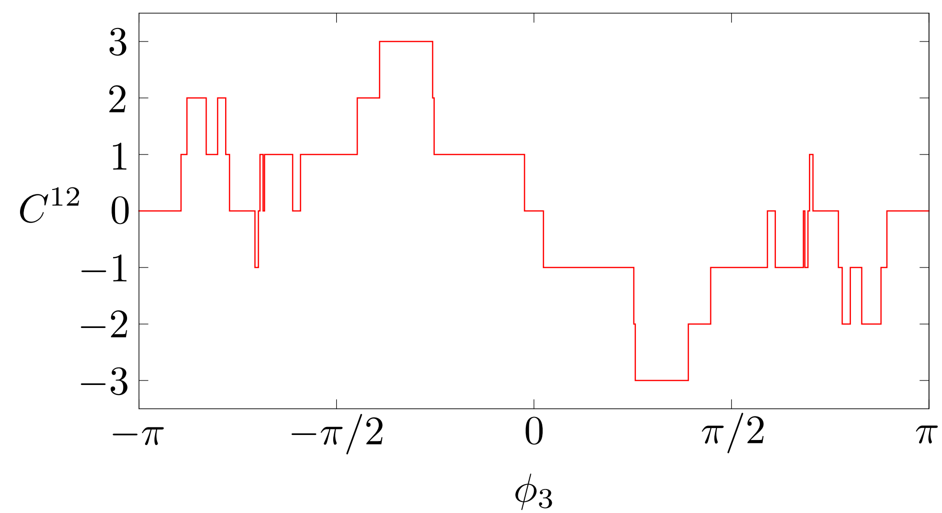
- 4 single-channel terminals:

  ~ 5% of random scattering matrices possess Weyl points

- 4 multi-channel terminals:

  example: $N_\alpha = 12, 11, 10, 9$

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# 4-terminal junctions: Occurence of Weyl points

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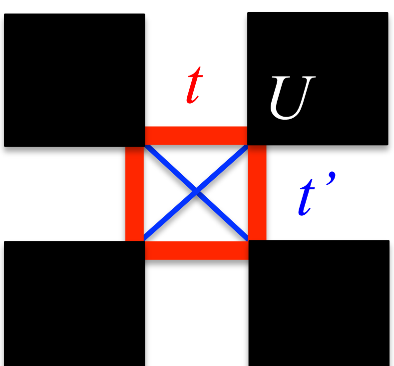
- symmetric structures:

$$H_N = H_{\text{leads}} + U \sum_\alpha \psi^\dagger_\alpha(0)\psi_\alpha(0)$$
$$+ t \sum_\alpha \psi^\dagger_\alpha(0)\psi_{\alpha+1}(0) + t' \sum_\alpha \psi^\dagger_\alpha(0)\psi_{\alpha+2}(0) + \text{h.c.}$$



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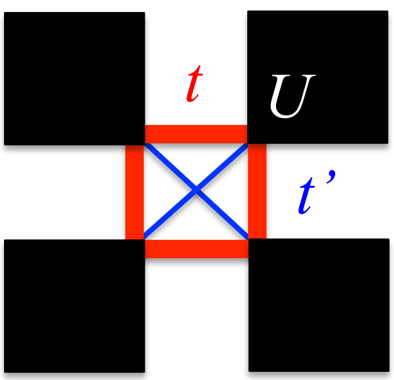


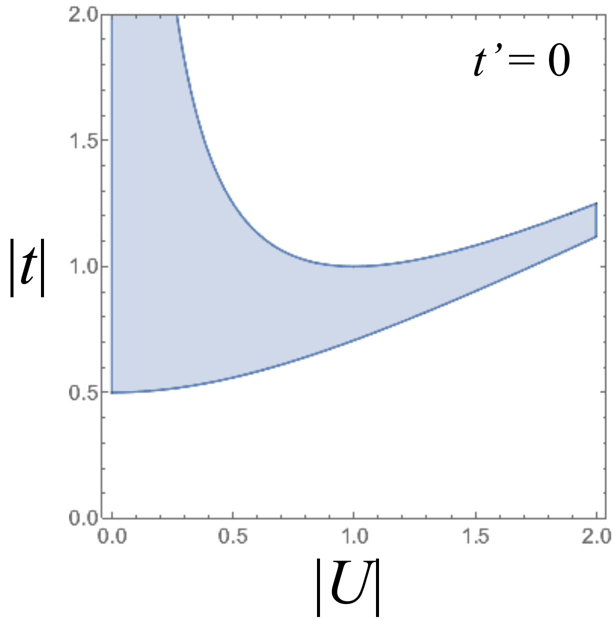


Weyl points of a given charge at $(0, \pi\pm\phi , \pm\phi , \pi)$
& of opposite charge at $(0, \pi , \pm\phi , \pi\pm\phi)$

# Consequences of Weyl singularities: The current

- current operator: $\hat{I}_\alpha = 2e\dfrac{\partial \hat{H}}{\partial \phi_\alpha}$

- use instantaneous eigenbasis $E_{A\nu}(t)\,|\psi_\nu(t)\rangle = \hat{H}(t)\,|\psi_\nu(t)\rangle$
  to compute expectation value for time-dependent phases:

contribution of ABS $$I_{\alpha\nu}(t) = \frac{2e}{\hbar}\frac{\partial E_{A\nu}(t)}{\partial \phi_\alpha} - 4e\sum_\beta \dot{\phi}_\beta\, \Im\langle \frac{\partial \psi_\nu}{\partial \phi_\alpha} | \frac{\partial \psi_\nu}{\partial \phi_\beta}\rangle$$

adiabatic supercurrent $I_{\alpha\nu}{}^0(t)$

first correction: $\delta I_{\alpha\nu}(t) = -2e\sum_\beta \dot{\phi}_\beta B_\nu^{\alpha\beta}$

with $B_\nu^{\alpha\beta} = 2\,\Im\left\langle \dfrac{\partial \psi_\nu}{\partial \phi_\alpha}\middle|\dfrac{\partial \psi_\nu}{\partial \phi_\beta}\right\rangle$ Berry curvature

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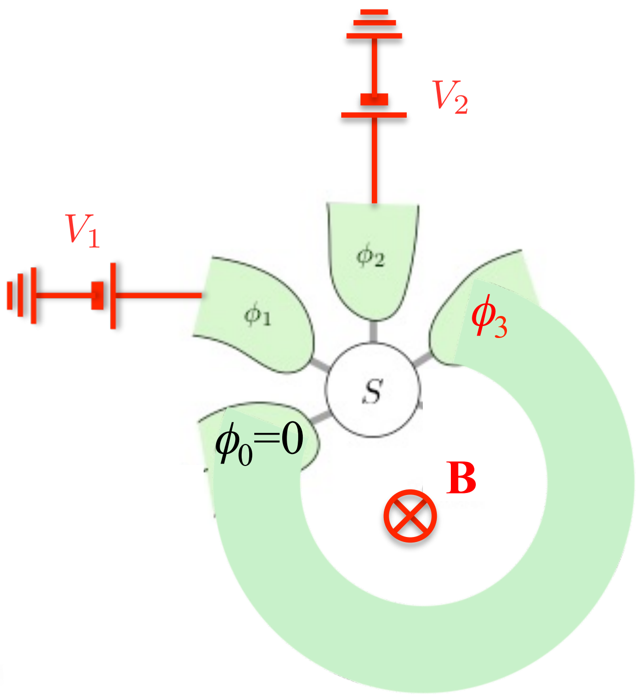
# Quantized transconductance

- total current:

$$I_\alpha(t) = \sum_{k,\sigma} I_{\alpha k}(t)\left(n_{k\sigma} - \frac{1}{2}\right) = I_\alpha^0(t) - 2e \sum_{k,\sigma,\beta} \dot{\phi}_\beta B_k^{\alpha\beta}\left(n_{k\sigma} - \frac{1}{2}\right)$$

- consider 2 voltage-biased leads: $\phi_\alpha = 2eV_\alpha t$



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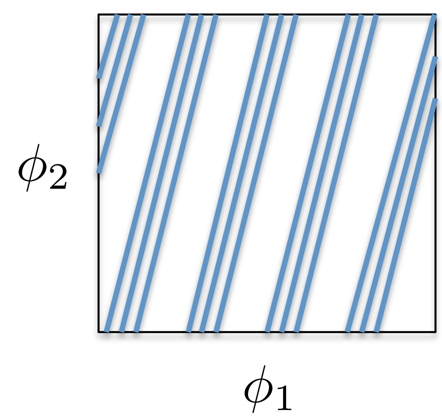
→ phase sweeps 2D “Brillouin zone”

($V_{\alpha,\beta}$ ¿ $\Delta$ incommensurate)



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- consider 2 voltage-biased leads: $\phi_\alpha = 2eV_\alpha t$

→ phase sweeps 2D "Brillouin zone"

→ time-averaged current in the ground state ($n_{k\sigma} = 0$):

$$\overline{I}_\alpha = G^{\alpha\beta} V_\beta \quad \text{with} \quad \boxed{G^{\alpha\beta} = -\frac{2e^2}{\pi\hbar} C^{\alpha\beta}}$$

where $C^{\alpha\beta} = -\frac{1}{2\pi} \sum_k \int_{-\pi}^{\pi} d\phi_\alpha \, d\phi_\beta \, B_k^{\alpha\beta}$ integer

= Chern number

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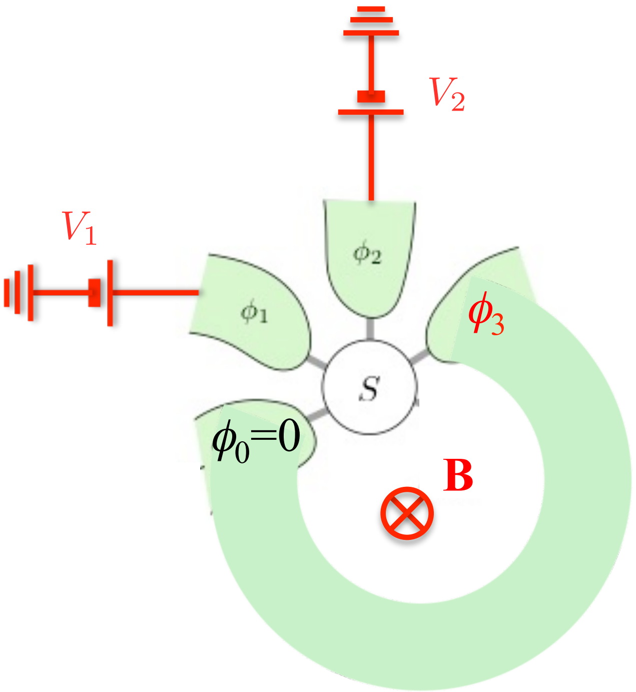
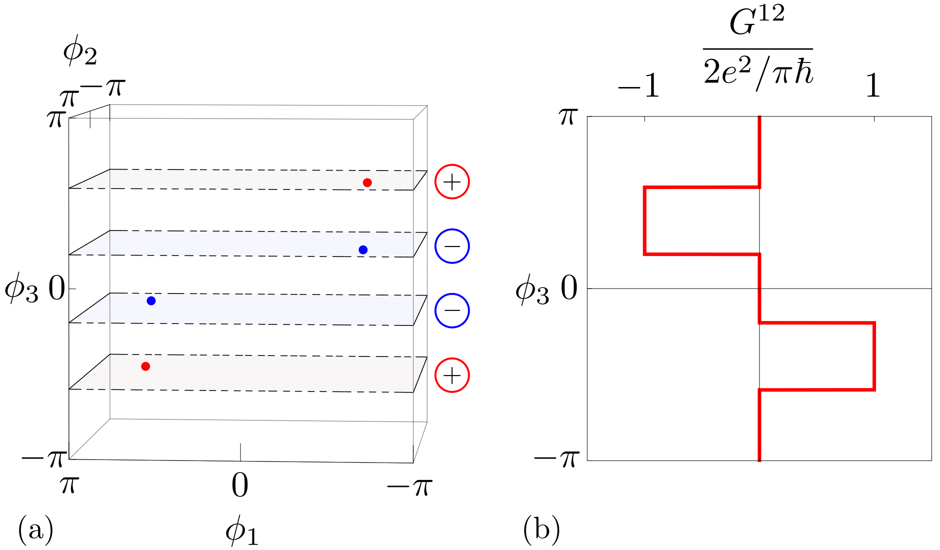
# Multiterminal junctions as topological matter




experimental manifestation:
quantized transconductance

$$\bar{I}_\alpha = G^{\alpha\beta} V_\beta \quad \text{with} \quad G^{\alpha\beta} = \frac{4e^2}{h} C^{\alpha\beta}$$

Chern number

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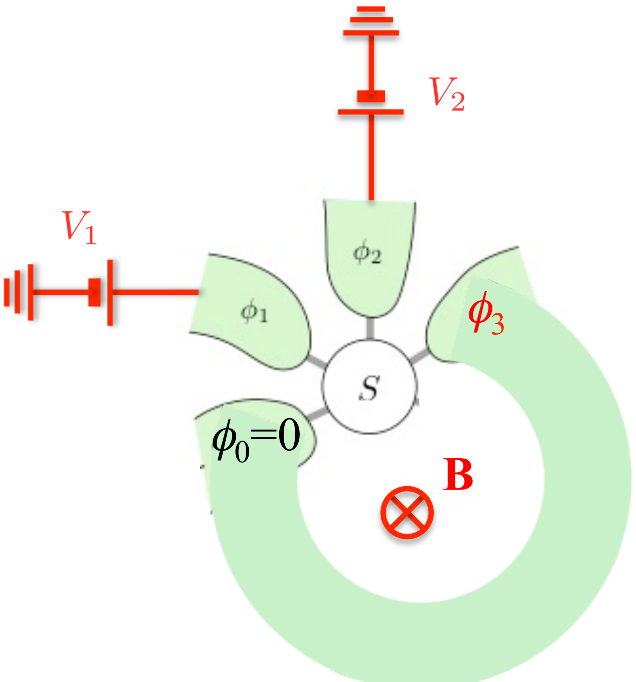


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Chern number

$$\bar{I}_\alpha = -\frac{4e^2}{h} V_\beta \sum_k C_k^{\alpha\beta} (n_{k\uparrow} + n_{k\downarrow} - 1)$$

ground state: $n_{k\sigma} = 0$
$\rightarrow$ poisoning ? (Landau-Zener …)

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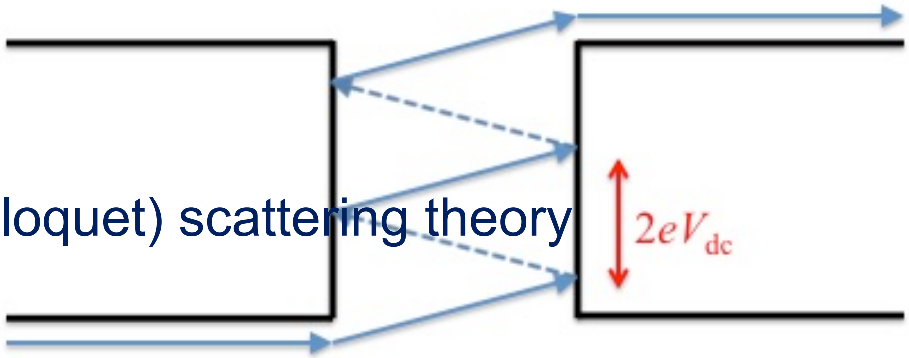
# Beyond the adiabatic regime




- multiple Andreev reflections

→ compute the currents using (Floquet) scattering theory

specific scattering matrix
with Weyl points at ±(1.7, -1.9, -2.8, 0) and ±(2.7, -1.8, 1.0, 0)

- choose $\phi_1 = 2en_1Vt + \chi$

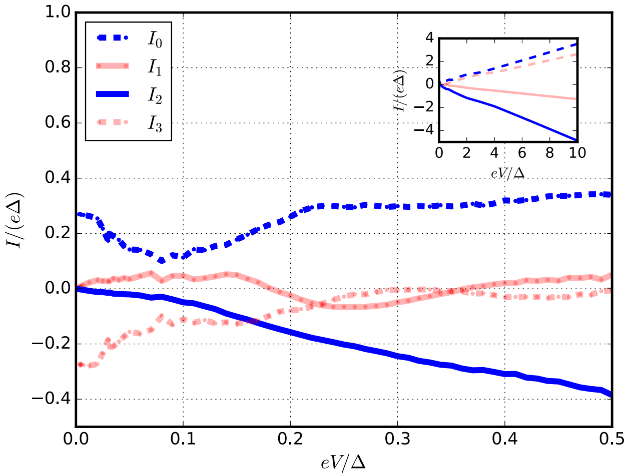
  $\phi_2 = 2en_2Vt$
- commensurate voltages → average over $\chi$
- obtain conductances from 2 sets of voltages: $(n_1, n_2) = (1, 3)$ and $(2, 3)$

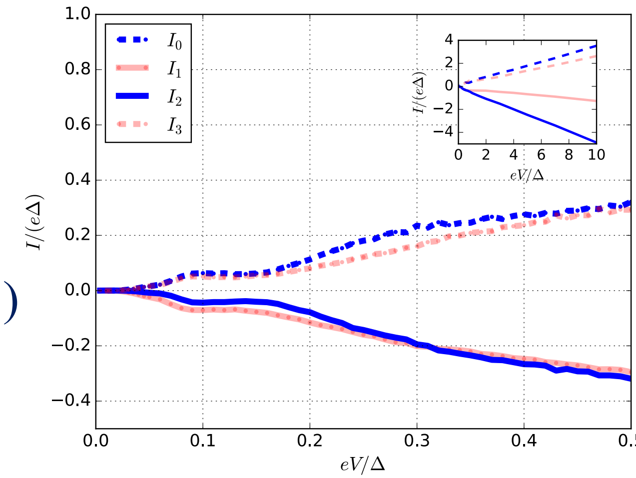
$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

- account for inelastic relaxation with a Dynes parameter $\Gamma$ in the leads

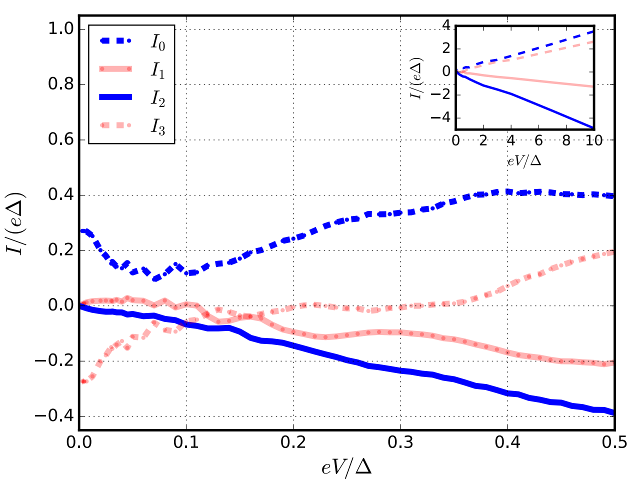
# Beyond the adiabatic regime

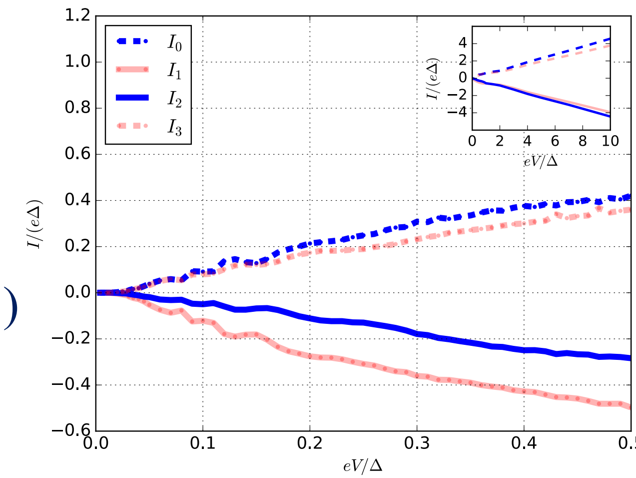
currents as a fct of $V$ at fixed $\phi_0$ ( $\Gamma = 0.002\Delta$ ):




$(n_1, n_2) = (1, 3)$



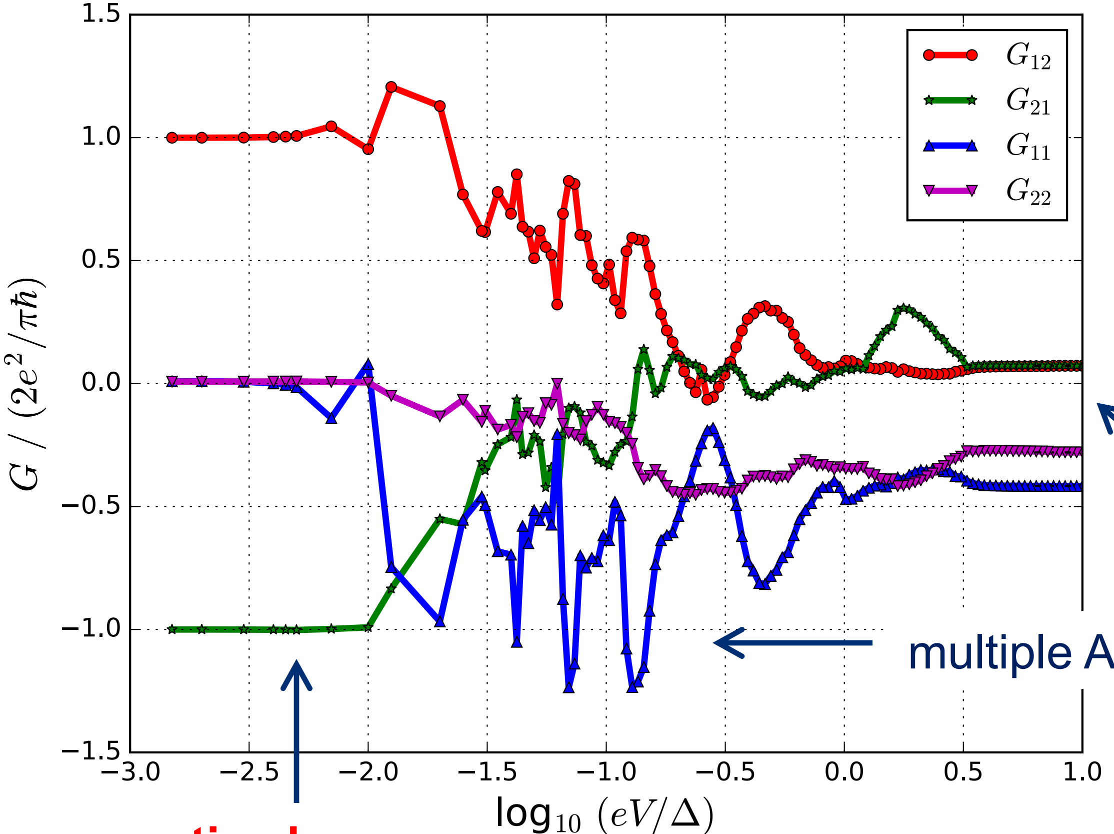

$(n_1, n_2) = (2, 3)$

**$\phi_0 = 2.21$ (topological)**

**$\phi_0 = 0$ (trivial)**

# Beyond the adiabatic regime

conductances as a fct of $V$ at fixed $\phi_0 = 2.21$ ( $\Gamma = 0.002\Delta$ ):




normal state conductances ($G_{12} = G_{21}$)

multiple Andreev reflections

**quantized transconductances**

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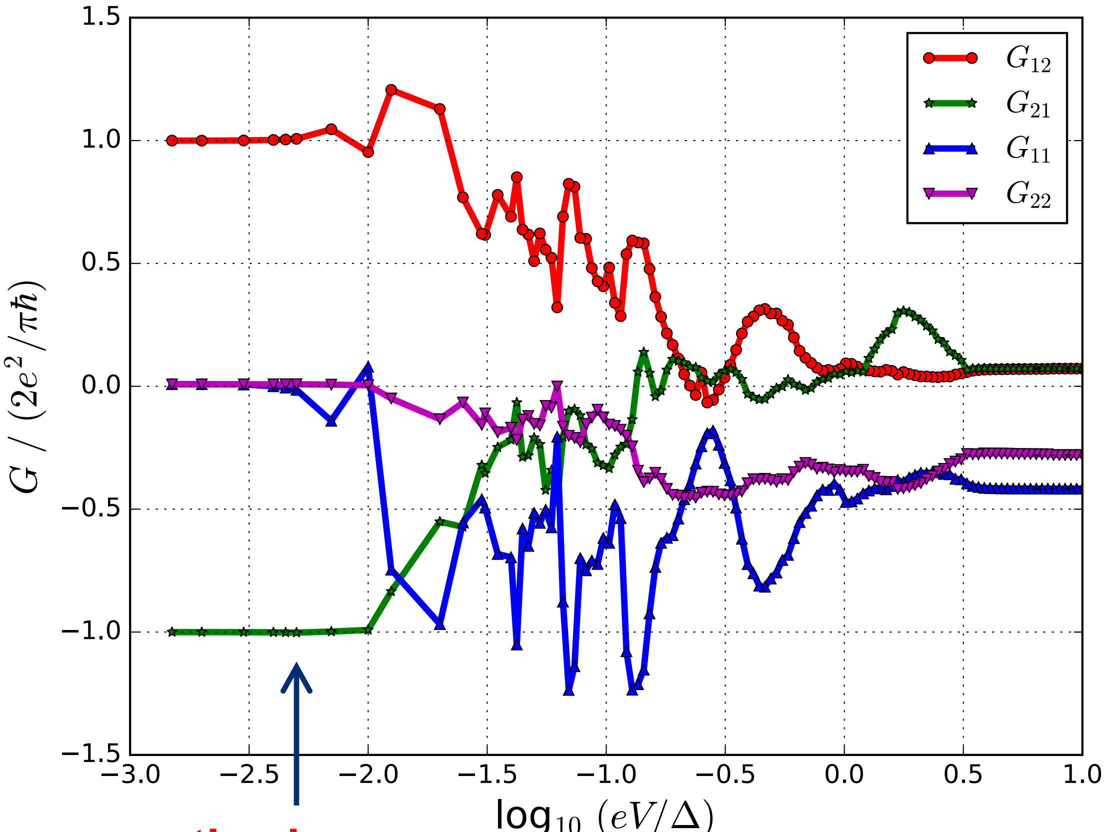


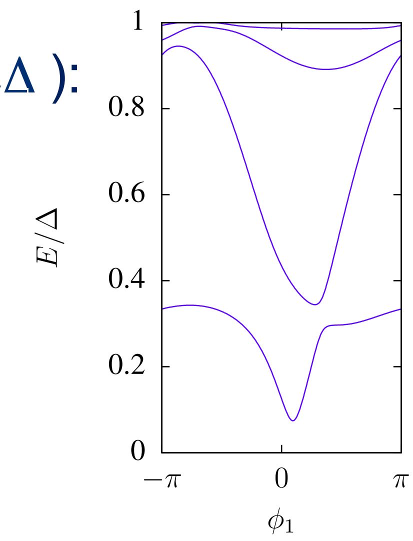


**quantized transconductances**

quantization requires fixed parity:

$$eVe^{-E_A/eV} < \Gamma$$

$$\rightarrow eV < eV_\star \sim \frac{E_A}{\log(E_A/\Gamma)}$$

where

$$E_A = \min_{\phi_1,\phi_2} E_A(\phi_0,\phi_1,\phi_2)$$

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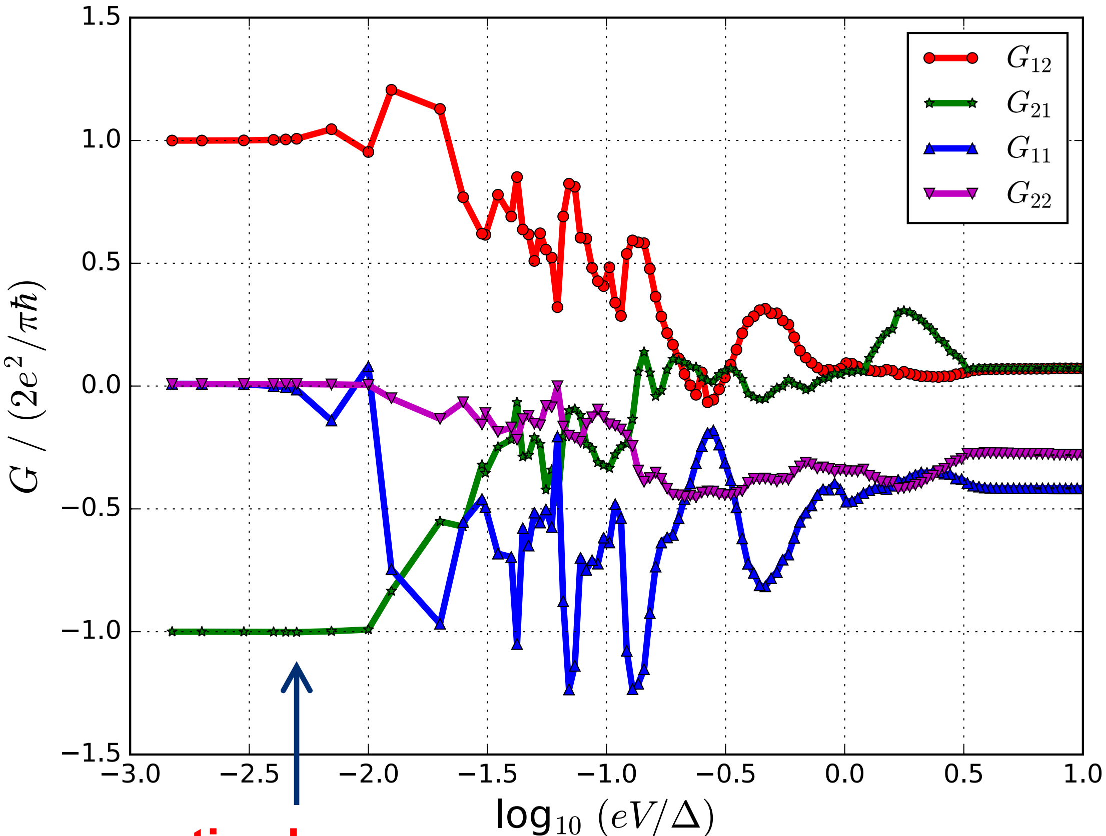
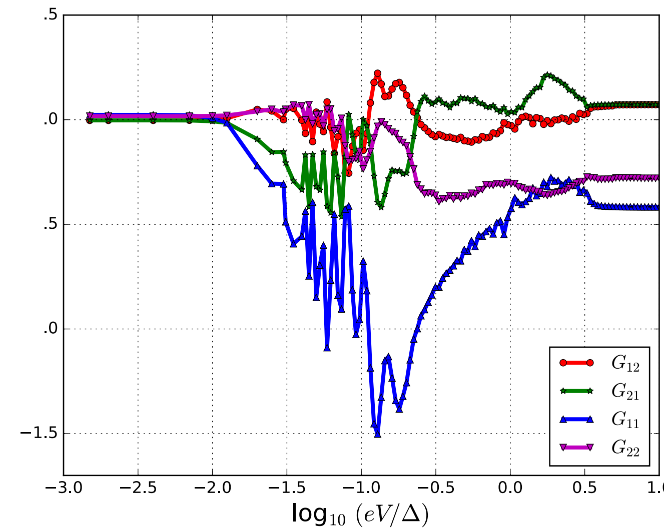


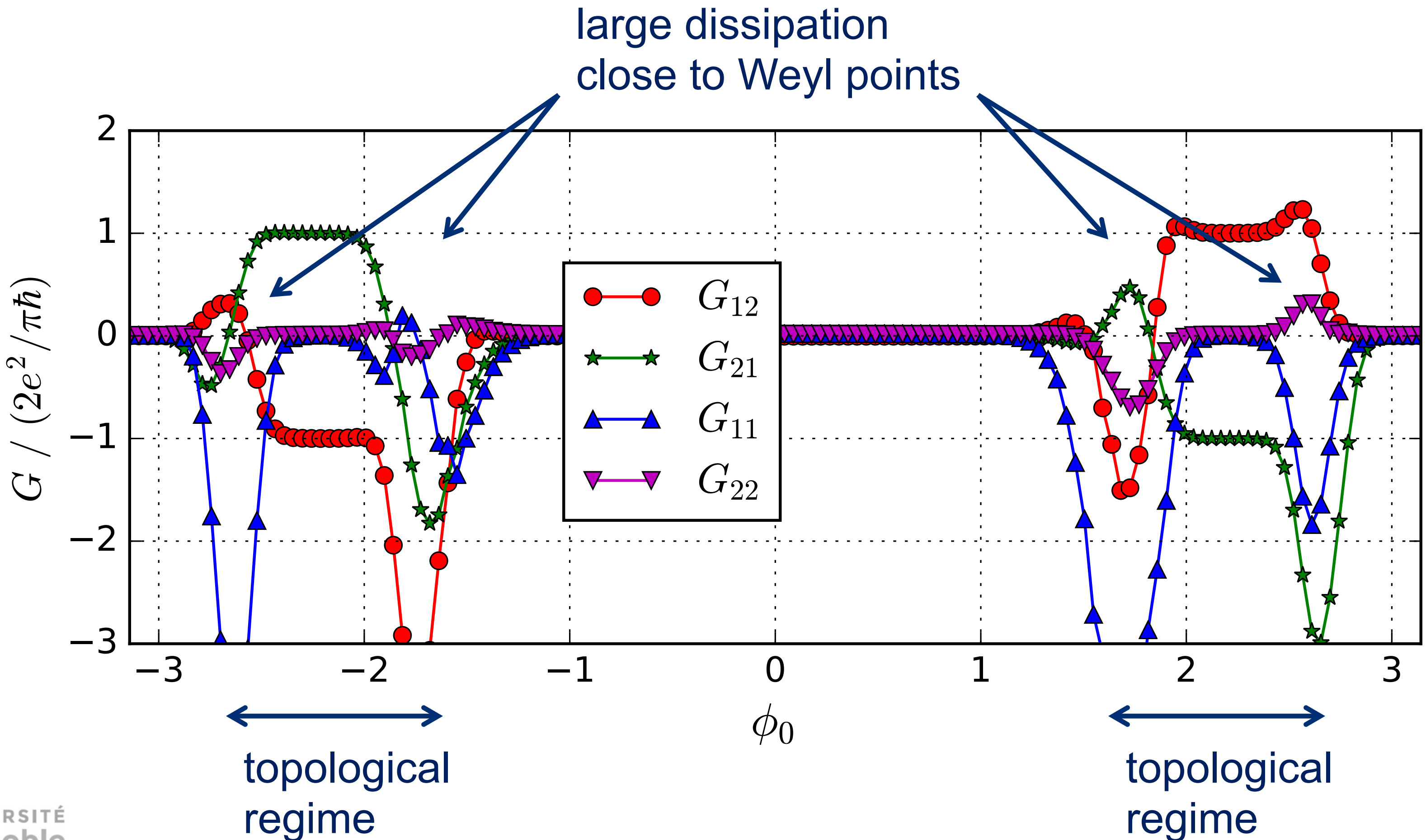

**quantized transconductances**

for comparison: $\phi_0 = 0$



# Beyond the adiabatic regime

conductances as a fct of $\phi_0$ at fixed $V = 0.0003\Delta/e$ :



# Conclusion

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  of multi-terminal Josephson junctions
  without any fine-tuning

- superconducting phase = quasi-momenta

- transconductance
  between 2 voltage-biased terminals
  probes Chern number

$$\overline{I}_\alpha = G^{\alpha\beta} V_\beta \quad \text{with} \quad G^{\alpha\beta} = -\frac{2e^2}{\pi\hbar} C^{\alpha\beta}$$

multi-terminal Josephson junction
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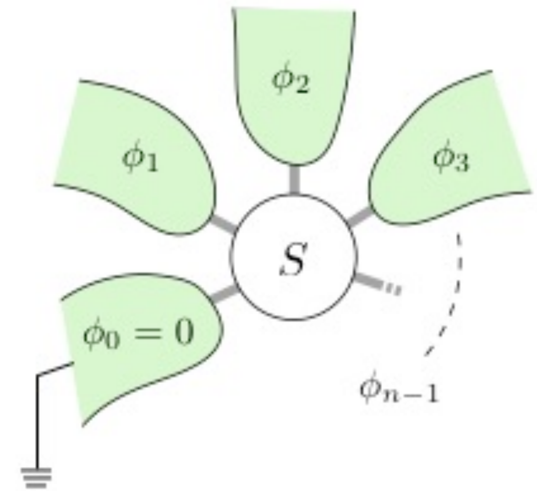


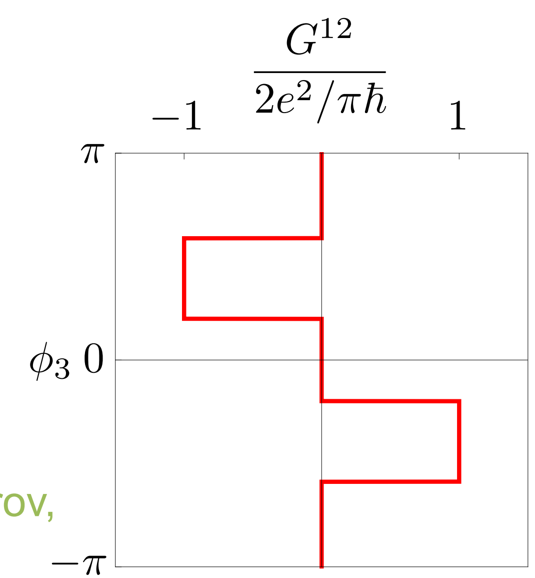



R.-P. Riwar, M. Houzet, JSM, and Y.V. Nazarov, Nat. Commun. **7**, 11167 (2016);
E. Eriksson *et al.*,in preparation

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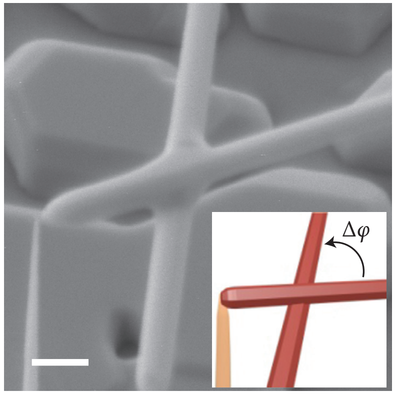
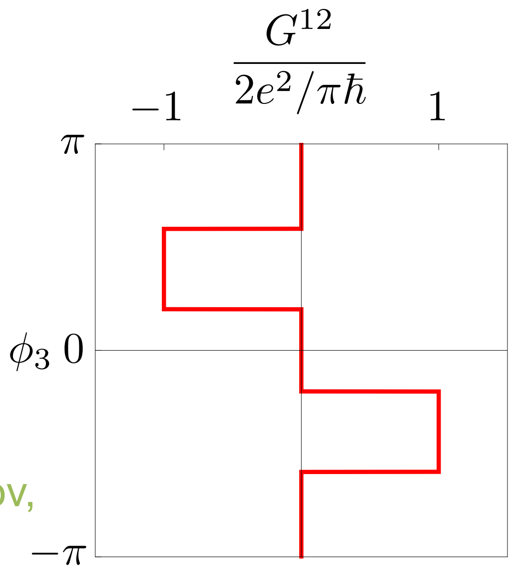



InSb nanocrosses ?
Plissard *et al*. (2013)

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# Thank you!

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