A short introduction to Topological insulators

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Outline

I) Semimetallic graphene: massless Dirac fermions

II) h-BN and Haldane model: massive Dirac fermions
trivial vs Chern insulators

III) Kane-Mele model: massive Dirac fermions
with spin-orbit coupling
Z2 topological insulators

IV) Experimental realizations of topological insulators

C. Kane and Z. Hasan RMP 2010; X-L Qi and S-C Zhang RMP 2012

0- Brief reminder about Dirac and Weyl equations in high-energy physics

Dirac equation (1928)

Wave-equation for relativistic et quantum particle (electron)

$$E^2 = p^2c^2 + m^2c^4$$

$$i\hbar\frac{\partial\Psi}{\partial t} = c\left(-i\hbar\alpha_i\frac{\partial}{\partial x_i} + \beta mc\right)\Psi$$

$$E\Psi = c\left(\alpha_i p_i + \beta mc\right)\Psi$$

$$\{\alpha_i, \beta\} = 0 \qquad \{\alpha_i, \alpha_j\} = 2\delta_{ij}$$

Clifford algebra

$$E\Psi = c\left(\alpha_i p_i + \beta mc\right)\Psi$$

Coefficients must be Hermitian anti-commuting matrices squaring to identity

Pauli matrices (2x2): only 3 matrices which is not enough in 3D for a massive particle, but is enough for a massless particle

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

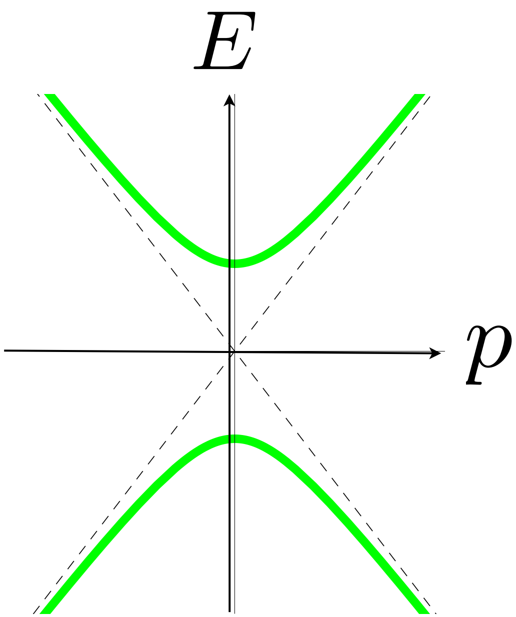
Dirac matrices (4x4): 15 matrices (more than needed)

Implies 4 components wave-functions: spin x particle/antiparticle

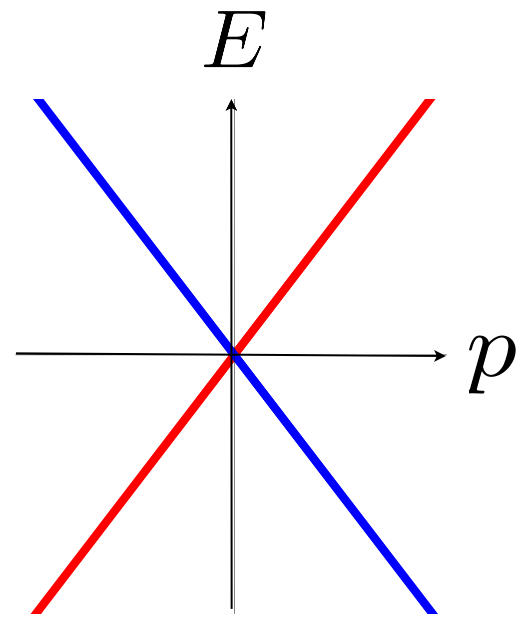
Dirac and Weyl equations (1928)

$$E^2 = p^2c^2 + m^2c^4$$

$$E = \pm cp$$



Electron's spin is tied
to its orbital motion



At low velocity, one gets the non relativistic Schrödinger equation and the spin gets decoupled from the motion up to small corrective spin-orbit terms

I- Graphene:
Massless Dirac fermions

Graphene: Dirac-like equation

Carbon is a light element: spin-orbit coupling is weak
spin is basically decoupled from motion

But:

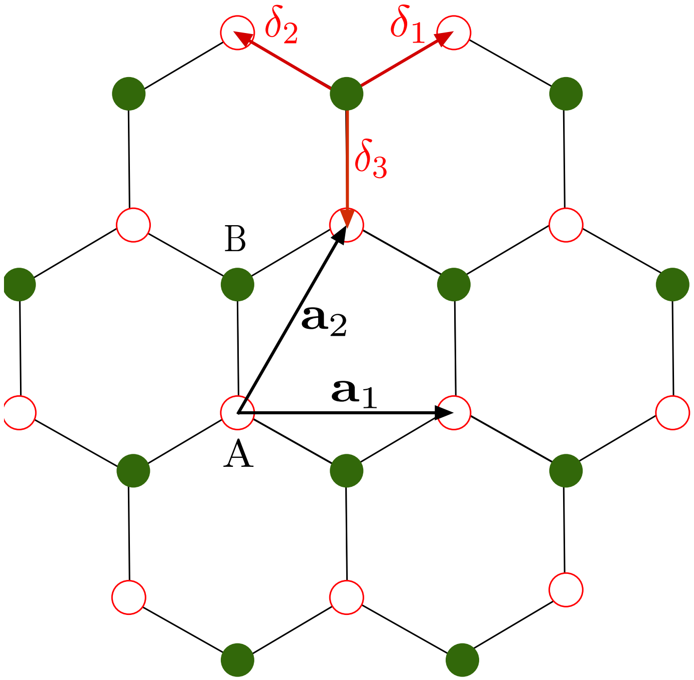
Graphene has a particular lattice structure which leads to an additional internal degree of freedom: the sublattice isospin.

It turns out that this sublattice isospin is tied to electronic motion.

Low energy electrons are described by a « Dirac-like » equation.

Graphene: honeycomb structure

$d_{C-C} = 0.142\,\mathrm{nm}$



Atomic monolayer

2 sites/unit cell: 2 sublattices: *A* et **B**

3 sp2 bondings + 1 pz orbital per atom

Bloch wave-function:

$$\left|\Psi_{\vec{k}}\right\rangle = \frac{1}{\sqrt{N}} \sum_{\vec{R}_j} \left(a_{\vec{k}} \left|\vec{R}_j, A\right\rangle + b_{\vec{k}} \left|\vec{R}_j, B\right\rangle \right) e^{i\vec{k}.\vec{R}_j}$$

internal degree of freedom: sub lattice isospin (a_k, b_k)

Tight-binding (TB) model

Schrödinger equation + projections on A and B orbitals

$$H\left|\Psi_{\vec{k}}\right\rangle = E_k\left|\Psi_{\vec{k}}\right\rangle$$

TB Hamiltonian in real space (only nearest neighbors hopping terms)

$$H_0 = -t\sum_{\vec{R},\vec{\delta}_\alpha} c^\dagger_B(\vec{R})c_A(\vec{R}+\vec{\delta}_\alpha) + H.c. \qquad t = 2.7\ \mathrm{eV}$$

with:

$$-t = \int d^3r\,\phi^*(\vec{r}-\vec{R}_A-\vec{\delta}_3)(V(\vec{r})-V_{\mathrm{ato}}(\vec{r}-\vec{R}_B))\phi(\vec{r}-\vec{R}_B)$$

Diagonalization

$$H_0 = -t \sum_{\vec{k},\vec{\delta}_\alpha} e^{i\vec{k}.\vec{\delta}_\alpha} c_B^\dagger(\vec{k}) c_A(\vec{k}) + H.c.$$

$$= -t \sum_{\vec{k}} c_\alpha^\dagger(\vec{k}) [h_0]_{\alpha\beta} c_\beta(\vec{k}) \qquad \alpha, \beta = A, B$$

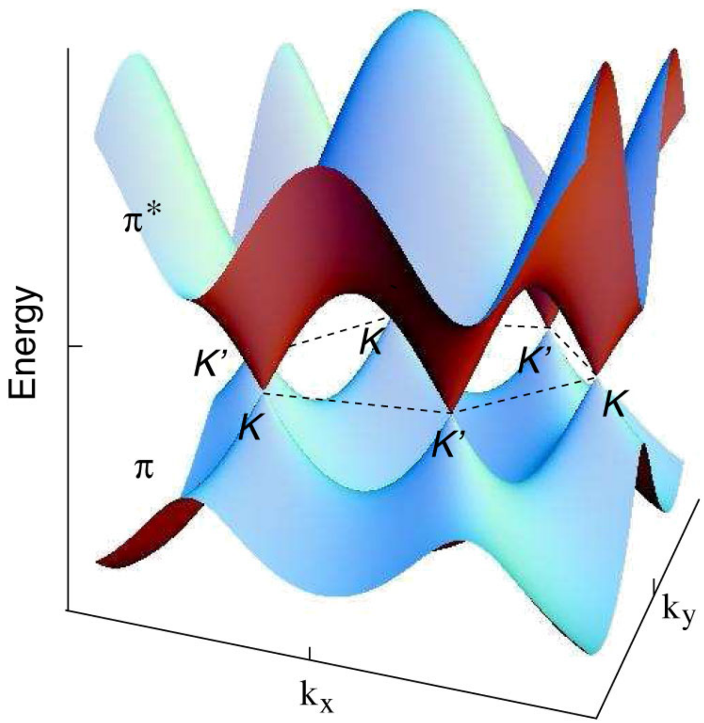
$$h_0(\vec{k}) = -t \sum_{\vec{k},\vec{\delta}_\alpha} \left(\cos(\vec{k}.\vec{\delta}_\alpha)\sigma_1 + \sin(\vec{k}.\vec{\delta}_\alpha)\sigma_2 \right)$$

even function — odd function

Both functions periodic in reciprocal space

Electronic structure

Two bands (2 sites/unit cell)



$$h_0(\vec{k}) = d_1(\vec{k})\sigma_1 + d_2(\vec{k})\sigma_2$$

$$E(\vec{k}) = \pm \, || \, \vec{d}(\vec{k}) \, ||$$

Band touching points (two valleys)

$$d_1(\vec{k}) = d_2(\vec{k}) = 0 \rightarrow \vec{k} = \pm\vec{K}$$

1 electron/orbital (half-filling)

Electrostatic doping: it is possible to raise/lower the Fermi level in a reversible manner and without adding impurities.

Linearization near Dirac points

$$\vec{k} = \xi \vec{K} + \vec{q} \qquad \xi = \pm 1 \quad \text{(Valley index)}$$

$$h_0(\xi \vec{K} + \vec{q}) \simeq \hbar v_F \left(\xi q_x \sigma_1 + q_y \sigma_2\right)$$

Linear dispersion (Dirac cone): electrons (or holes) behave as if they were massless

$$E(\xi \vec{K} + \vec{q}) = \hbar v_F \sqrt{q_x^2 + q_y^2}$$ valley independent

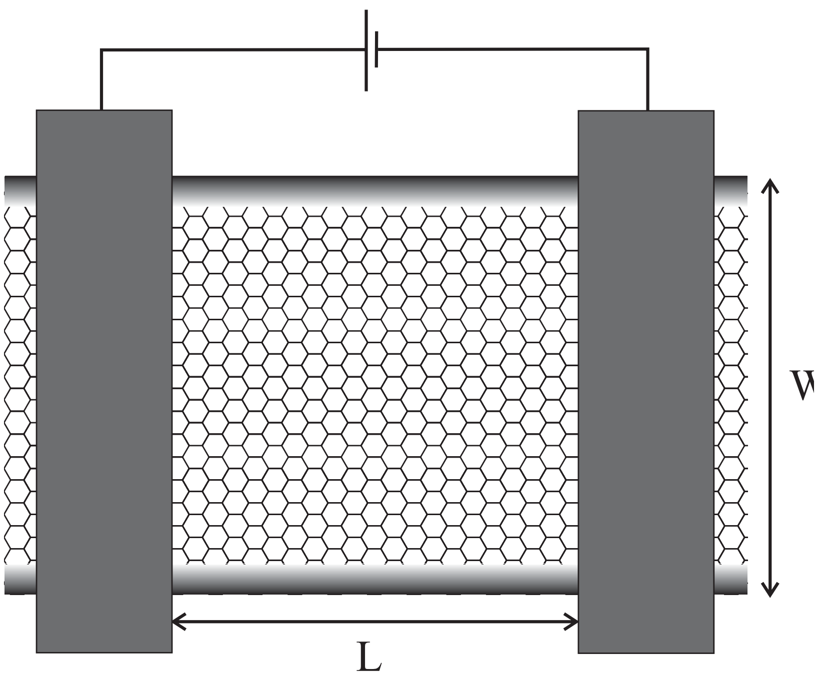
<u>Fermi velocity:</u> $$v_F = 3at/2\hbar \simeq 10^6 \mathrm{m.s}^{-1}$$

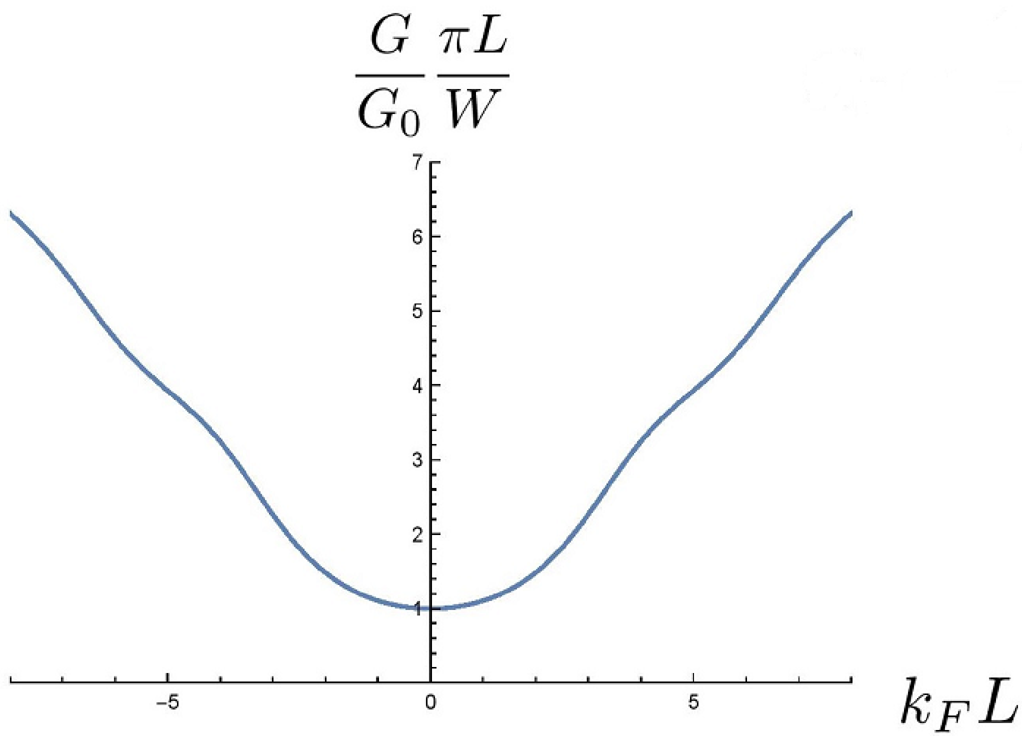
Transport in ballistic graphene

$$E_F = \hbar v_F q_F$$

typically 100 meV

electron/hole densities $n \simeq 10^{-12}\, cm^{-2}$



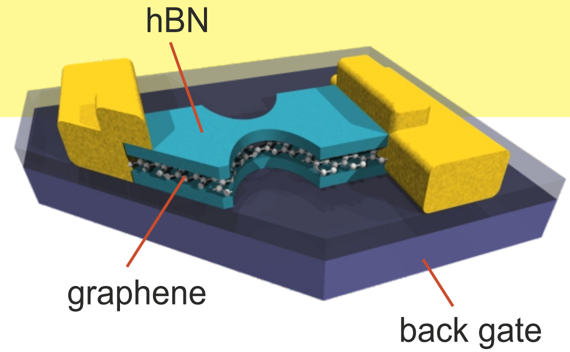


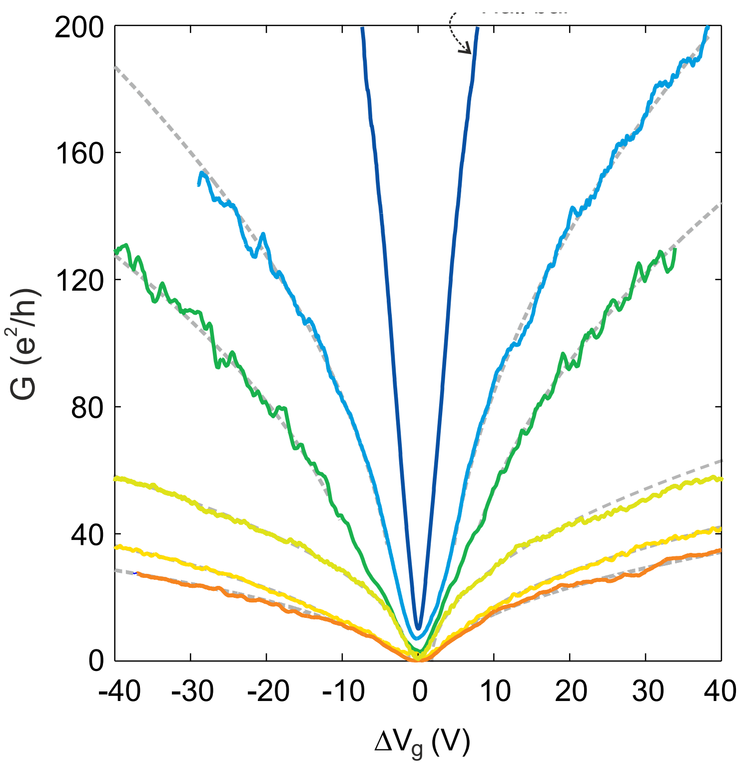
Tworzydlo et al. , PRL 2006

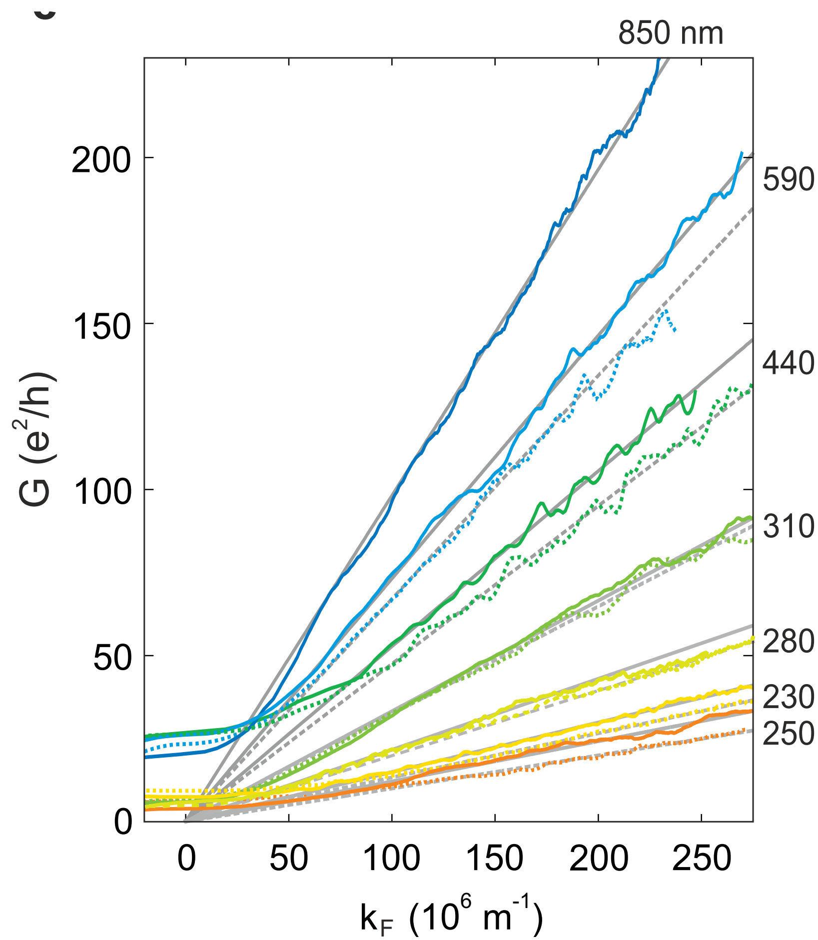
No off-state !

Electronic transport

From C. Stampfer's group (Aachen)

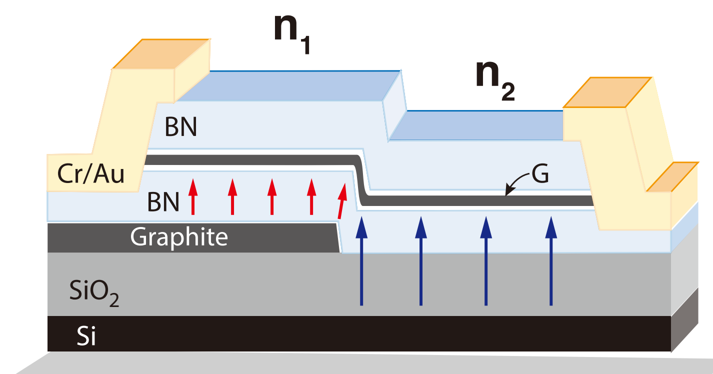






Electronic optics

From Cory Dean's group (Columbia), Science 2016



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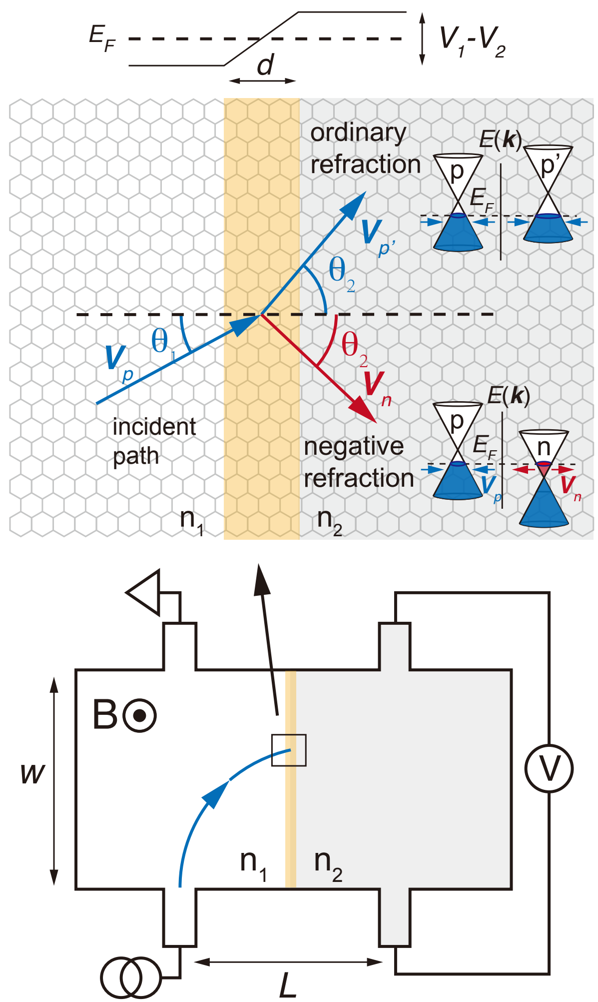
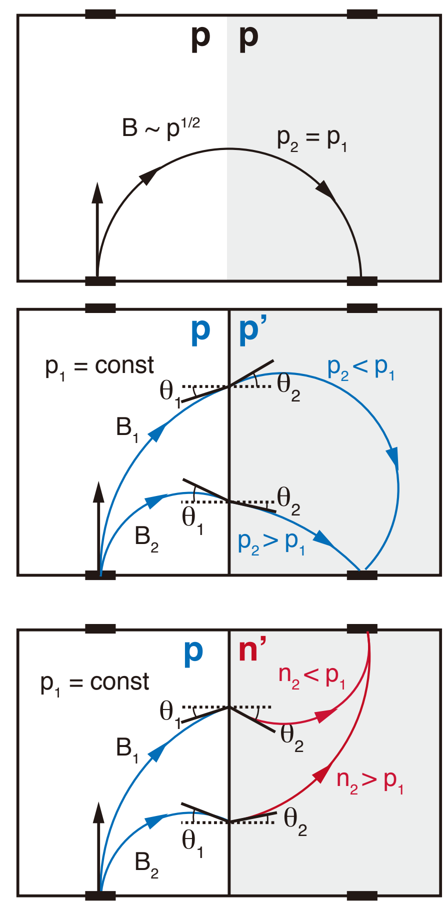




Protection of Dirac points

The possible perturbations: only the third Pauli matrix can open a gap at the Dirac points

$$h_0(\vec{k}) = d_1(\vec{k})\sigma_1 + d_2(\vec{k})\sigma_2 + d_3(\vec{k})\sigma_3$$

Inversion symmetry enforces the relation:

$$h_0(\vec{k}) = \sigma_1 h_0(-\vec{k})\sigma_1 \longrightarrow d_3(\vec{k}) = -d_3(-\vec{k})$$

Time-reversal symmetry (for spinless electrons) enforces the relation:

$$h_0(\vec{k}) = h_0^*(-\vec{k}) \longrightarrow d_3(\vec{k}) = d_3(-\vec{k})$$

Hence if both T and P are satisfied: $d_3(\vec{k}) = 0$

II)- Graphene:
Massive spinless Dirac fermions

How to transform a semimetal into an insulator ?

How to provide a mass to Dirac fermions ?

Add a perturbation that anticommutes with graphene's kinetic Hamiltonian and breaks either T or P

$$h_0(\vec{k}) = d_1(\vec{k})\sigma_1 + d_2(\vec{k})\sigma_2 + d_3(\vec{k})\sigma_3$$

Dirac mass 1: Semenov model (1984)

The simplest perturbation is a staggered potential on A/B sites (Semenov, PRL 1984): +M on A sites and -M on B-sites

$$d_3(\vec{k})\sigma_3 = M\sigma_3$$

This k-independent perturbation breaks inversion symmetry (A and B orbitals are no longer identical)

The resulting insulator is a trivial band insulator

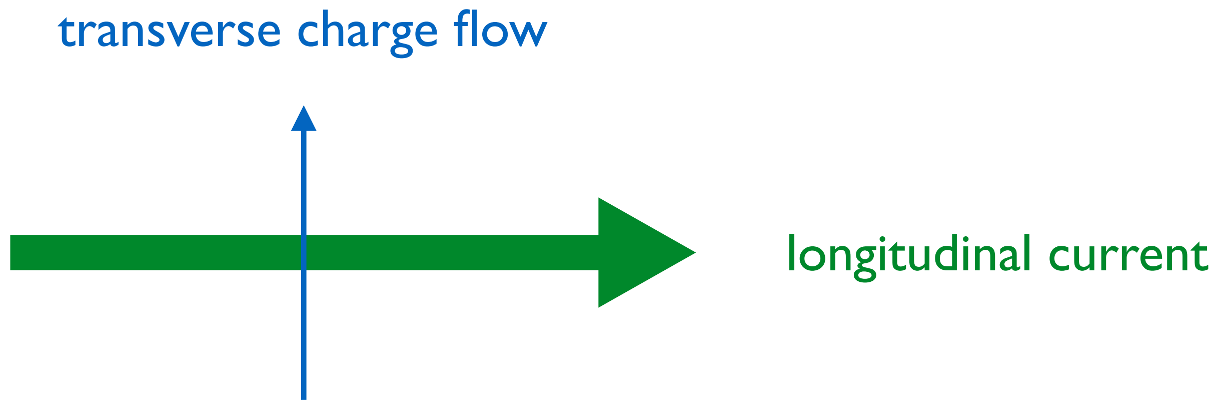
Relevant for hexagonal boron-nitride (h-BN) which is 2D insulator with a large gap (around 5 eV)

Dirac mass 2: Haldane model (1988)

A (far) less evident perturbation was proposed by D. Haldane. His initial motivation was to induce Quantum Hall effect in 2D lattice without Landau levels

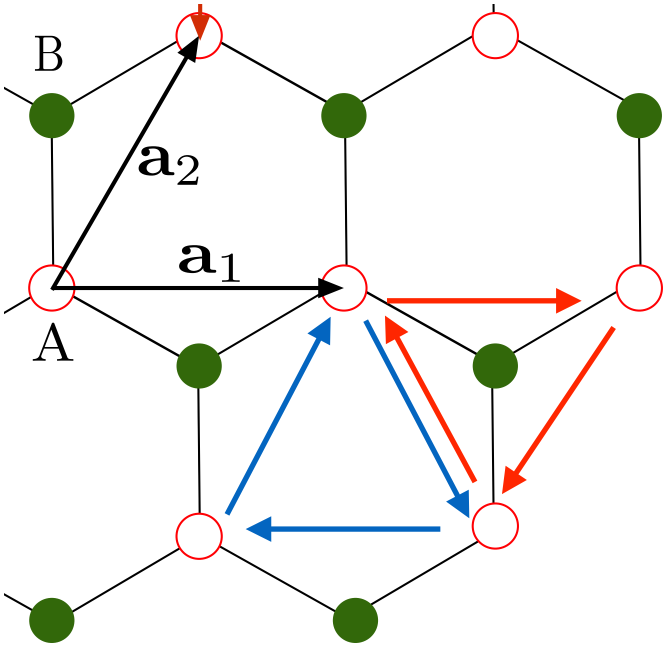
Ingredients are :

- 2D crystal: graphene
- Break time-reversal symmetry (to generate Quantum Hall Effect)
- No net magnetic flux per unit cell (to avoid Landau Levels)



Haldane found that complex valued local fluxes with zero net average value meet all these criteria and do the job (PRL 1988).

Complex second-neighbor hopping

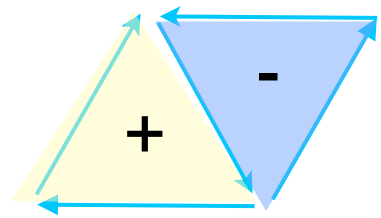


Red and blue arrows represent an electron jumping from A site to nearest A sites

$t_2 e^{-i\varphi}$ hopping with a B on the its right

$t_2 e^{i\varphi}$ hopping with a B on the its right

Same pattern on B sites



This staggered local fluxes pattern has the symmetry of the Bravais lattice and therefore one has a band insulator with states labelled by a quasi-momentum

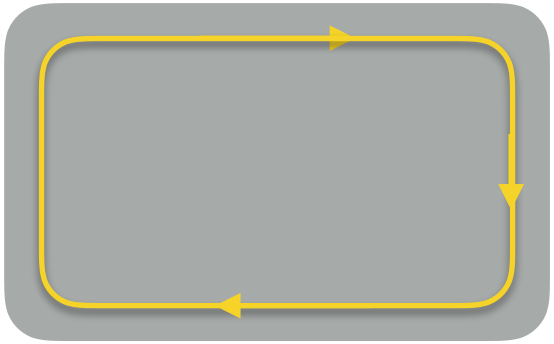
Haldane mass

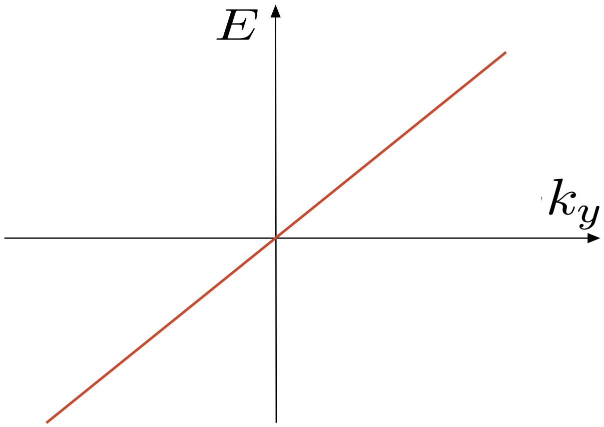
$$d_3(\vec{k}) = 2t_2 \sin\varphi \sum_{i=1}^{i=3} \sin(\vec{k}.\vec{b}_i)$$

This perturbation depends on momentum and breaks time-reversal symmetry

- The resulting band insulator exhibits a non trivial winding of the wave functions as gaps/masses are opposite at +K and -K

- This non trivial winding implies a chiral edge state





Hall response

Hamiltonian

$$H = d_x(\vec{k})\sigma_x + d_y(\vec{k})\sigma_y + d_z(\vec{k})\sigma_z$$

Current operator:

$$j_i = \frac{\partial \mathcal{H}(\mathbf{k})}{\partial k_i} = \frac{\partial \varepsilon_0(\mathbf{k})}{\partial k_i}\mathbf{I}_{2\text{x}2} + \sum_j^3 \frac{\partial \mathbf{d}(\mathbf{k})}{\partial k_i}.\boldsymbol{\sigma}$$

Hall conductivity (Kubo formalism)

$$\sigma_{xy} = \frac{e^2}{4\pi h}\int_{\text{BZ}} d^2\mathbf{k}(f_+(\mathbf{k}) - f_-(\mathbf{k}))\left(\frac{\partial \hat{\mathbf{d}}(\mathbf{k})}{\partial k_x} \times \frac{\partial \hat{\mathbf{d}}(\mathbf{k})}{\partial k_y}\right).\hat{\mathbf{d}}(\mathbf{k})$$

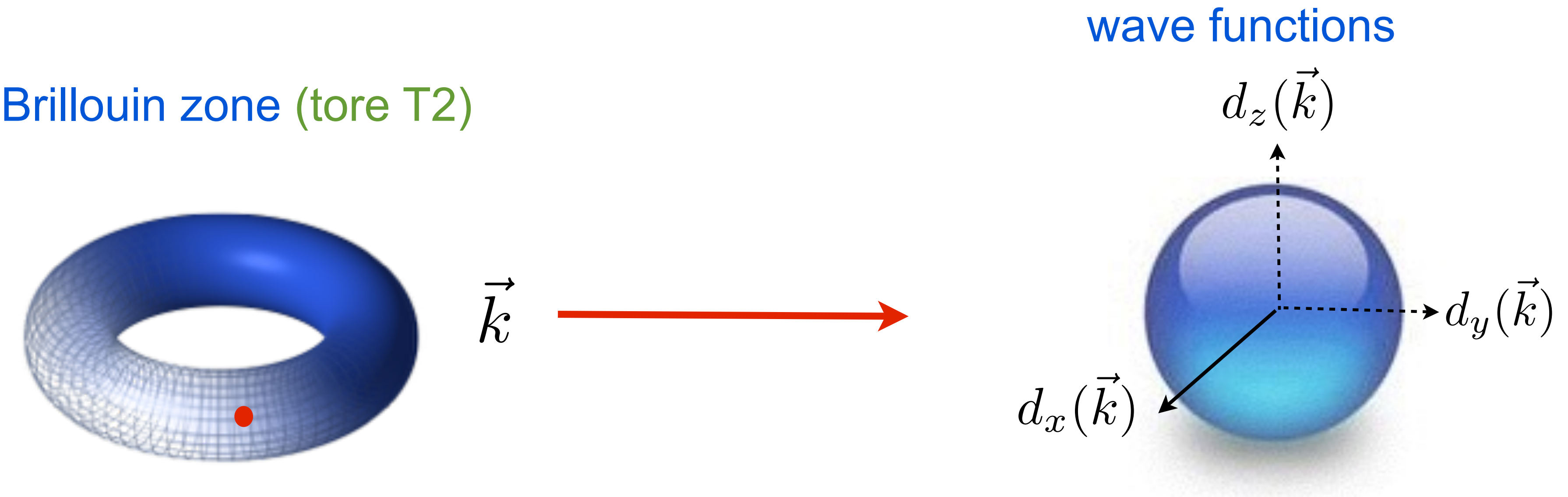
Insulator at T=0:

$$\sigma_{xy} = \frac{e^2}{h} n_w$$

$$n_w = \frac{1}{4\pi}\int_{\text{BZ}} d^2\mathbf{k}\left(\frac{\partial \hat{\mathbf{d}}(\mathbf{k})}{\partial k_x} \times \frac{\partial \hat{\mathbf{d}}(\mathbf{k})}{\partial k_y}\right).\hat{\mathbf{d}}$$

This winding number is often zero, and has to be an integer

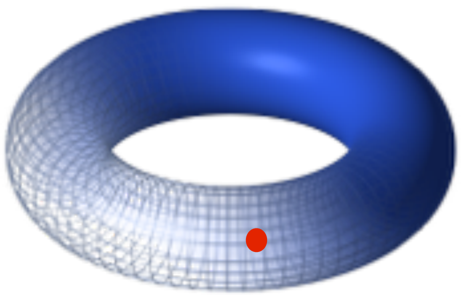
Topology: mapping BZ to BS



$$\hat{d}(\vec{k}) = \frac{d(\vec{k})}{|\vec{d}(\vec{k})|}$$

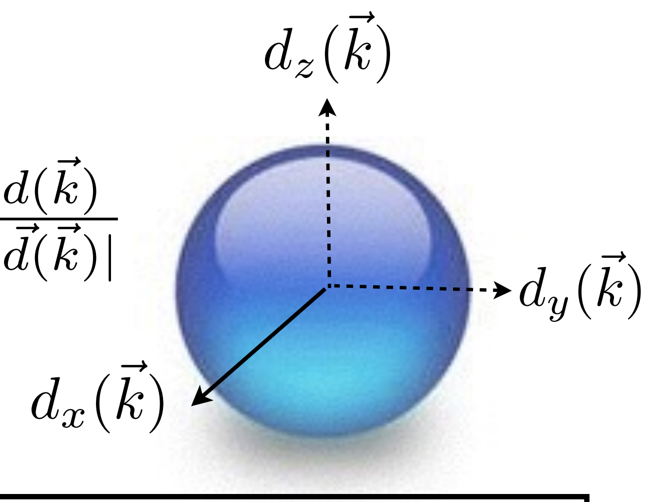
Topological invariant

Brillouin zone (torus T2)

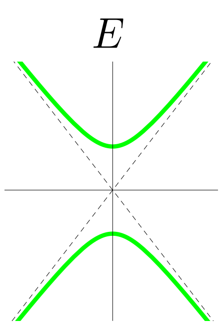


$$\vec{k} \longrightarrow \hat{d}(\vec{k}) = \frac{d(\vec{k})}{|\vec{d}(\vec{k})|}$$

Wavefunctions



For an isolated Dirac crossing with mass M:



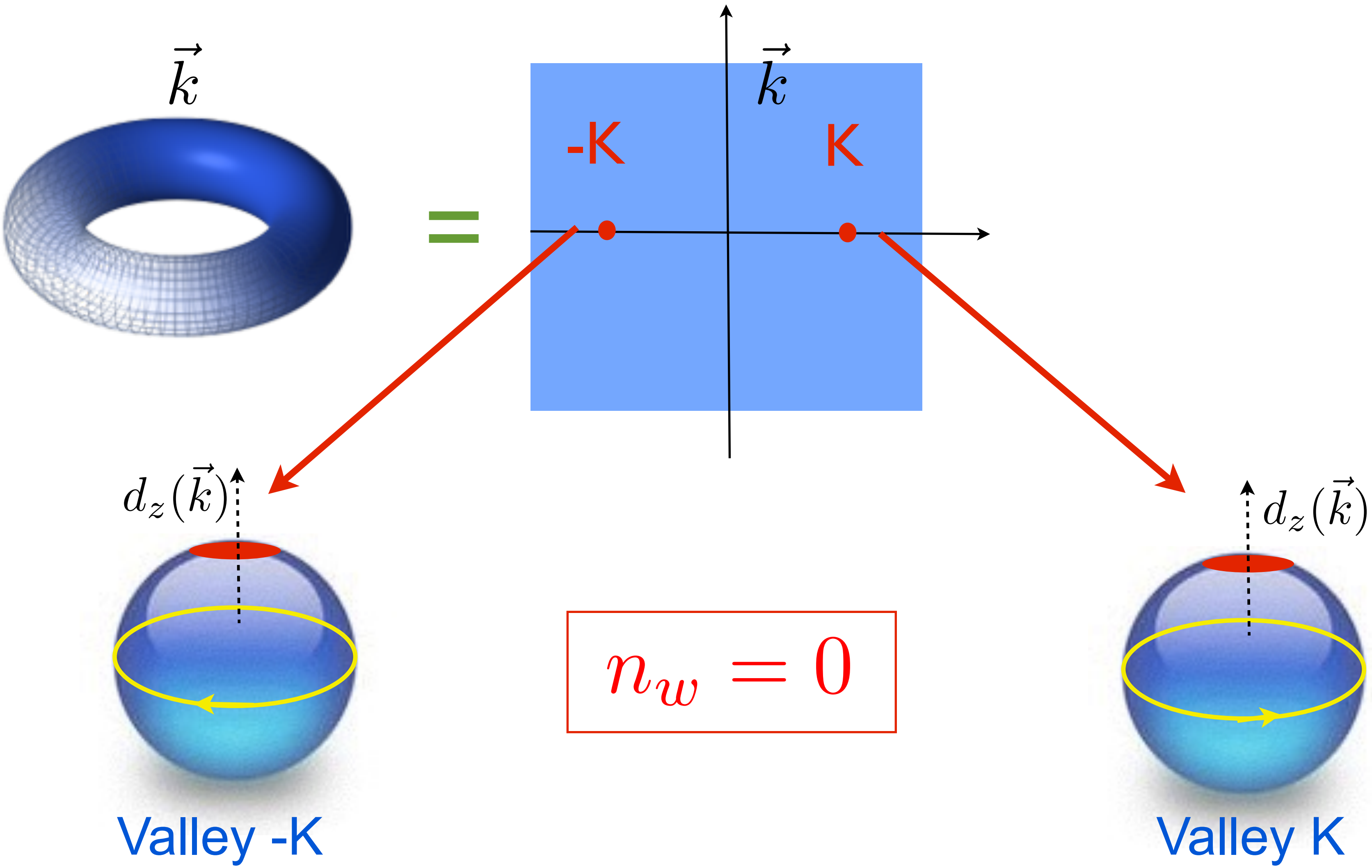
$$H = k_a A_{ab} \sigma_b + M \sigma_z$$

$$\downarrow$$

Winding number $n_w = sign(M) sign(Det A)/2$

Trivial insulator (TR invariant)

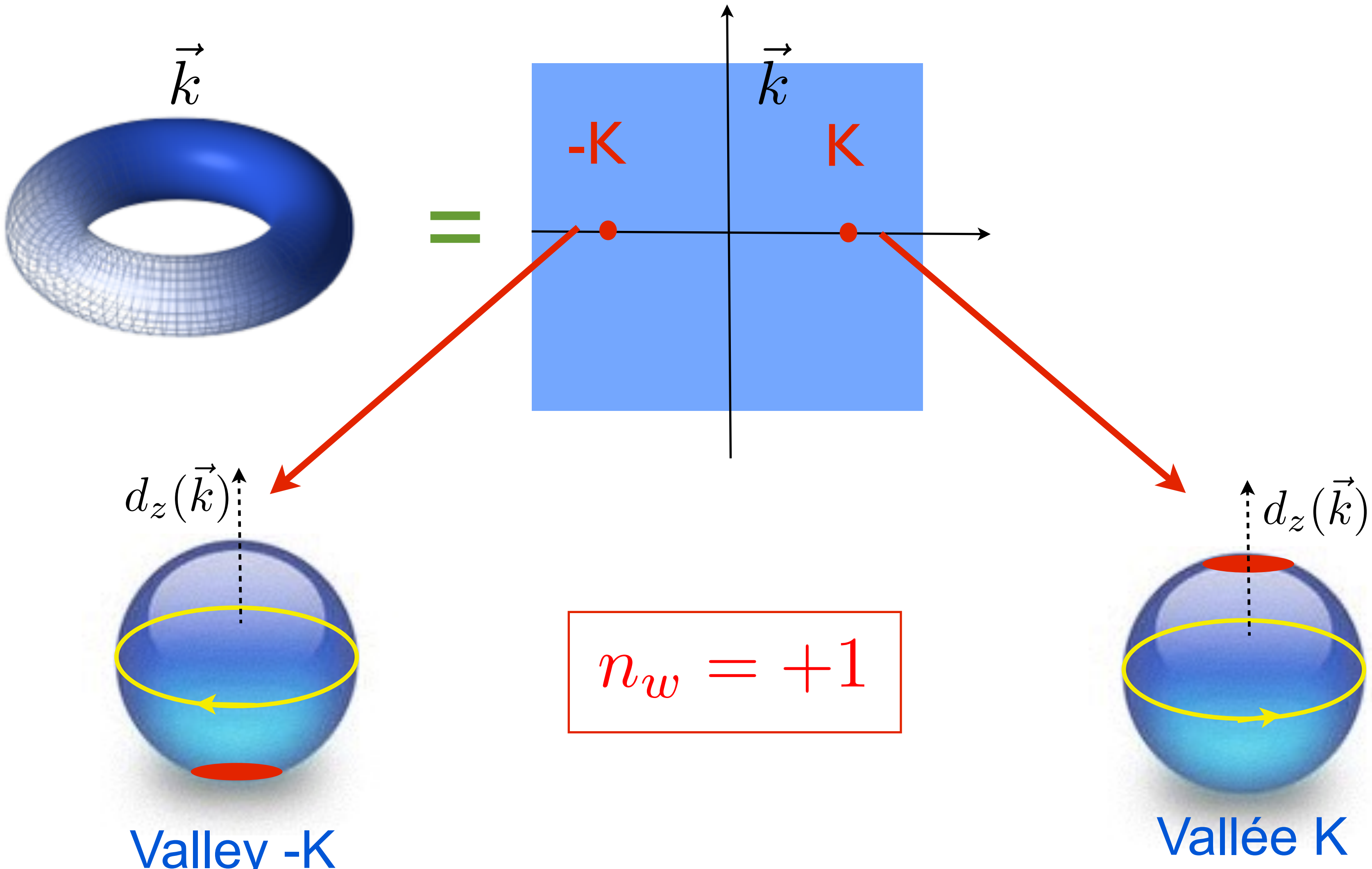
$$H = v_F\left(\pm p_x\sigma_x + p_y\sigma_y\right) + d_z(\pm\vec{K})\sigma_z$$



$$n_w = 0$$

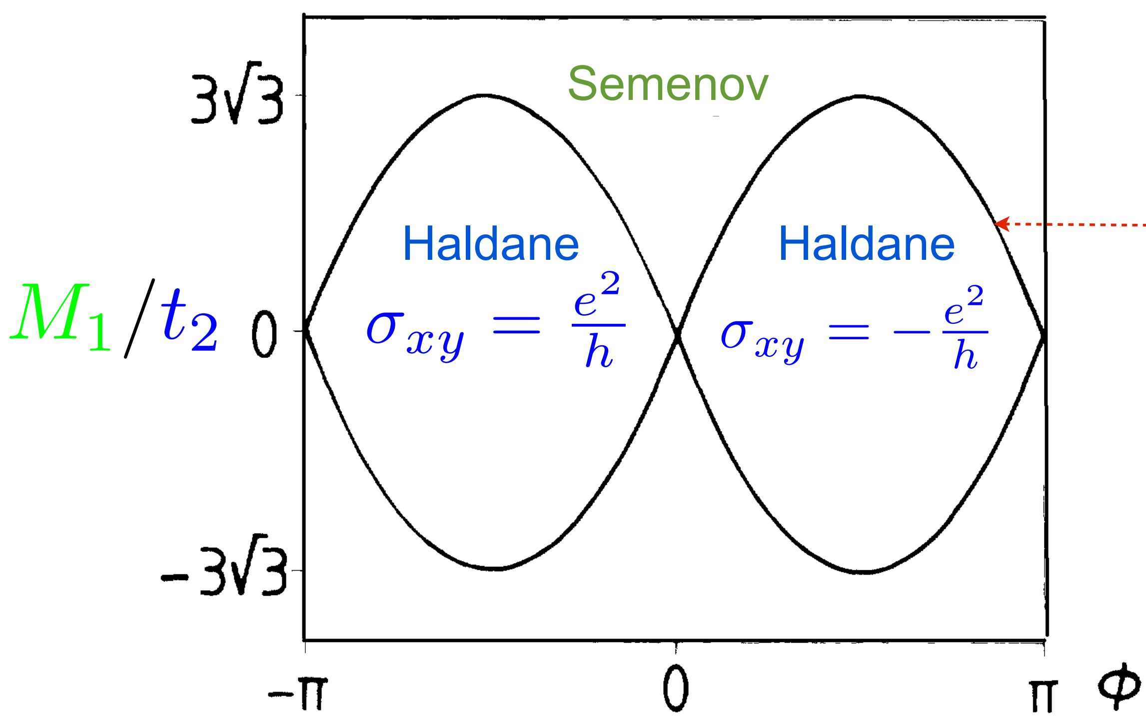
Haldane insulator

$$H = v_F\left(\pm p_x\sigma_x + p_y\sigma_y\right) + d_z(\pm\vec{K})\sigma_z$$

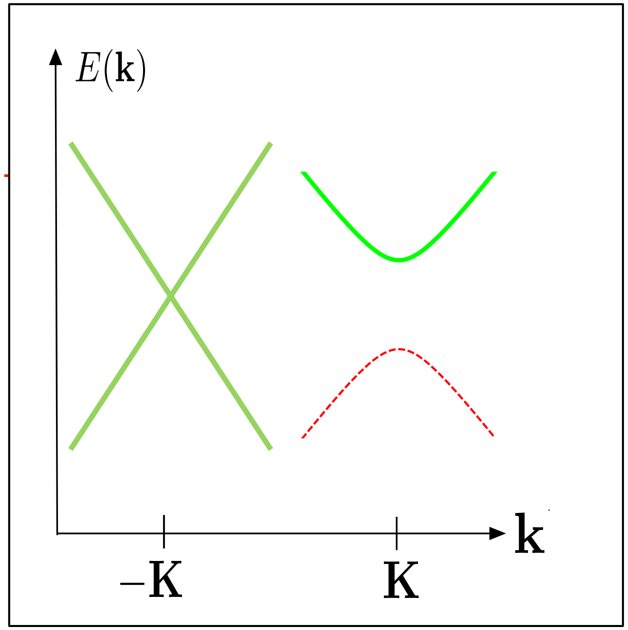


Bulk: « phase diagram » competing Semenov and Haldane masses

$$d_z(\pm\vec{K}) = M_1 \mp 3\sqrt{3}t_2 \sin\phi$$

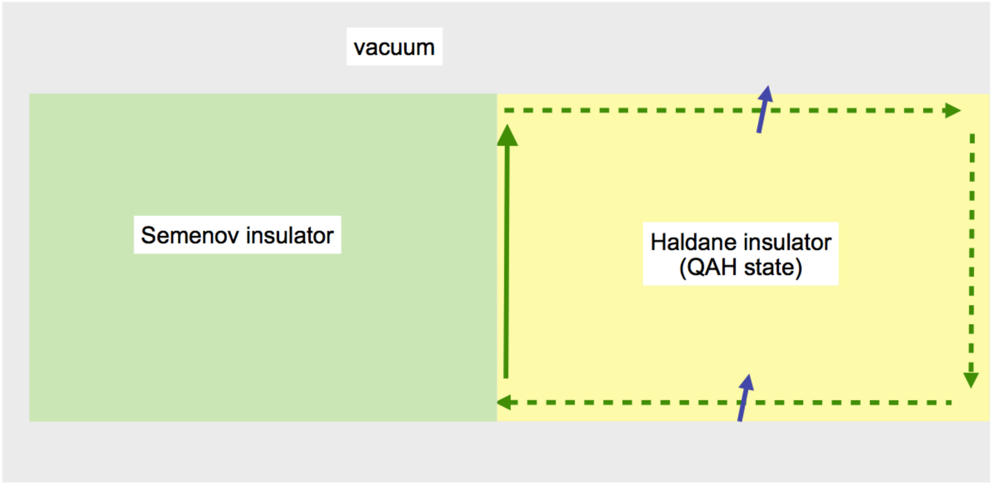


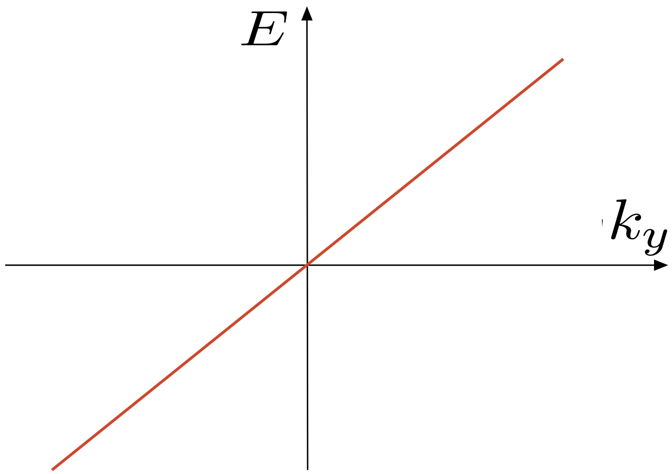
On the transition line



Transition: gap closing leading to a Dirac point jump in the Hall conductance

Interfaces between insulators





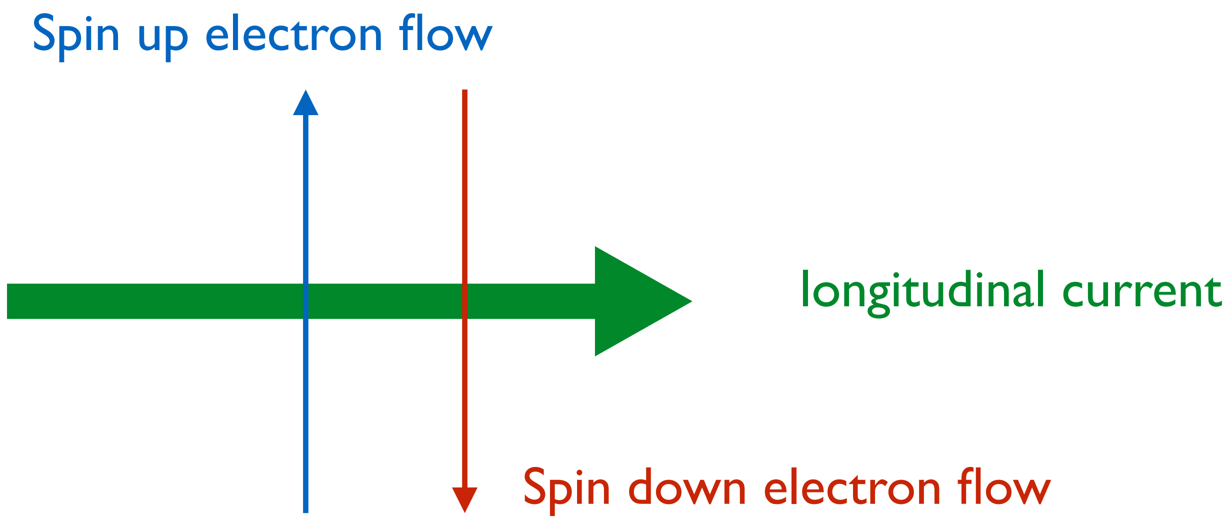
III)- Graphene:
massive Dirac fermions with spin

How to provide a mass to Dirac fermions
without breaking T or P ?

Dirac mass 3: Kane-Mele model

In 2004, Kane and Mele realized that it is possible to open gaps without breaking any of the fundamental symmetries (P, T).

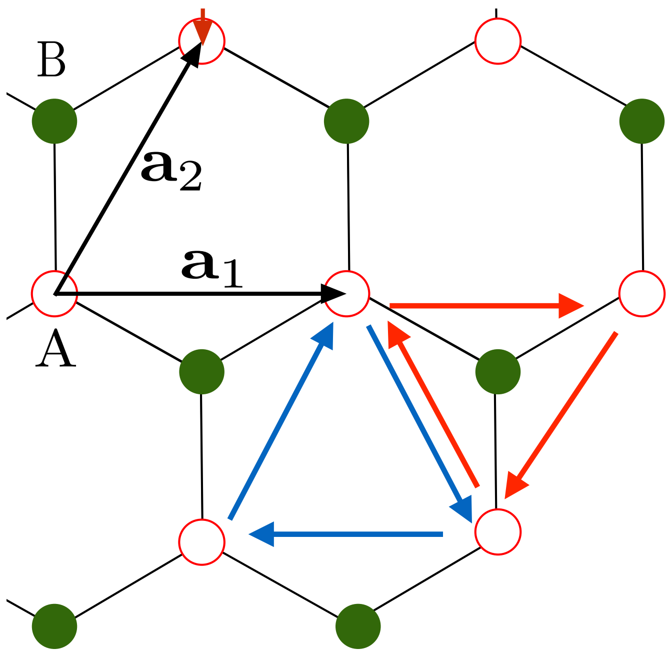
<u>Their initial motivation</u>: combining spin Hall effect + graphene



Spin-dependent complex hopping

The idea is to restore time-reversal invariance by gathering two copies of the Haldane model

$$\mathcal{T} = i s_y K$$



Spin-up	Spin-down
$t_2 e^{-i\varphi}$	$t_2 e^{i\varphi}$
$t_2 e^{i\varphi}$	$t_2 e^{-i\varphi}$

This corresponds to spin-orbit coupling

Same pattern on B sites

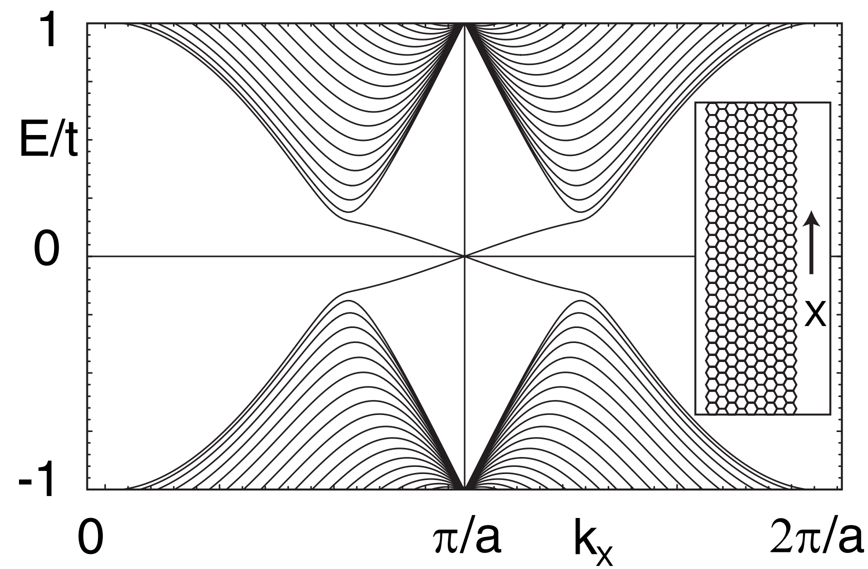
Kane-Mele model

Spin-orbit coupling

$$H = t \sum_{\langle i,j \rangle} c^{\dagger}_{i\alpha} c_{j\alpha} + i t_2 \sum_{\langle\langle i,j \rangle\rangle} \nu_{ij} c^{\dagger}_{i\alpha} (s_3)_{\alpha\beta} c_{j\beta}$$

Spin-conserving model: two copies of Haldane model

Two counter propagating edge states



Quantum Spin Hall insulator

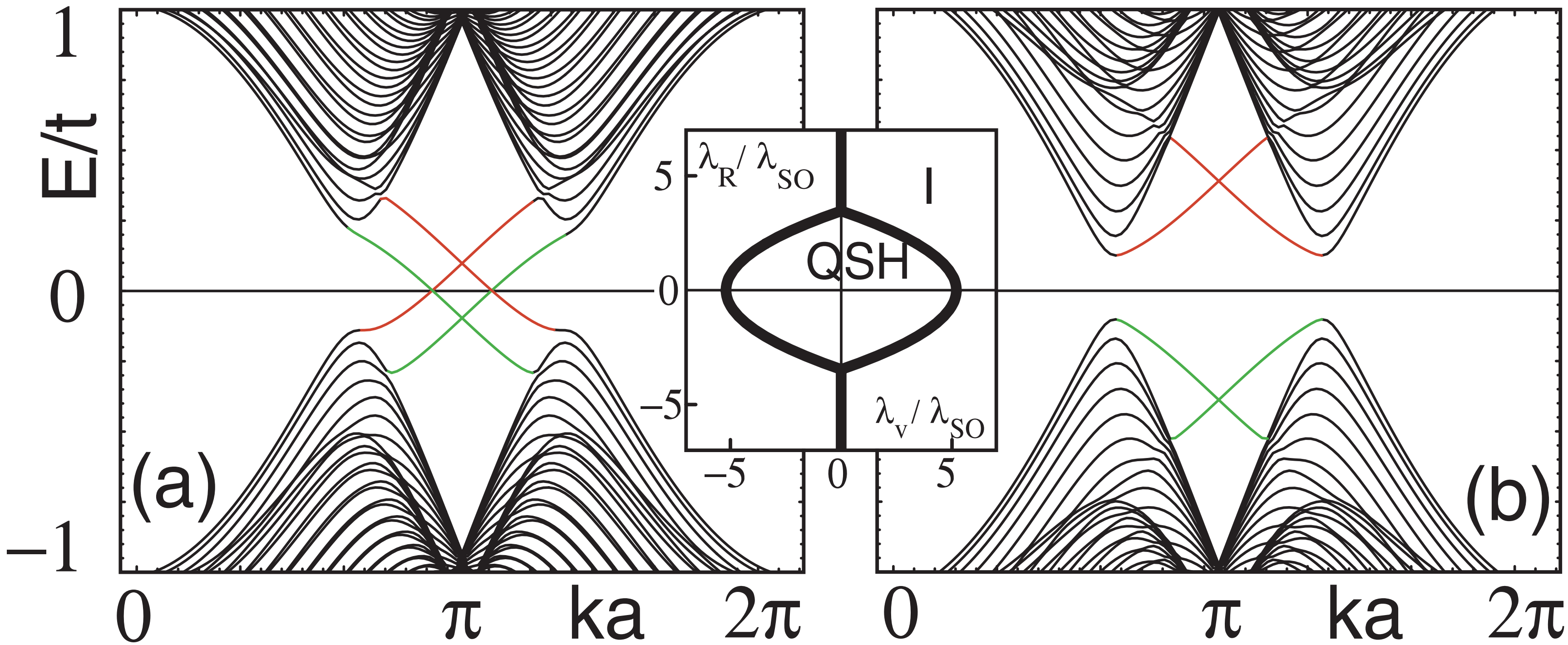
Robustness of the QSH insulator

One can add time-reversal invariant perturbations

Rashba spin mixing: λ_R

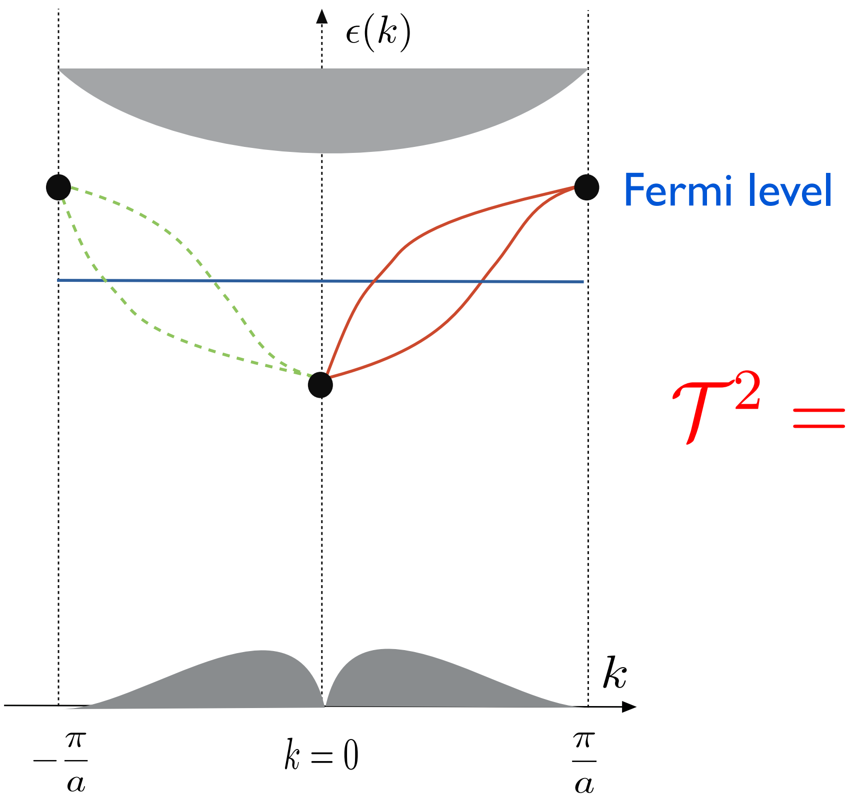
Staggered potential: λ_v

FIG. 1 (color online). Energy bands for a one-dimensional "zigzag" strip in the (a) QSH phase $\lambda_v = 0.1t$ and (b) the insulating phase $\lambda_v = 0.4t$. In both cases $\lambda_{SO} = .06t$ and $\lambda_R = .05t$. The edge states on a given edge cross at $ka = \pi$. The inset shows the phase diagram as a function of λ_v and λ_R for $0 < \lambda_{SO} \ll t$.



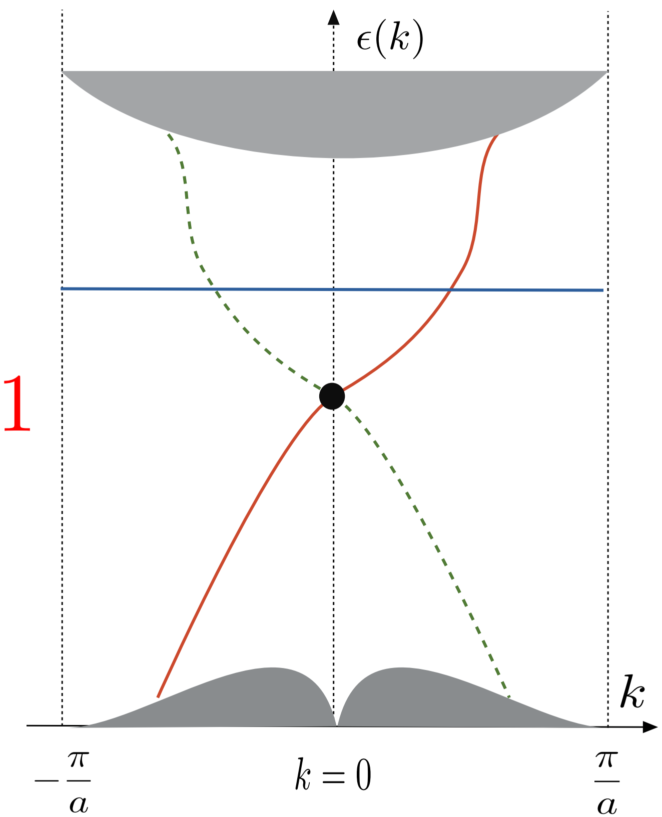
Kramer's pairs

standard case: 2 pairs



non trivial: case one pair

$$\mathcal{T}^2 = -1$$

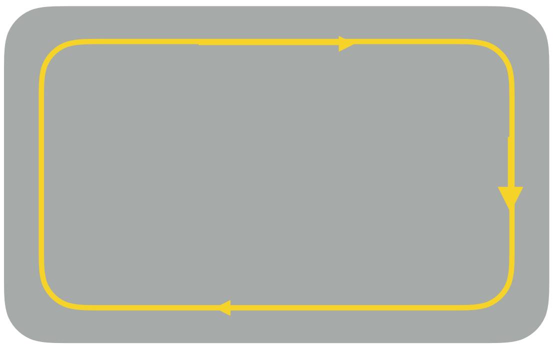
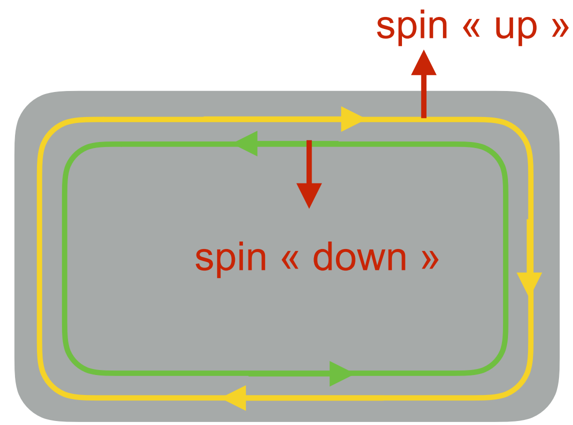


Kramer's pairs number at Fermi level

Symmetry protection if this number is odd

Z2-type topological invariant

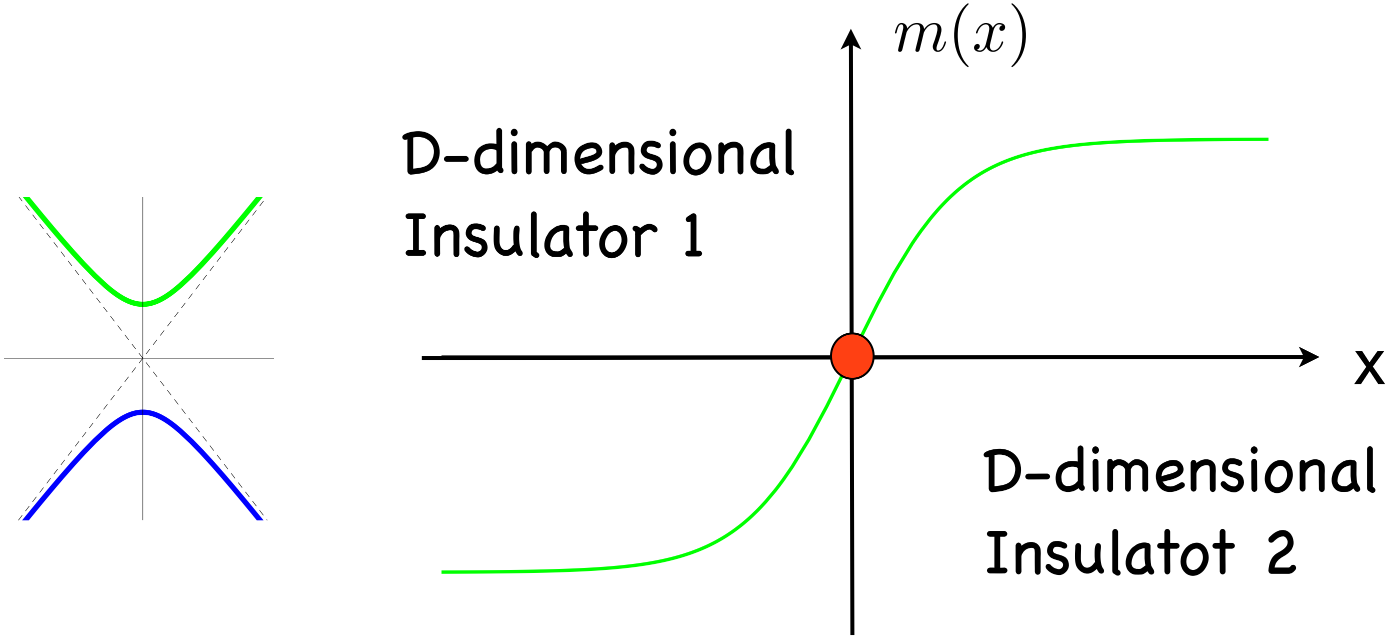
Comparison Chern versus Z2 insulators

Chern insulator	Z2 insulator
	
Chiral edge state	helical edge state
Time-reversal breaking	Time-reversal symmetric
Bulk topological invariant: Z	Bulk topological invariant: Z2
Minimal bulk model: 2 bands	Minimal bulk model: 4 bands
$h_0(\vec{k}) = d_1(\vec{k})\sigma_1 + d_2(\vec{k})\sigma_2 + d_3(\vec{k})\sigma_3$	$\mathcal{H}(\mathbf{k}) = \sum_{a=1}^{5} d_a(\mathbf{k})\Gamma^a + \sum_{a<b=1}^{5} d_{ab}(\mathbf{k})\Gamma^{ab}$

Topology and Dirac mass

- Jackiw-Rebbi mechanism (1976):

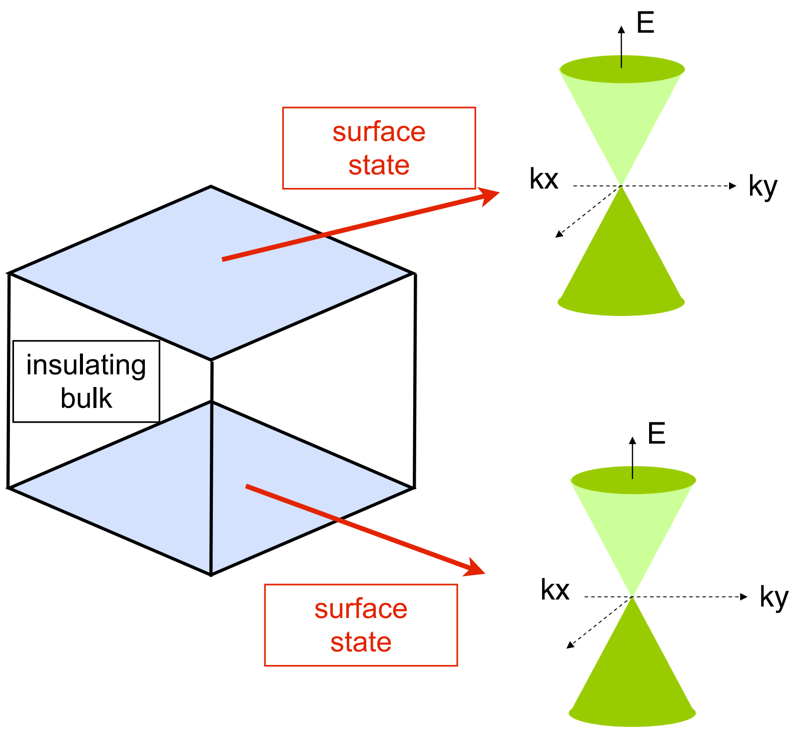
$$i\hbar\partial_t\Psi(x,t) = \left(-i\hbar\partial_x\sigma_1 + m(x)\sigma_2\right)\Psi(x,t)$$

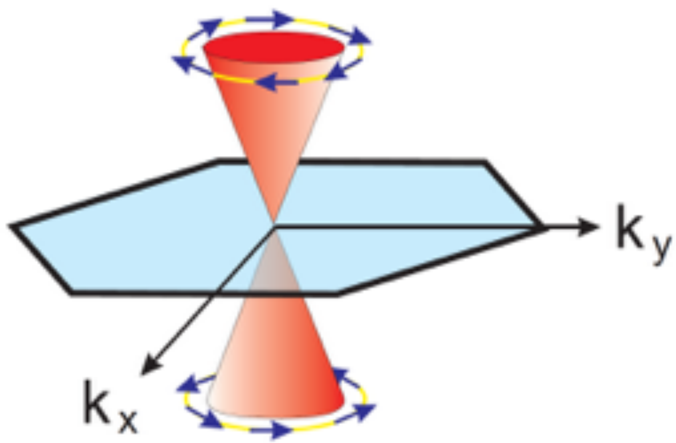


- D-1 interfacial state E=0, localised at x=0 between two insulators

3D topological insulators

- Semiconductor: weak gap and strong SO coupling
- Dirac surface state (ARPES, STM, transport)





Each Bloch state k has only one spin direction

Surface state is spin-polarized

IV)- Topological insulators in real materials

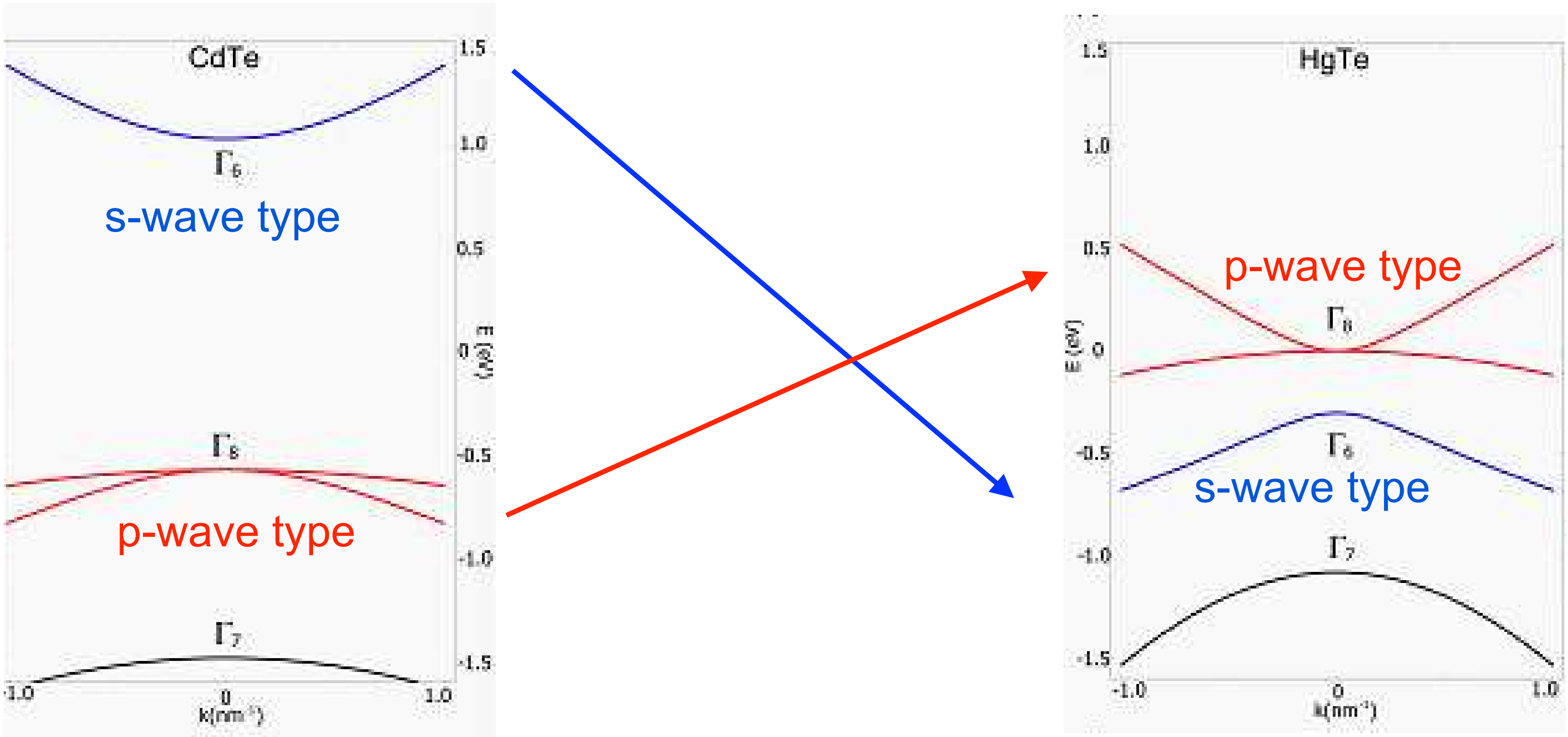
Spin orbit is too weak in graphene

HgTe and Bi-based compounds: strong spin-orbit and close to band inversion

HgTe/CdTe

CdTe spectrum

HgTe spectrum



3D semiconductor

3D semimetal:
no gap

This can be described by 8 (or 6) band model (within k.p theory)

4-band model

« Ab-initio » 6-band model + envelope functions leads to BHZ model

Symmetry allowed terms (both linear and quadratic in momentum)

$$H_{\text{eff}}(k_x, k_y) = \begin{pmatrix} H(k) & 0 \\ 0 & H^*(-k) \end{pmatrix}$$

$$d_1 + id_2 = A(k_x + ik_y) \equiv Ak_+$$

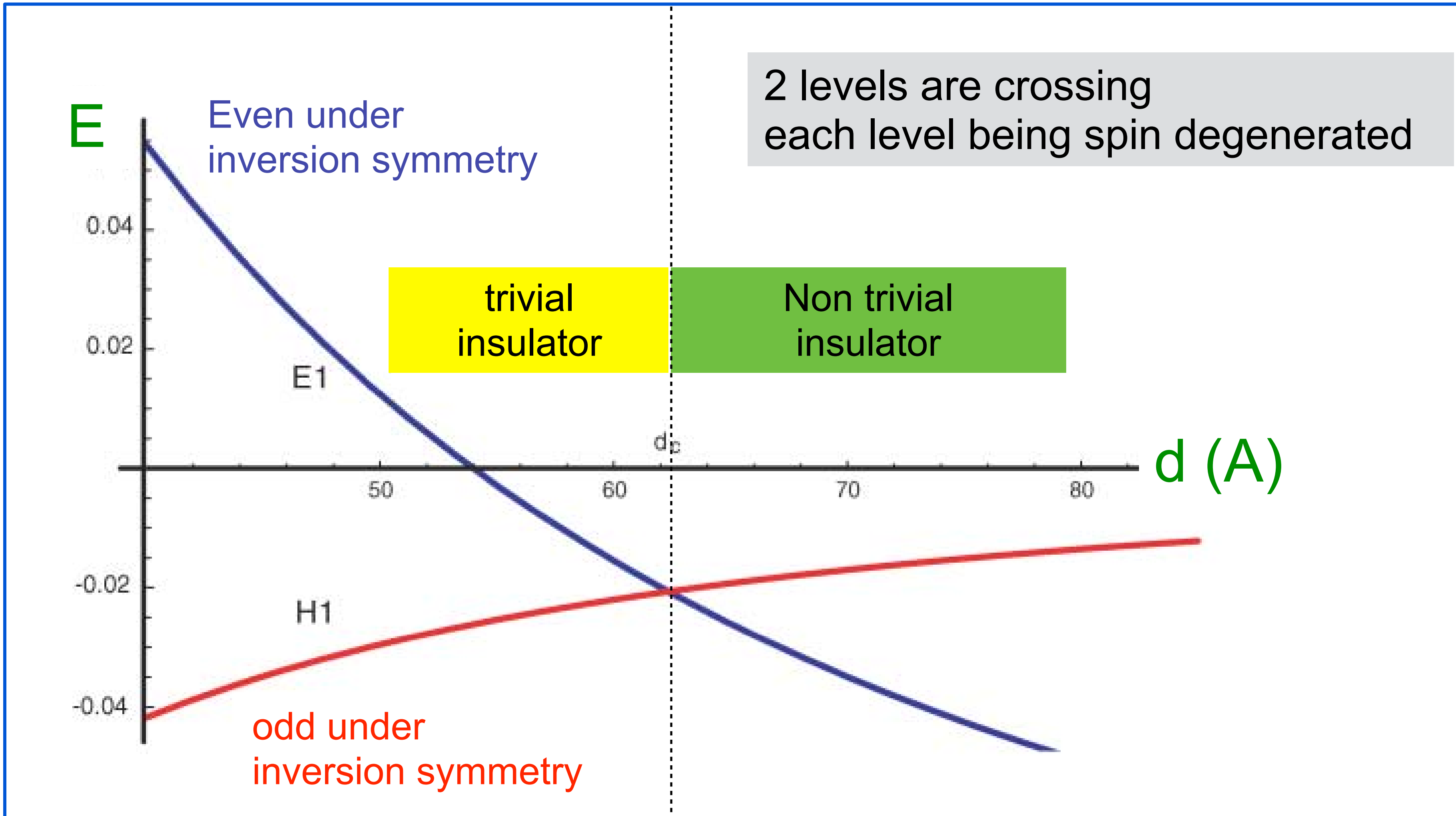
$$d_3 = M - B(k_x^2 + k_y^2)$$

$$\varepsilon(k) = C - D(k_x^2 + k_y^2)$$

HgTe/CdTe quantum well

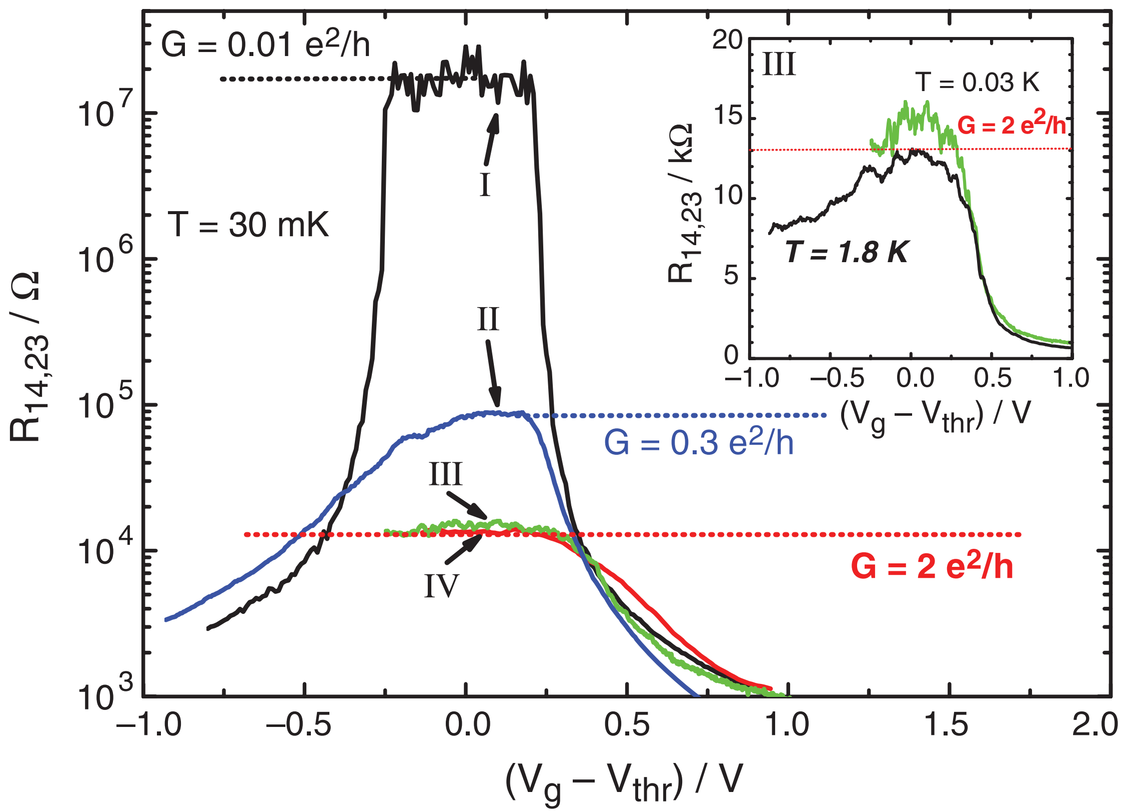
From B.A. Bernevig, T.L. Hughes, and S.C. Zhang, Science **314**, 1757 (2006)

$$M(d_c) = 0$$



Experiments

Koenig, Wiedmann, Brune, Buhmann, Molenkamp, Qi and Zhang, Science 2007



I. d=5.5nm (non inverted) insulating in the gap

II-IV d=7.3nm (inverted) conducting in the gap

L= 20 microns

L= 1 micron W=1 micron

L= 1 micron W=0.5 micron

Edge conduction in III-IV cases (suppressed by magnetic field)

Theory: *B.A. Bernevig, T.L. Hughes, and S.C. Zhang, Science* ***314****, 1757 (2006)*

Experiments:

Konig, Wiedmann, Brune, Buhmann, Molenkamp, Qi and Zhang, Science 2007

Roth, Brune, Buhmann, Molenkamp, Maciejko, Qi and Zhang, Science 2009

Experiments

Roth, Brune, Buhmann, Molenkamp, Maciejko, Qi and Zhang, Science 2009

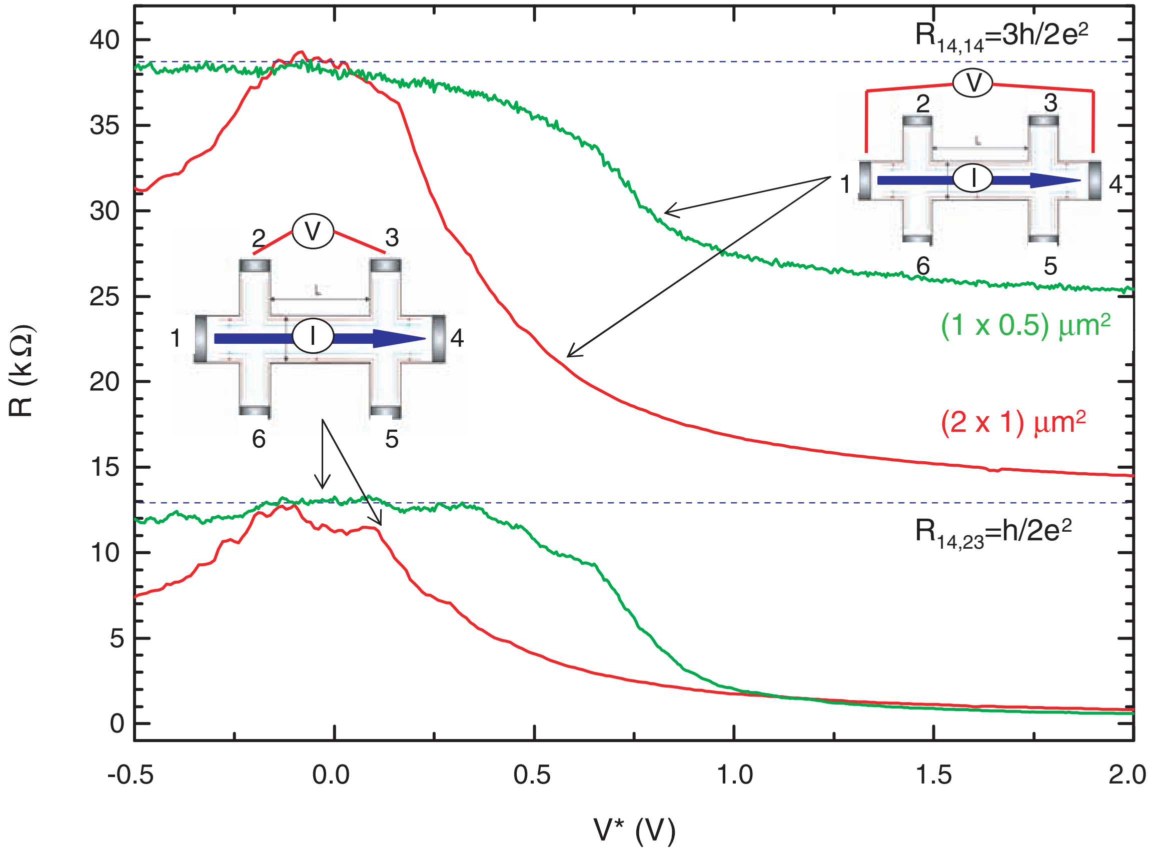
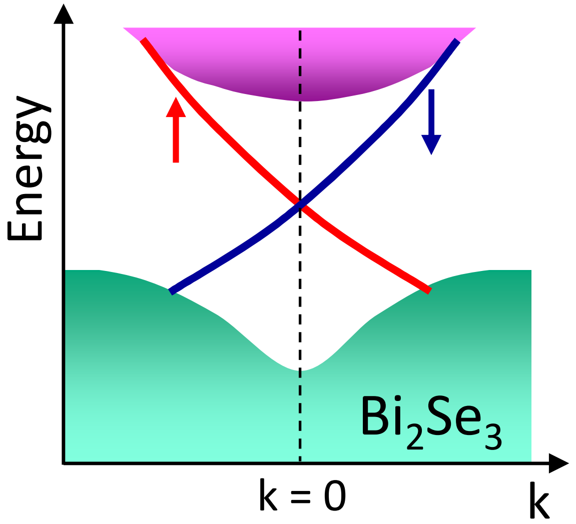
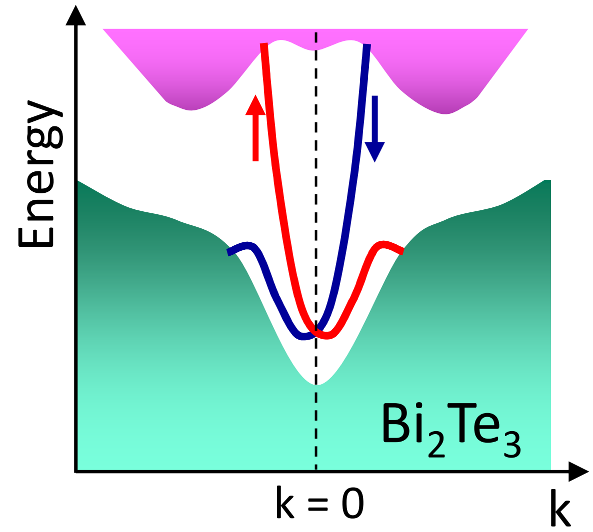


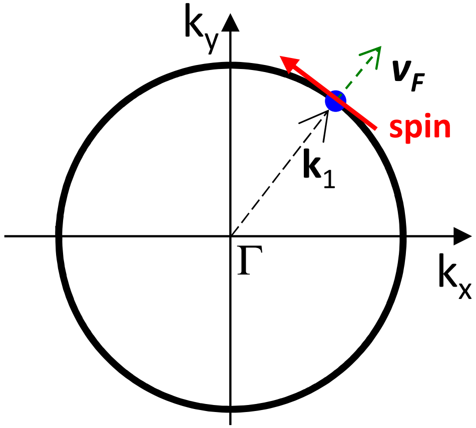
Fig. 1. Two-terminal ($R_{14,14}$) (top two traces) and four-terminal ($R_{14,23}$) (bottom traces) resistance versus (normalized) gate voltage for the Hall bar devices D1 and D2 with dimensions (length × width) as indicated. The dotted blue lines indicate the resistance values expected from the Landauer-Büttiker approach.

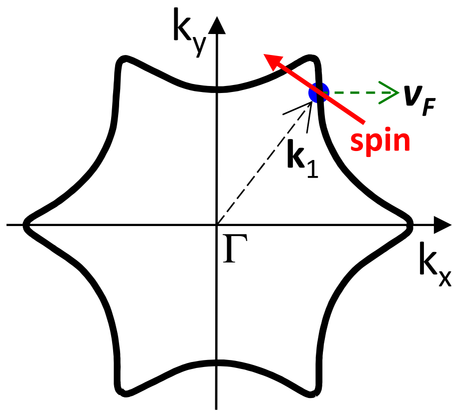
Landauer-Buttiker analysis

Bi_2Se_3 and Bi_2Te_3



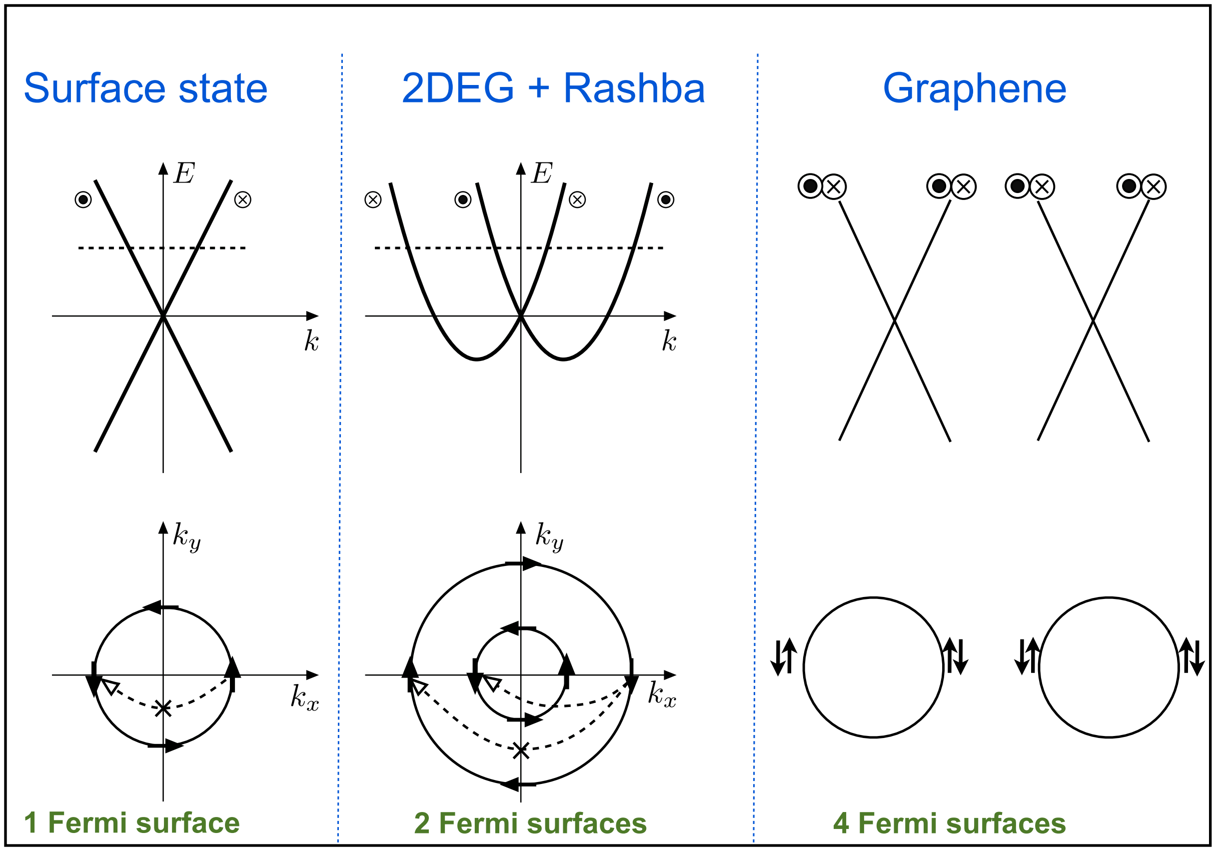






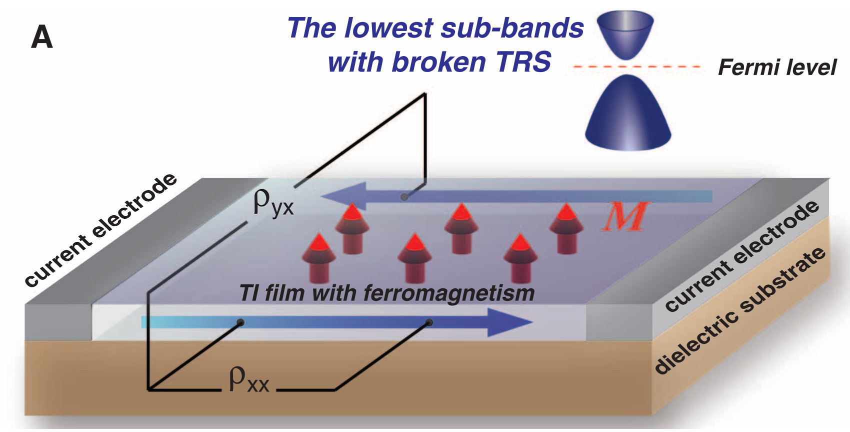
from Y. Ando, JSPJ review 2013

2D Surface states versus graphene

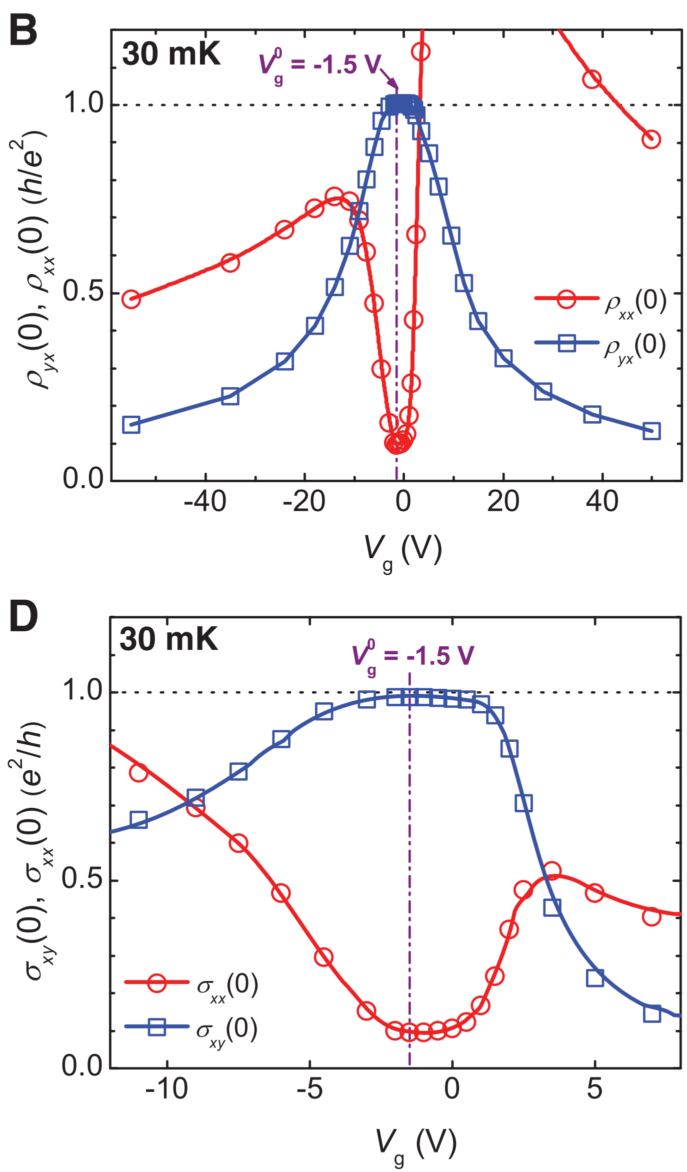


from J. Bardarson and J. Moore, Rep. Prog. Phys 2012

Chern insulator: experiment



$Cr_{0.15}(Bi_{0.1}Sb_{0.9})_{1.85}Te3$



Conclusions

Edge states provide new low dimensional conductors (1D) which differ from previously known 1D systems: nanowires, 1D organic conductors, carbon nanotubes.

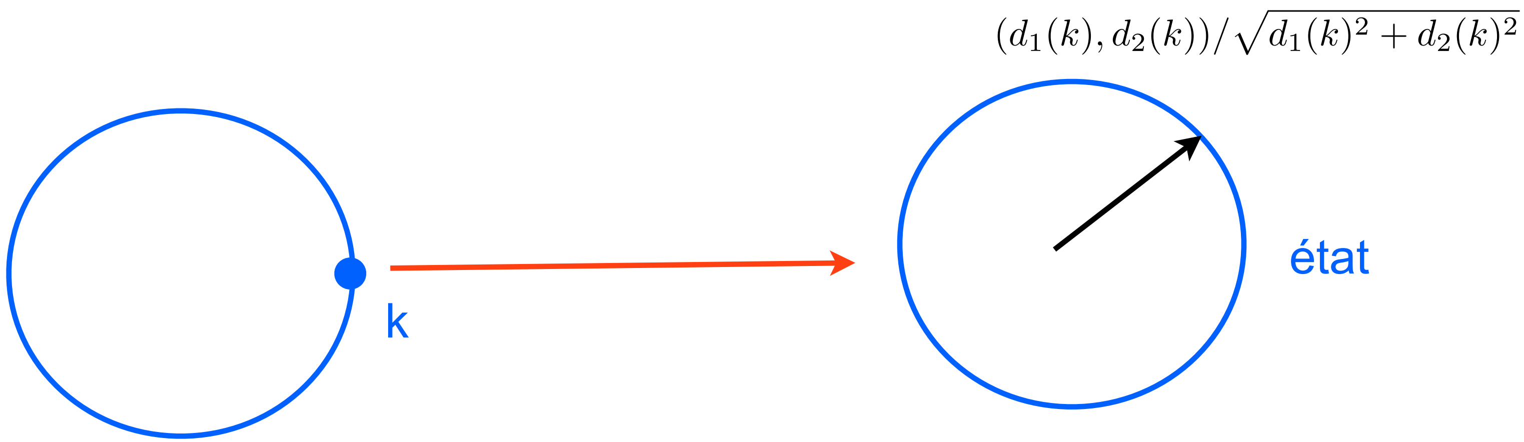
The spin-momentum locking leads to:

- suppression of some scattering channels
- absence of backscattering by non magnetic impurities
- unusual proximity effect with superconductors: realization of exotic phases (Majorana quasiparticles)

Surface states of TIs also provide new low dimensional conductors (2D) which differ from previously known 2D systems: graphene, 2DEG trapped in heterojunctions.

Topologie: enroulement de Bloch 1D

- Nombre d’enroulement S1 vers S1:



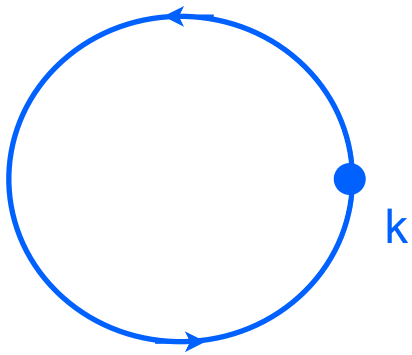
Zone de Brillouin = cercle

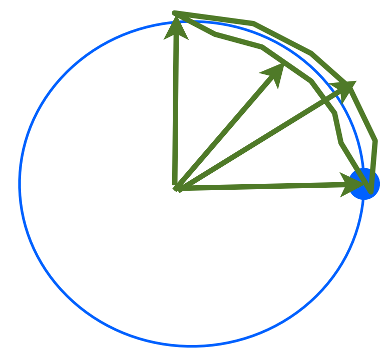
Fonctions d’ondes
parametrées par un seul angle

$$H(k) = d_1(k)\sigma_1 + d_2(k)\sigma_2$$

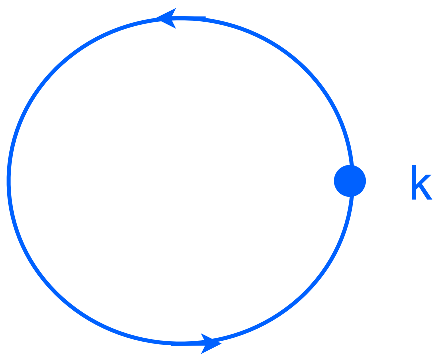
Topologie: enroulement de Bloch 1D

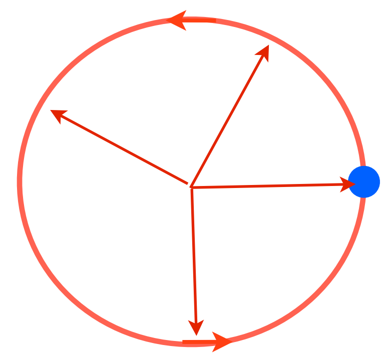
• Enroulement trivial des états de Bloch





• Enroulement non trivial

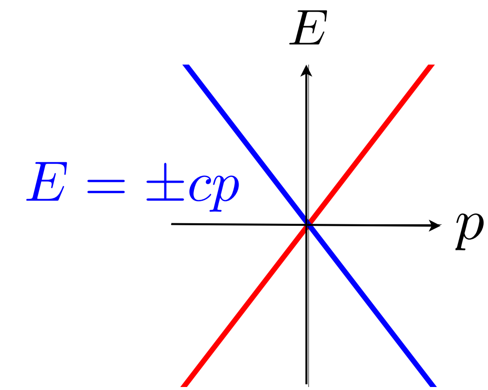




Weyl equation (1929)

Massless particle:

$$i\left(\partial_t - c\vec{\sigma}.\partial_{\vec{r}}\right)\psi_L = 0$$
$$i\left(\partial_t + c\vec{\sigma}.\partial_{\vec{r}}\right)\psi_R = 0$$



Masse de Dirac:

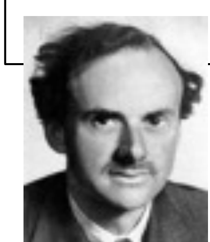
4 nombres complexes

$$i\left(\partial_t - c\vec{\sigma}.\partial_{\vec{r}}\right)\psi_L = m\psi_R$$
$$i\left(\partial_t + c\vec{\sigma}.\partial_{\vec{r}}\right)\psi_R = m\psi_L$$

Masse de Majorana (1937):

2 nombres complexes (4 réels)

$$\psi_R = i\sigma_y\psi_L^*$$



Neutrinos massifs: Dirac ou Majoranas ?

BHZ model on the square lattice

2 blocks

Spin-up block describes a Chern insulator

Edge states on finite lattices

It is the realization of a Z2 topological insulator

BHZ model: topological properties

$$\mathcal{H}(k) = \sin k_x \sigma_1 + \sin k_y \sigma_2 + (M - 2B(2 - \cos k_x - \cos k_y))\sigma_3$$

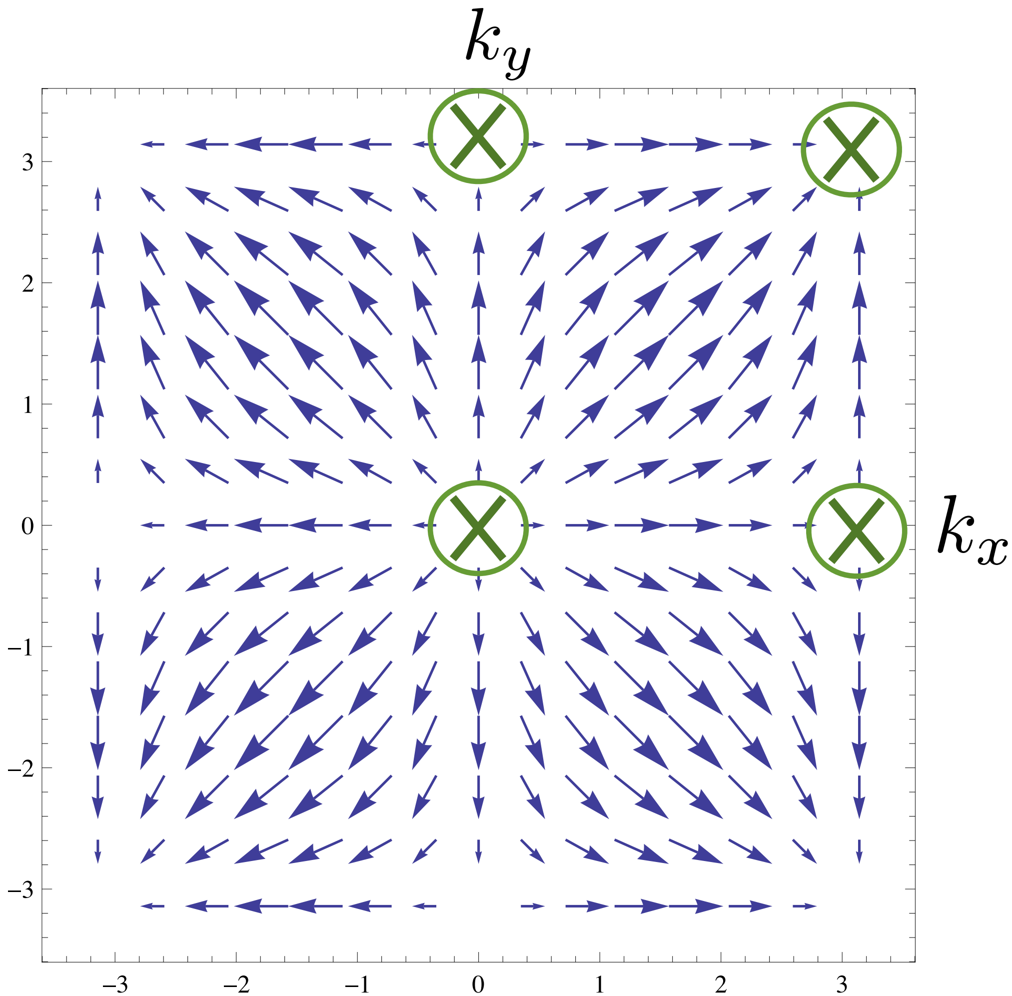
Réseau carré avec 2 orbitales (s et p) sur chaque site (électrons sans spin)

Dirac points	(0,0)	$(\pi,0)$	$(0,\pi)$	(π,π)	Ch
mass	M	$M-4B$	$M-4B$	$M-8B$	
chirality	$+$	$-$	$-$	$+$	
$M<0$	$-$	$+$	$+$	$-$	0
$M \in (0,4B)$	$+$	$+$	$+$	$-$	$+$
$M \in (4B,8B)$	$+$	$-$	$-$	$-$	$-$
$M>8B$	$+$	$-$	$-$	$+$	0

Modèle: *B.A. Bernevig, T.L. Hughes, and S.C. Zhang, Science* ***314****, 1757 (2006)*

Doru Sticlet et al., Phys. Rev. B **85**, 165456 (2012)

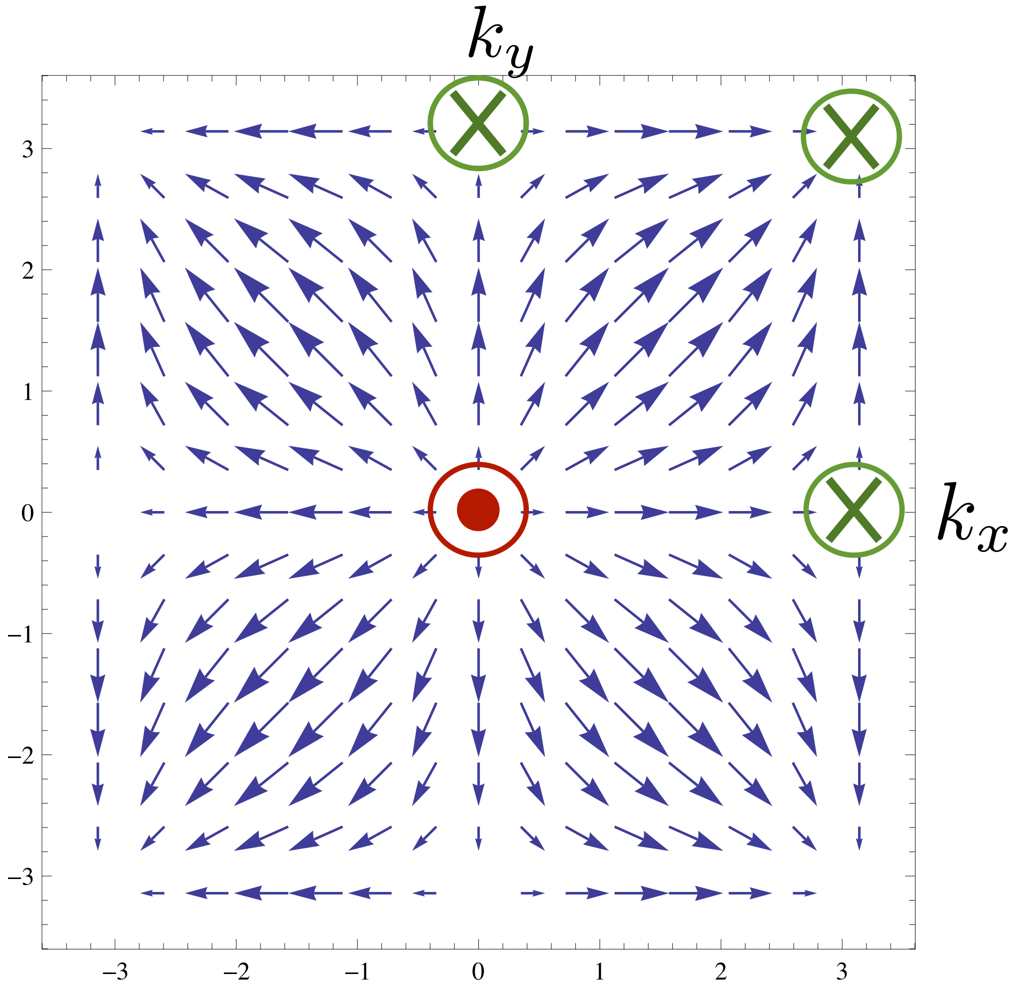
M<0



$$n_w = 0$$

Configuration "ferromagnétique" $d_3(\boldsymbol{k})$

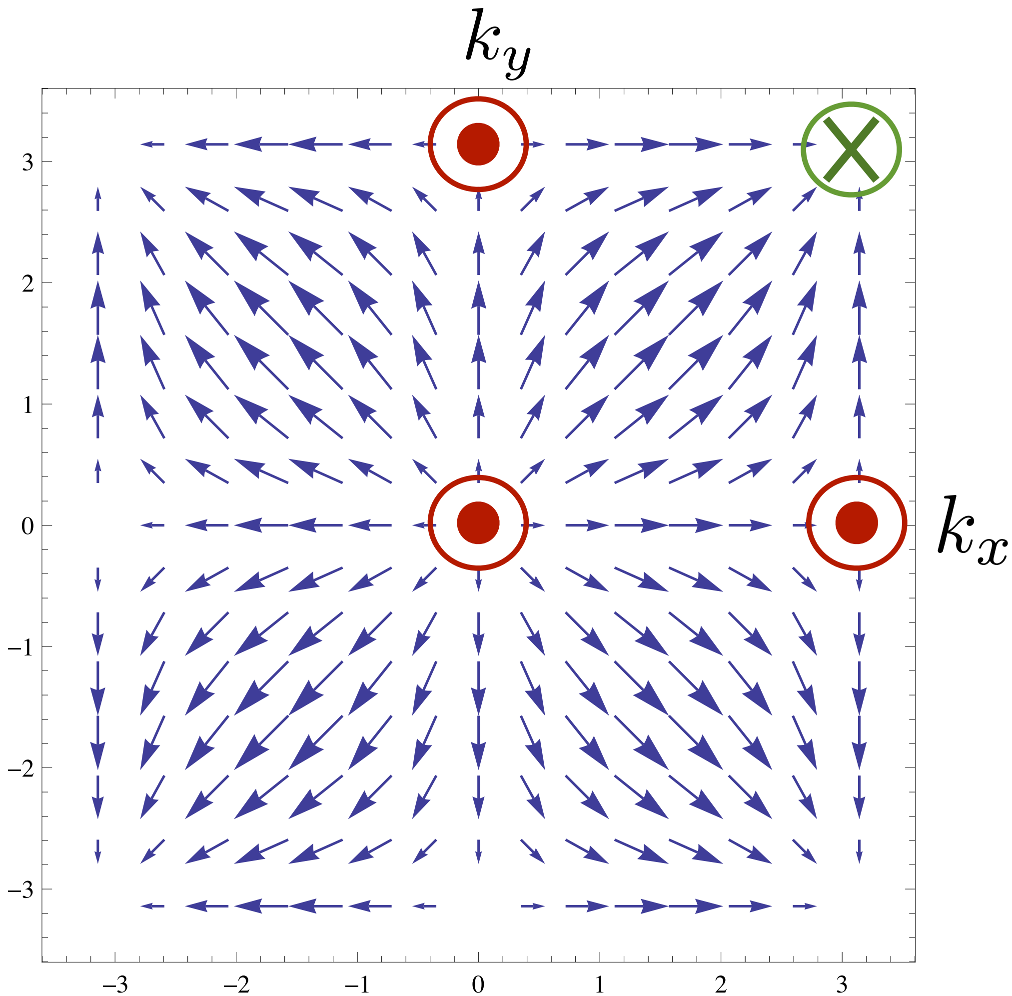
0<M<4B



$$n_w = 1$$

Configuration skyrmionique $\quad d_3(\boldsymbol{k})$

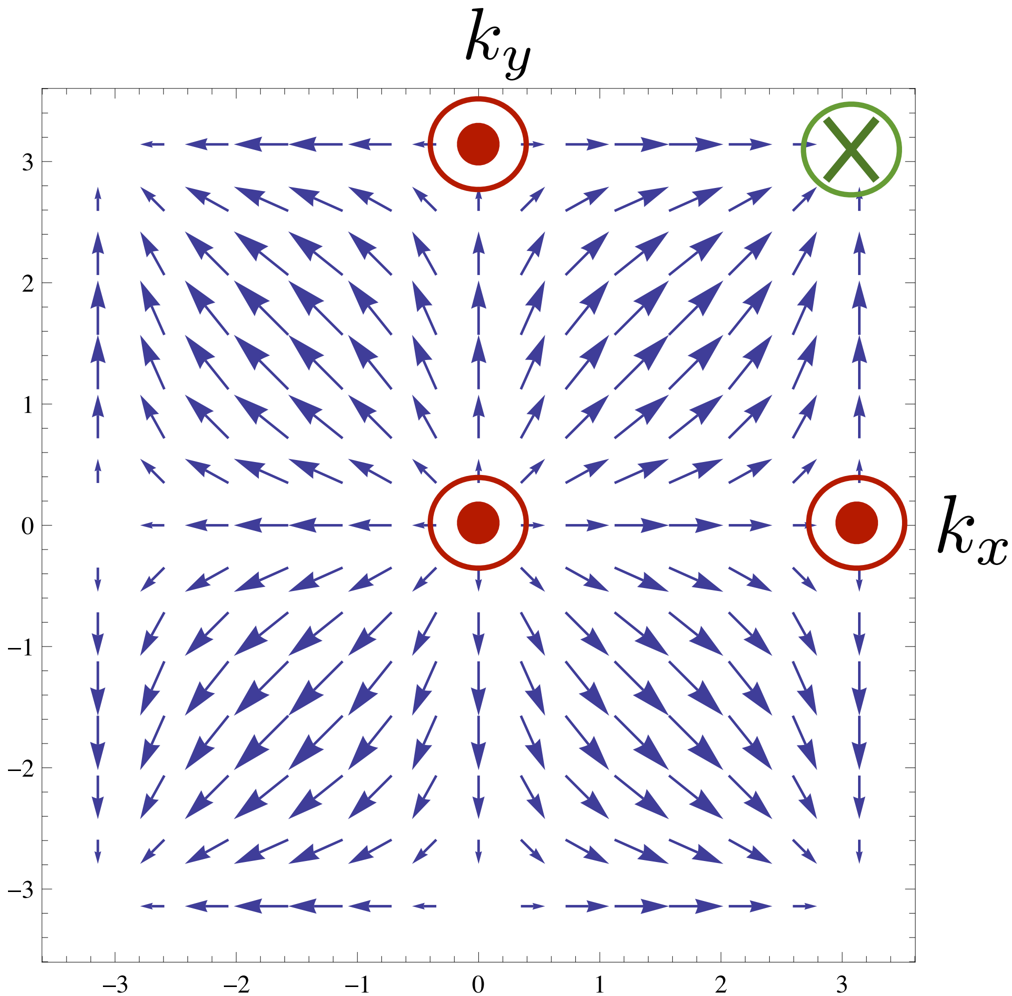
4B<M<8B



$$n_w = -1$$

Autre configuration skyrmionique $d_3(\boldsymbol{k})$

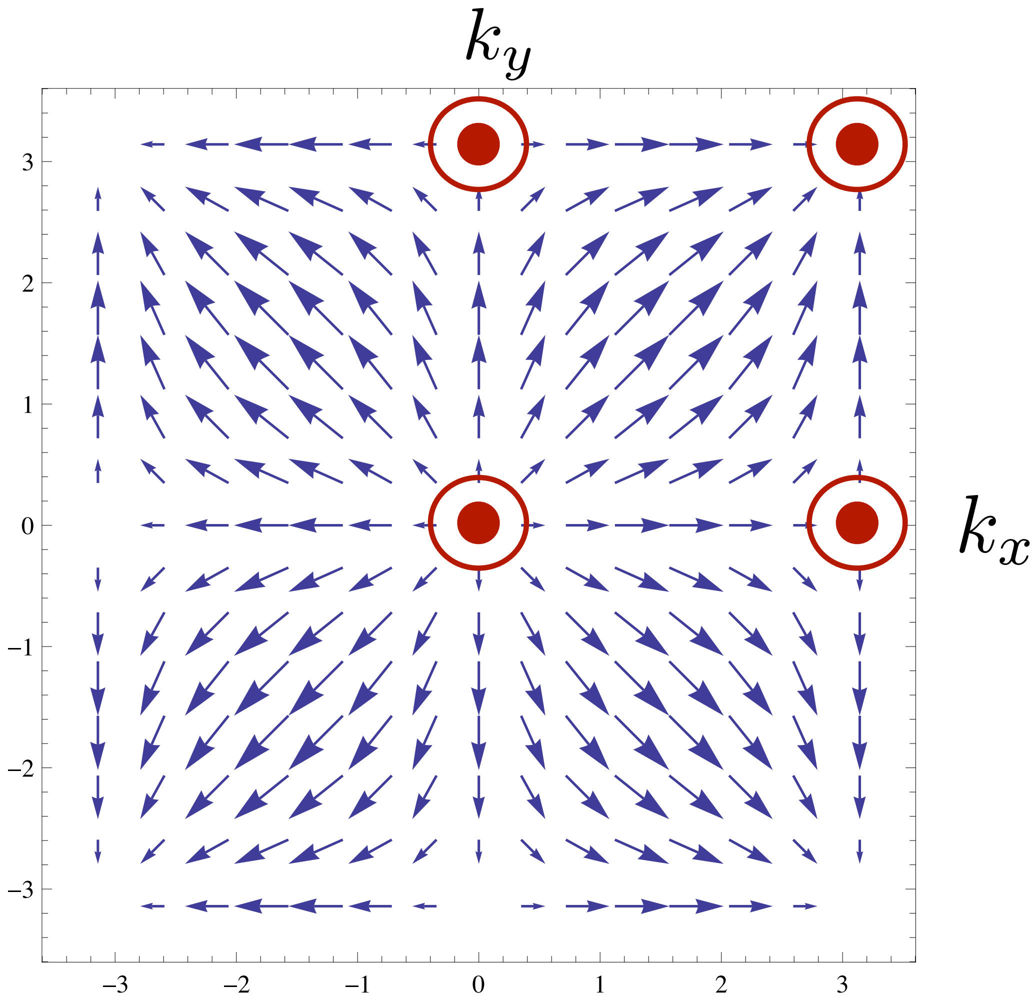
4B<M<8B



$n_w = -1$

Autre configuration skyrmionique $d_3(\boldsymbol{k})$

M>8B



$$n_w = 0$$

Configuration ferromagnétique $d_3(\boldsymbol{k})$

InAs/GaSb electron/hole bilayer

Transition can be driven by gating

$$H_0 = - t \sum_{\vec{R},\vec{\delta}_\alpha} c^\dagger_B(\vec{R}) c_A(\vec{R}+\vec{\delta}_\alpha) + H.c.$$

$$H_0 = - t \sum_{\vec{k},\vec{\delta}_\alpha} e^{i \vec{k} . \vec{\delta}_\alpha} c^\dagger_B(\vec{k}) c_A(\vec{k}) + H.c.$$

$$\, \, \, h_0 (\vec{k})= - t \sum_{\vec{k}, \vec{\delta}_\alpha} \left(\cos(\vec{k} . \vec{\delta}_\alpha) \sigma_1 + \sin(\vec{k} . \vec{\delta}_\alpha) \sigma_2 \right)$$

$$h_0 (\vec{k})= 0 \rightarrow \vec{k}= \pm \vec{K}$$

$$\, \, \, h_0 (\vec{k})= - t \sum_{\vec{k}, \vec{\delta}_\alpha} \left({\color{red} \cos(\vec{k} . \vec{\delta}_\alpha) }\sigma_1 + {\color{blue} \sin(\vec{k} . \vec{\delta}_\alpha) } \sigma_2 \right)$$

$$= - t \sum_{\vec{k}} c^\dagger_\alpha(\vec{k}) [h_{0}]_{\alpha \beta} c_\beta(\vec{k})$$

$$- t= \int d^3r \, \phi^*(\vec{r}-\vec{R}_A-\vec{\delta}_3) (V(\vec{r})-V_{\rm ato}(\vec{r}-\vec{R}_B)) \phi(\vec{r}-\vec{R}_B)$$

$$h_0 (\vec{k})= \sigma_1 h_0 (-\vec{k}) \sigma_1 \qquad h_0 (\vec{k})= h_0^* (-\vec{k})$$

$$E_F= \hbar v_F {\color{blue} q_F}$$

$$
{\color{red} d_3(\vec{k}) =- d_3(-\vec{k}) }

v_F = 3 at/2\hbar \simeq 10^6 {\rm m.s^{-1}}
$$

$$
E({\color{red} \xi } \vec{K} + {\color{blue} \vec{q} } \,) = \hbar v_F {\color{blue} \sqrt{q_x^2+q_y^2}}
$$

$$
\vec{k} = \vec{K} + {\color{blue} \vec{q} }
$$

$$
\, \, \, \, h_0 ({\color{red} \xi }\vec{K} + {\color{blue} \vec{q} } \,) \simeq \hbar v_F \left({\color{red} \xi } {\color{blue} q_x }\sigma_1 + {\color{blue} q_y } \sigma_2 \right)
$$

$$
\vec{k} = {\color{red} \xi} \vec{K} + {\color{blue} \vec{q} }
$$

$$
h_0 (\vec{k})=   h_0^* (-\vec{k})
$$

$$
{\color{blue} T^2 = - 1 }
$$