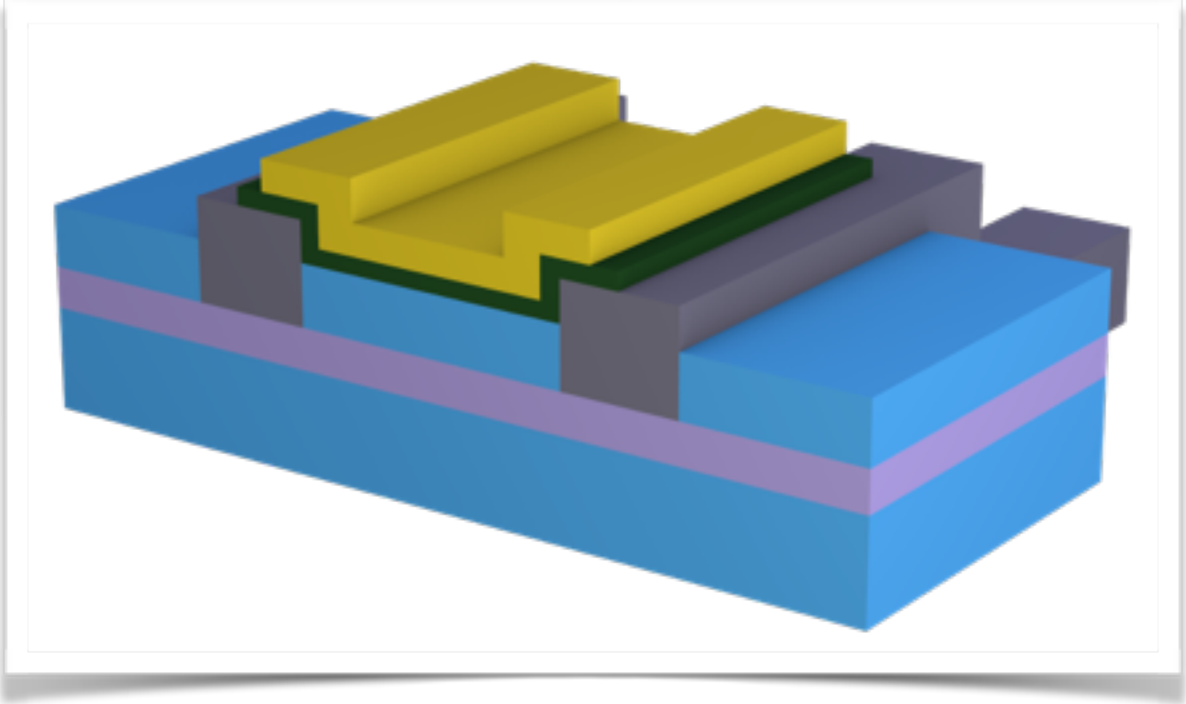
Erwann Bocquillon

Gapless Andreev bound states in HgTe-based topological Josephson junctions

TopoLyon 2016
03/10/2016



Physikalisches Institut (EP3)
Universität Würzburg, Am Hubland, D-97074 Würzburg
http://www.physik.uni-wuerzburg.de/EP3/

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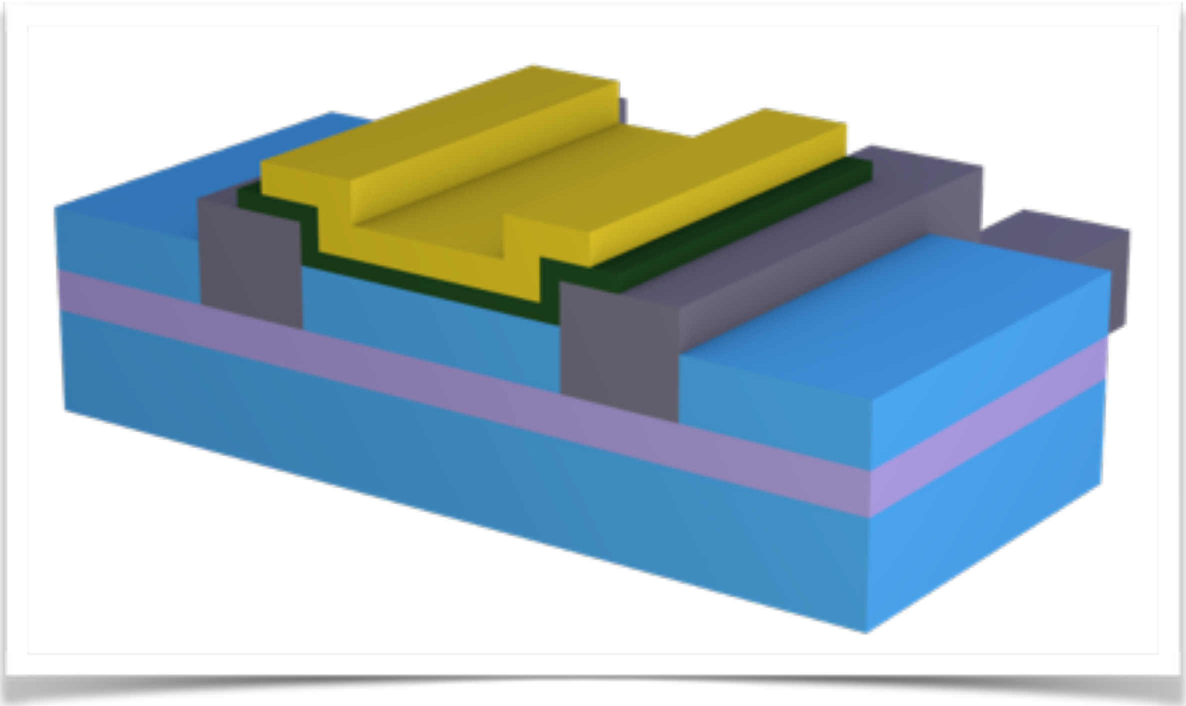

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People involved

Uni. Würzburg

- **PhD students :** J. Wiedenmann,
  P. Leubner
- **Staff :** C. Brüne
  H. Buhmann
  L.W. Molenkamp
- **Invited :** T.M. Klapwijk
- **Theory :** F. Domínguez, E.M. Hankiewicz

RIKEN, Tōkyō

- R.S. Deacon, K. Ishibashi, S. Tarucha

European Research Council
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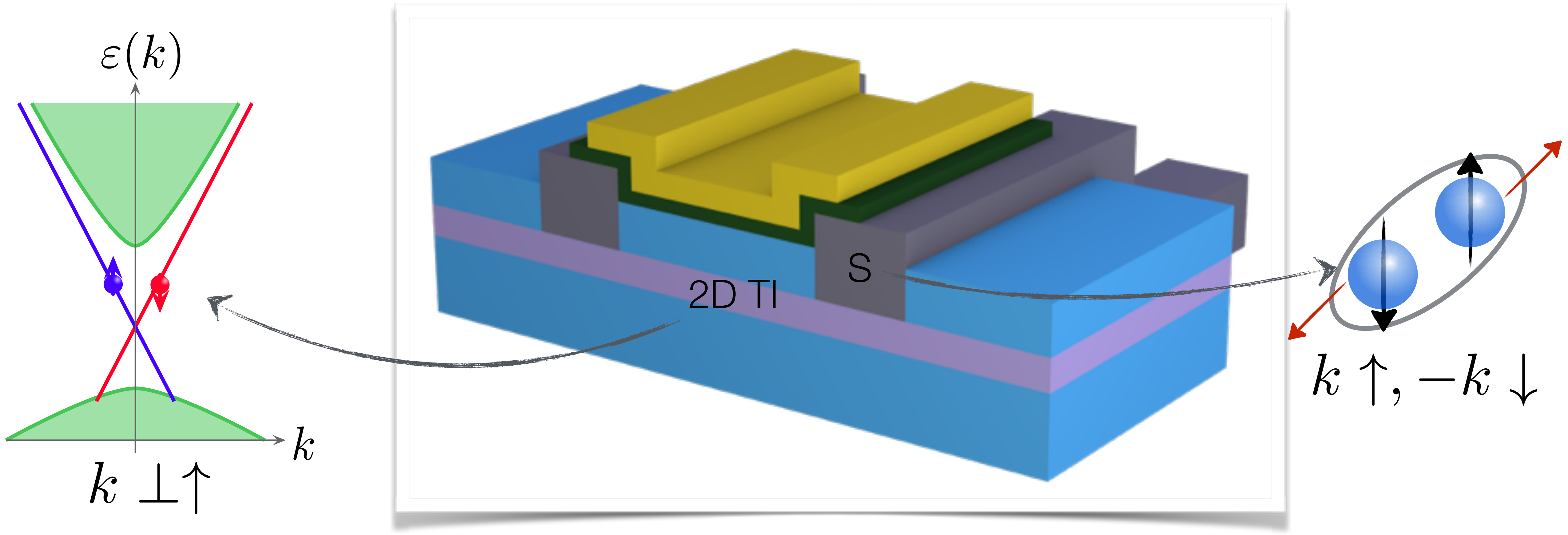
Stiftung / Foundation

Induced superconductivity in a 2D TI



Cooper pair of helical Dirac fermions
⇒ helical pairing
⇒ *p*-type correlations

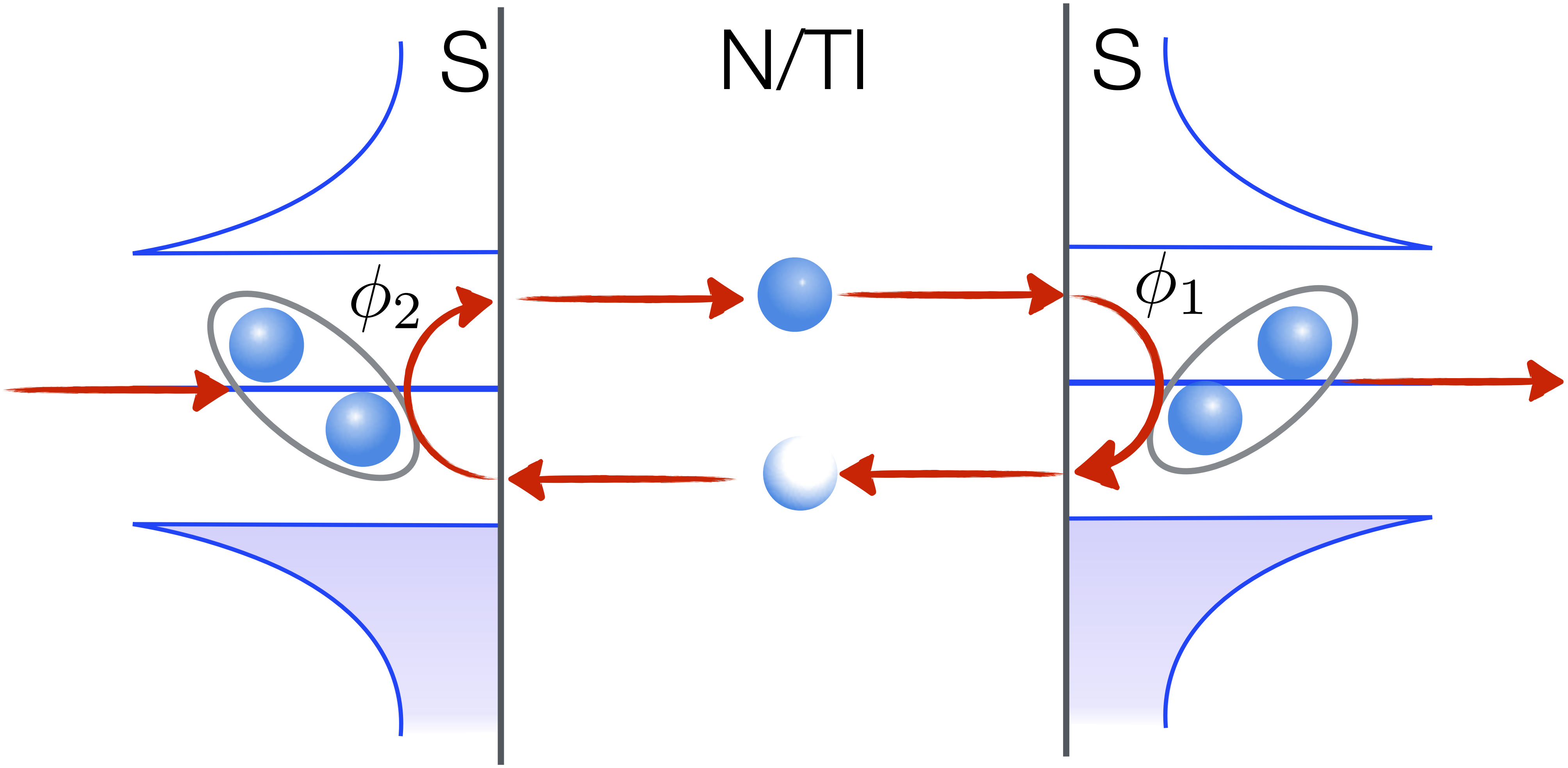
gapless Andreev bound states
⇒ Majoranas

Fu *et al.*, PRB **79**, 161408 (2009)

spin-orbit coupling
⇒ φ_0-junctions

Dolcini *et al.*, PRB **92**, 035428 (2015)

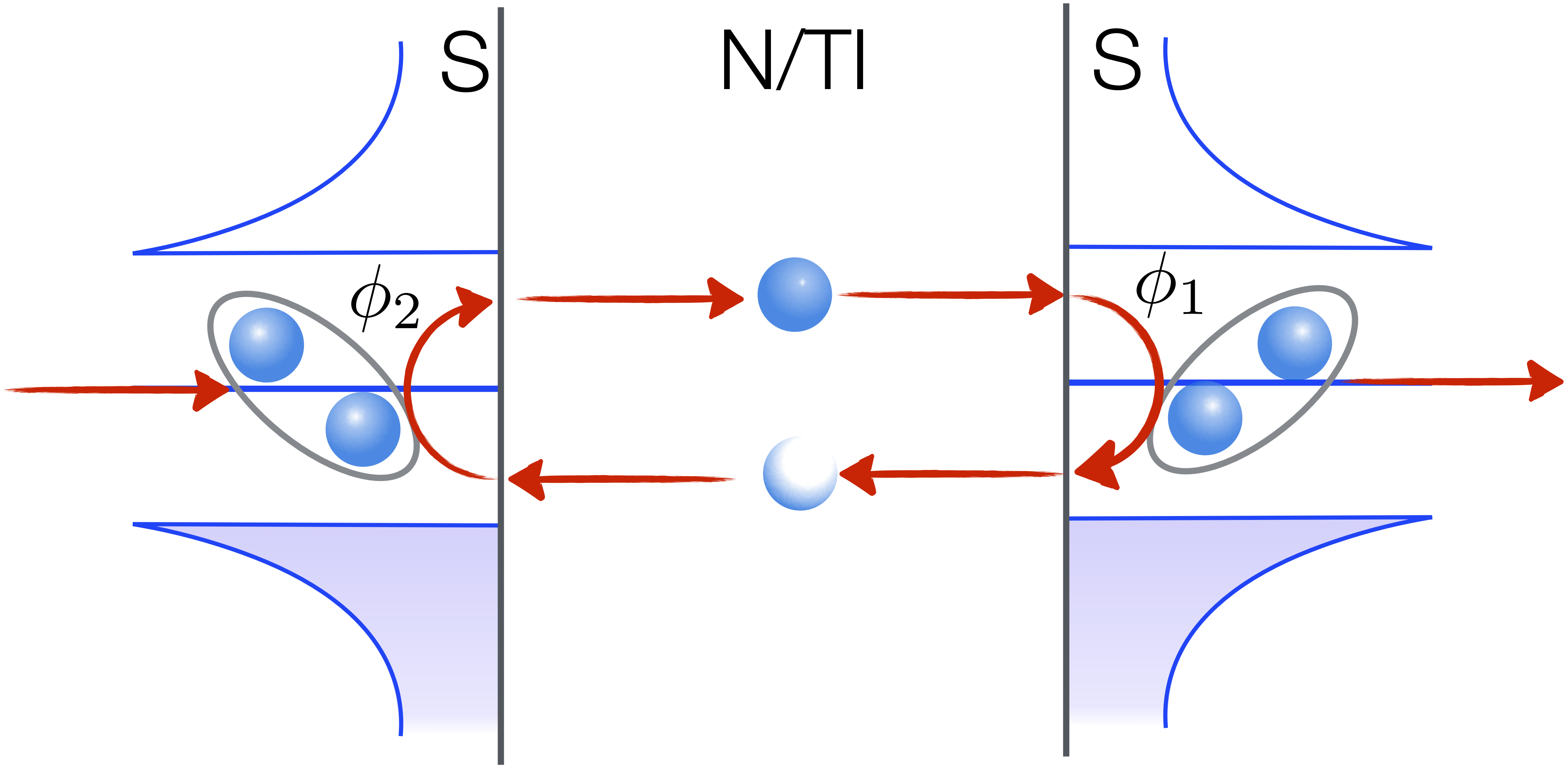
Andreev reflections



Andreev reflection

- ▹ Cooper pair created in S
- ▹ hole reflected in N
- ▹ phase coherence

Andreev reflections



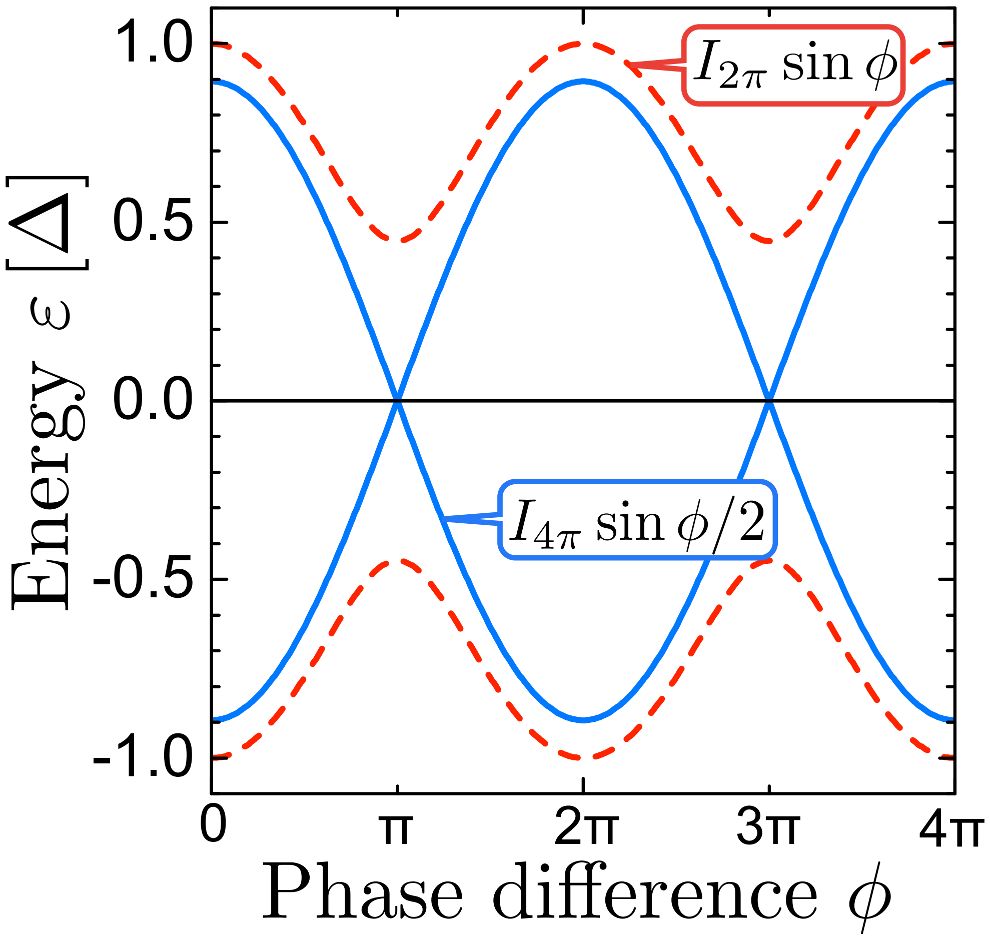
Andreev reflection

- ▹ Cooper pair created in S
- ▹ hole reflected in N
- ▹ phase coherence

Andreev bound states

- ▹ resonant modes of the ”cavity”
- ▹ energy levels $\varepsilon_n(\phi)$
  $\phi = \phi_1 - \phi_2$

Gapless Andreev bound states

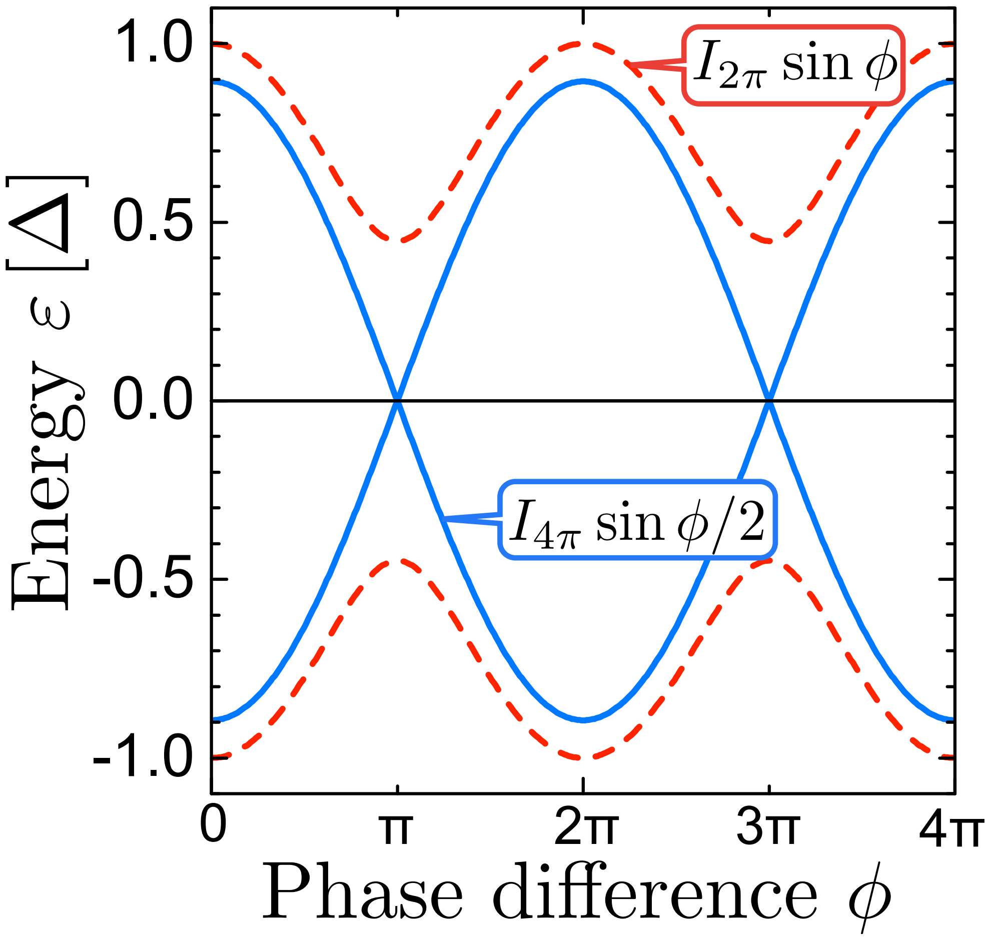


Andreev bound states

- 2π gapped states (bulk)
  $I_{2\pi} \sin\phi$ (+harmonics)
- 4π gapless topological state (edge)
  $I_{4\pi} \sin\phi/2$
- fractional Josephson effect

Kwon *et al.*, JLTP **30**, 613 (2004)
Fu *et al.*, PRB **79**, 161408 (2009)

Gapless Andreev bound states



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Kwon *et al.*, JLTP **30**, 613 (2004)
Fu *et al.*, PRB **79**, 161408 (2009)

Difficult detection

- ▹ 2π bulk states ⇒ 2π/4π mixture
- ▹ finite lifetime ⇒ 2π-periodicity restored
- ▹ interactions ⇒ 8π-periodicity
- ▹ Landau-Zener transitions ⇒ 4π-periodicity

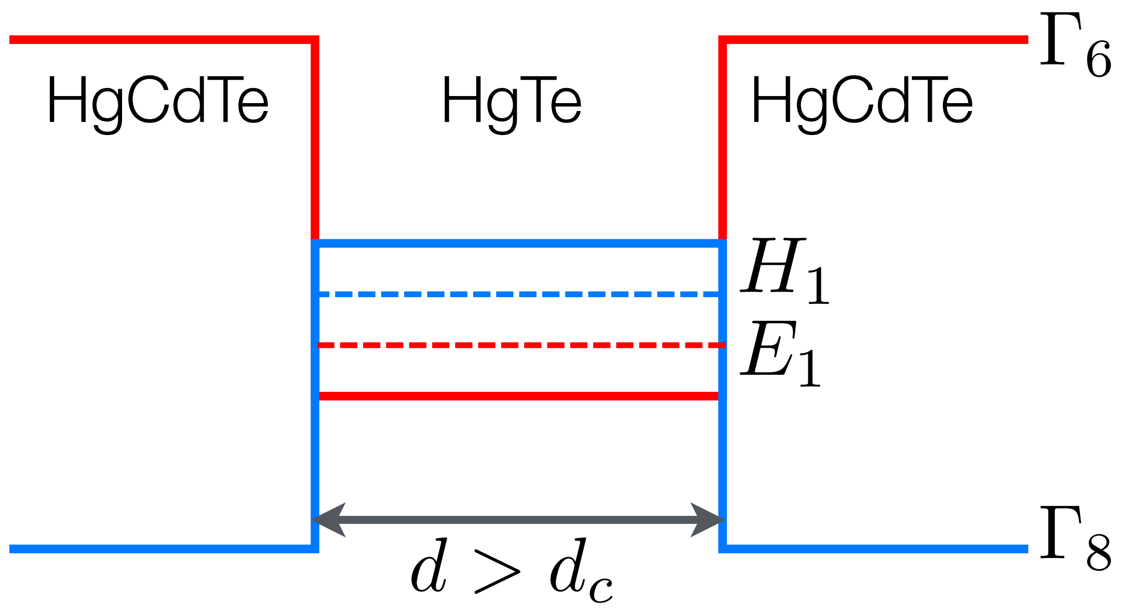
Pikulin *et al.*, PRB **86**, 140504 (2012)
Badiane *et al.*, CRP **14**, 840 (2013)
Zhang *et al.*, PRL **113**, 036401 (2014)
Peng *et al.*, ArXiv 1609.01896 (2016)
Hui *et al.*, ArXiv 1609.02909 (2016)

Outline

- ▹ Josephson junctions in a 2D TI
- ▹ Josephson emission
- ▹ Response to AC excitation (Shapiro steps)

Bocquillon *et al.*, Nat. Nano, DOI: 10.1038/NNANO.2016.159
Deacon *et al.*, ArXiv 1603.09611 (2016)

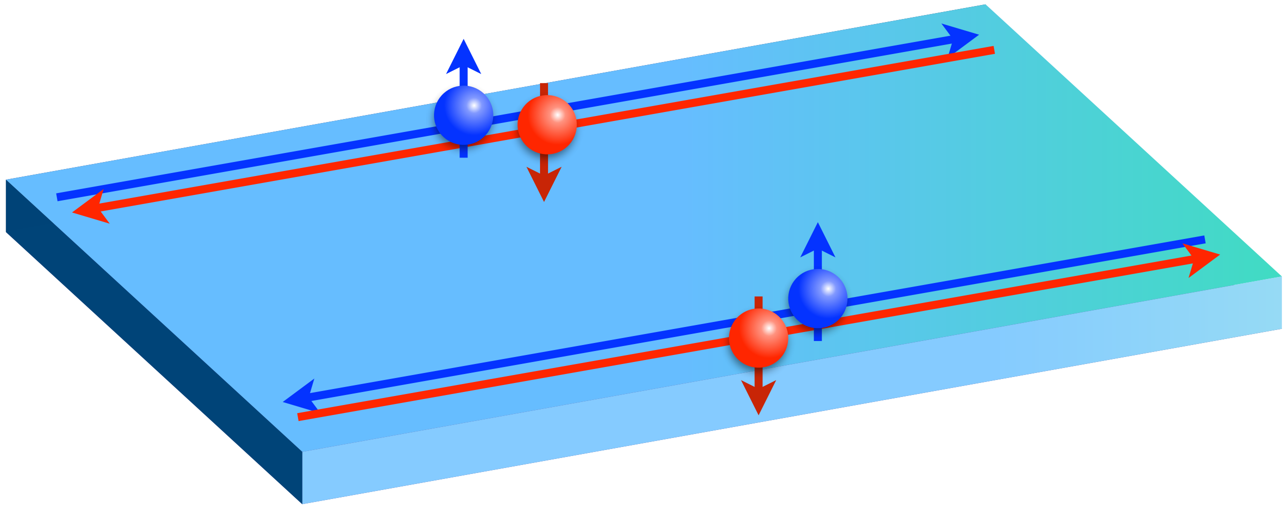
QSH effect in HgTe



- ▹ Band inversion in bulk HgTe

  ⇒ inverted QW

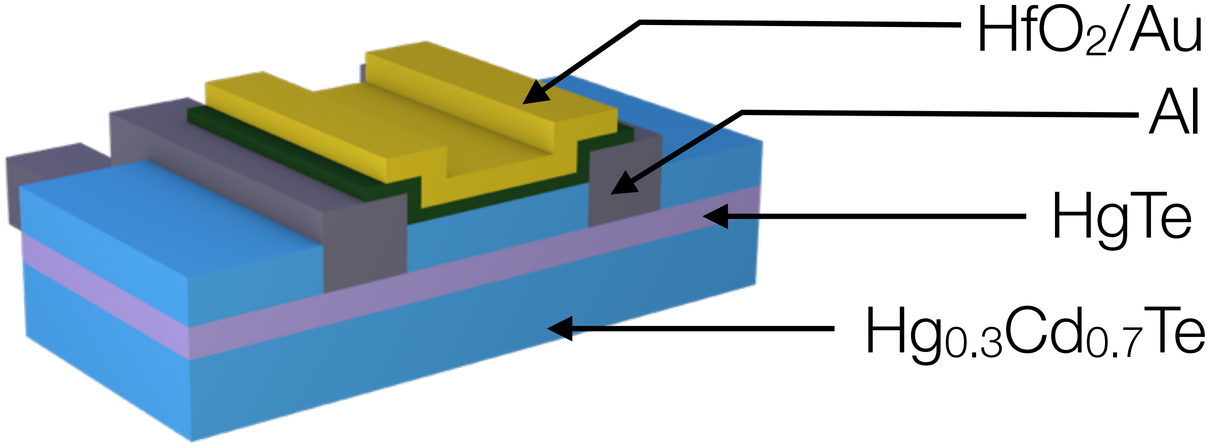
  ⇒ Quantum Spin Hall effect

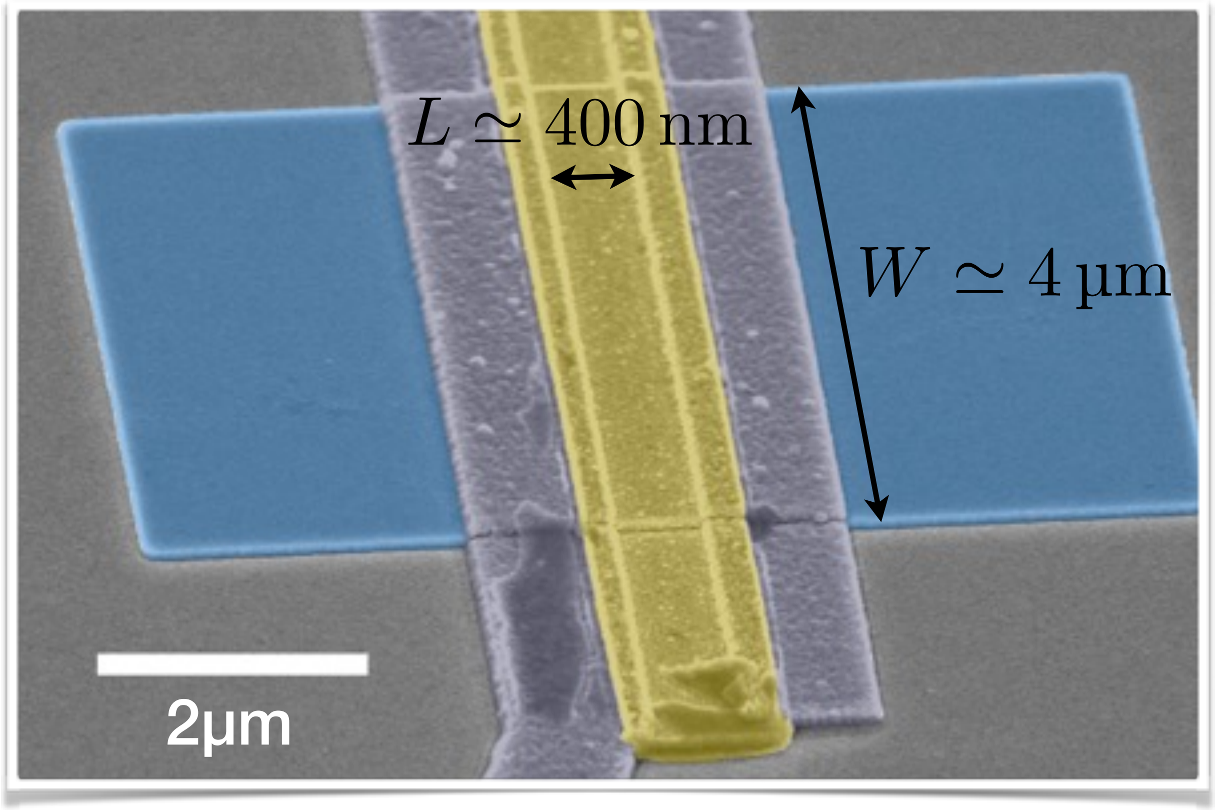


- ▹ MBE growth, μ ≃ 3 10^5 $cm^2V^{-1}s^{-1}$
- ▹ QSH if $d > d_c \simeq 6.3$ nm at $B = 0$!
- ▹ trivial if $d < d_c$

Bernevig *et al.*, Science **314**, 1757 (2006)
König *et al.*, Science **318**, 766 (2007)

Quantum spin Hall junctions



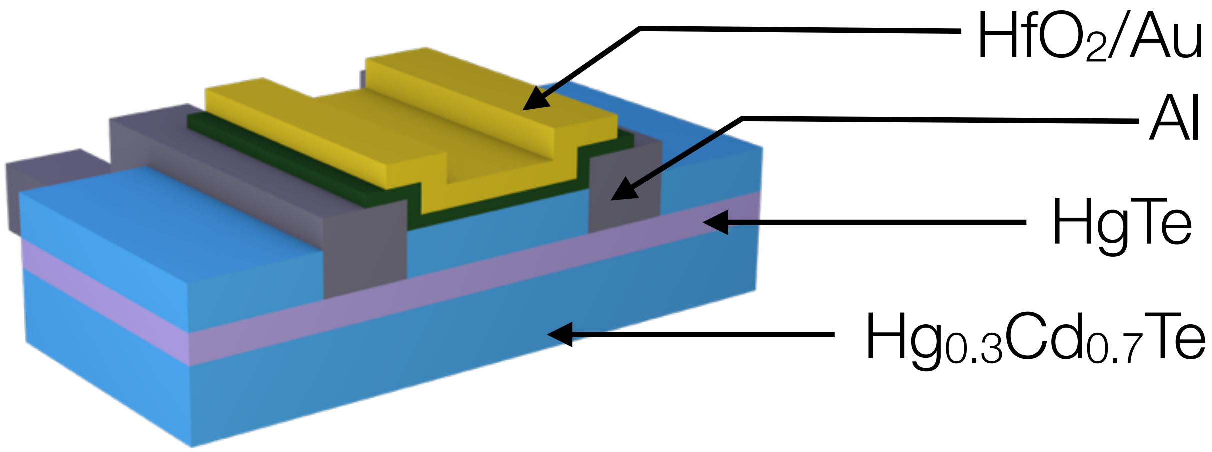


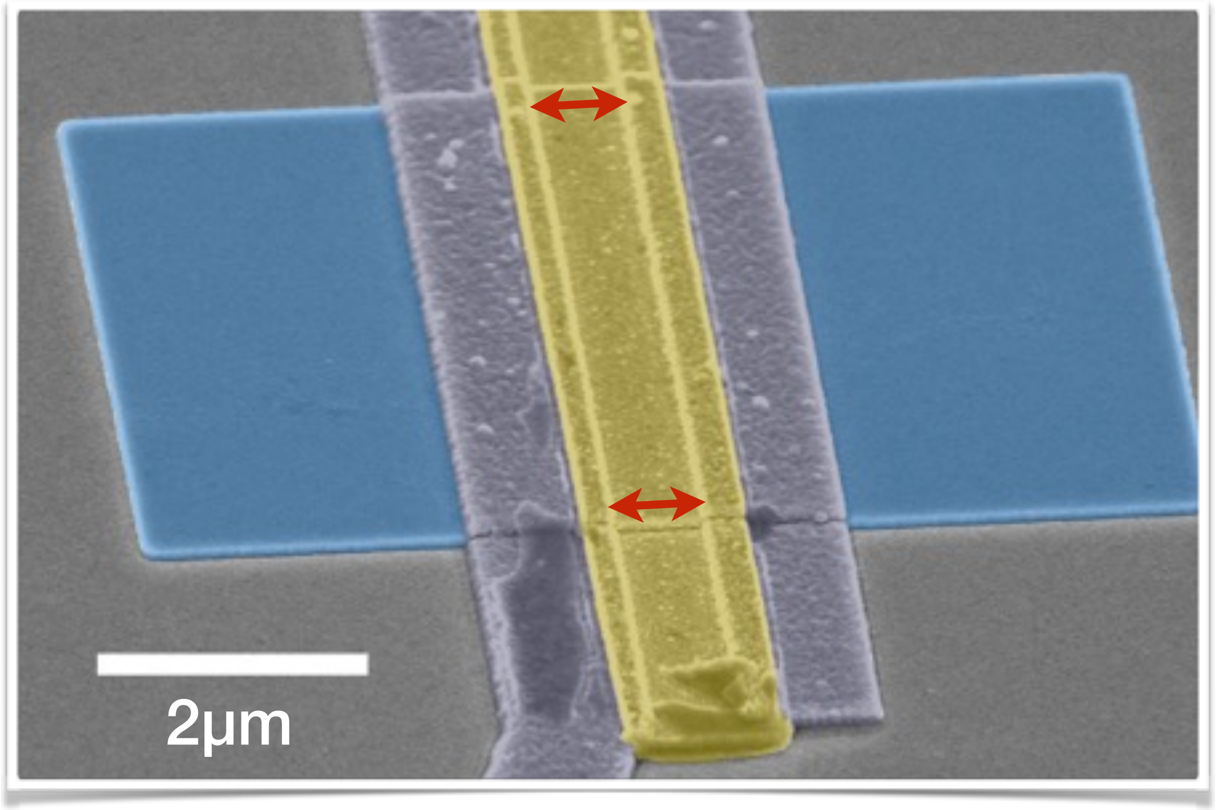
Josephson junctions

- ▹ $\mu \simeq 3\ 10^5\ \mathrm{cm^2V^{-1}s^{-1}}$
- ▹ Al contacts (in situ)
- ▹ HfO_2 /Au gate
- ▹ no overlap of edge states
- ▹ ballistic / intermediate

$$L \ll l \qquad L \lesssim \xi$$

Quantum spin Hall junctions





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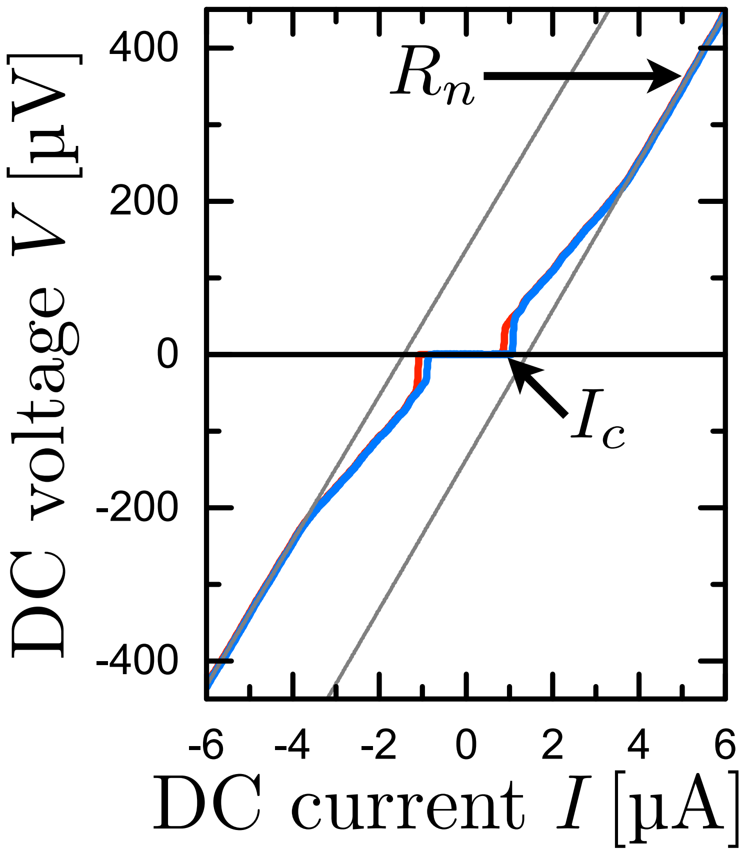
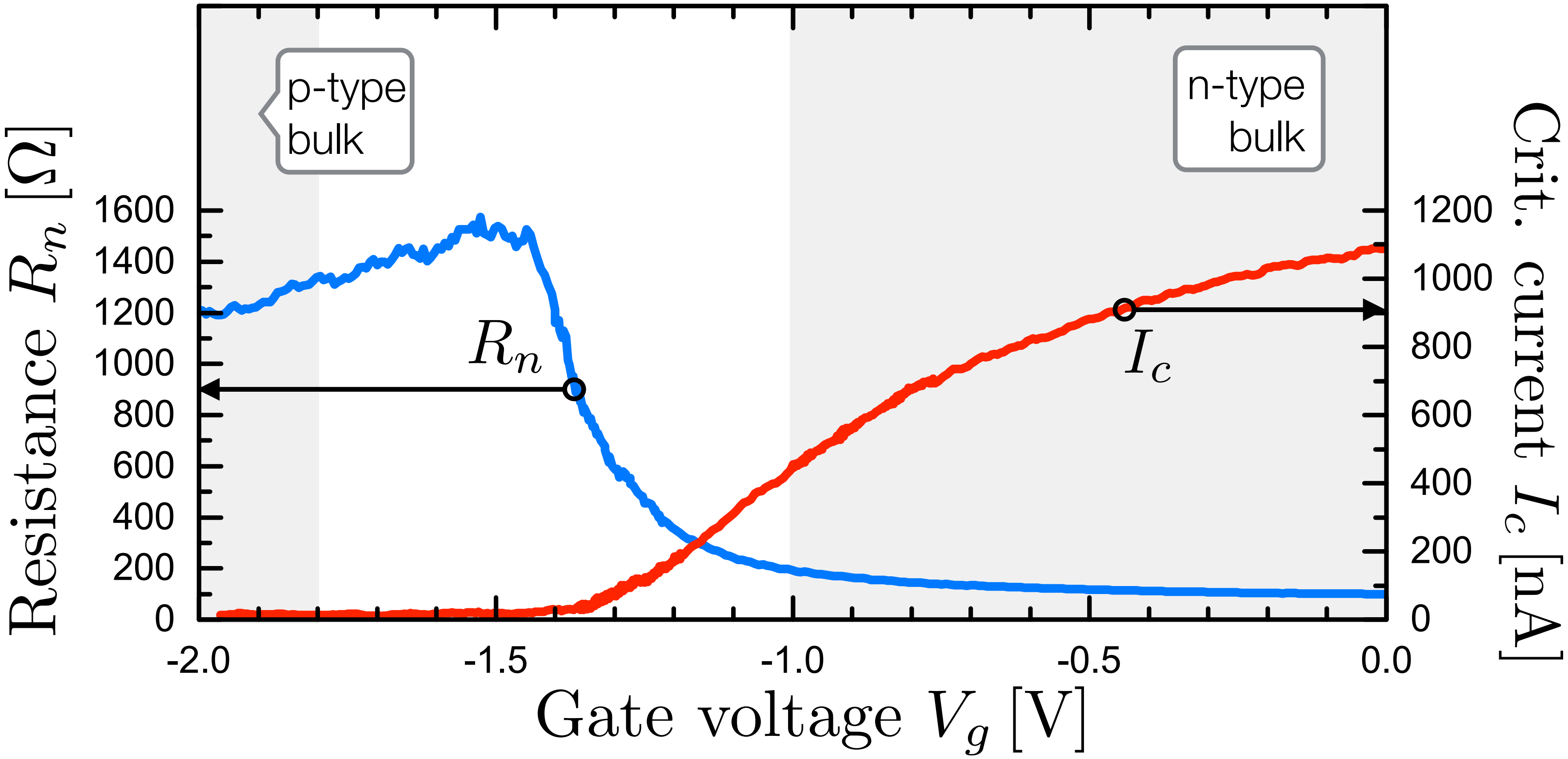
$$L \ll l \qquad L \lesssim \xi$$

First properties





I-V curve

- ▹ weak hysteresis visible
- ▹ excess current
  ⇒ Andreev reflections

Gate dependence

- ▹ 3 regimes : *p*, *n*, and QSH
- ▹ asymmetry between *n* and *p*

Blonder *et al.*, PRB **25**, 4515 (1982)

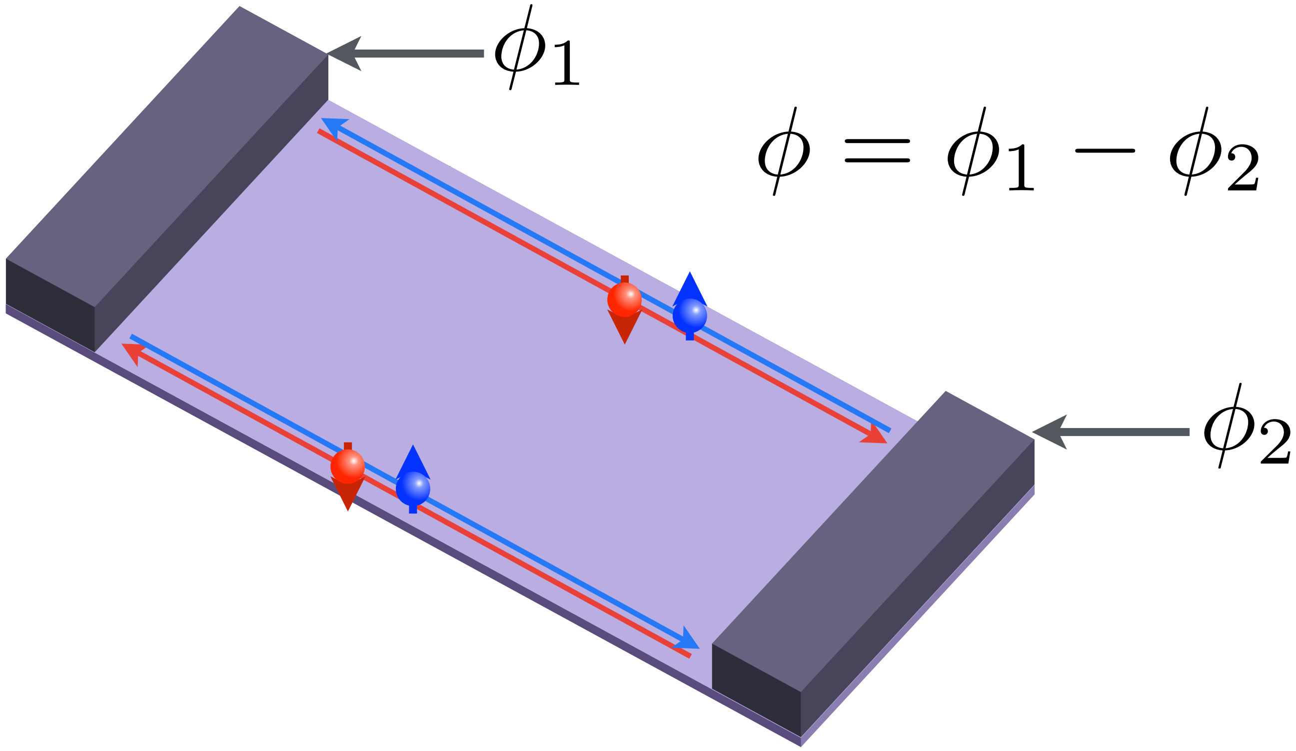
(Fractional) Josephson effect

Josephson equations

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$

$$I_S(\phi) = I_c \sin\phi$$

⇒ Josephson frequency $f_{\mathrm{J}} = \frac{2eV}{\hbar}$



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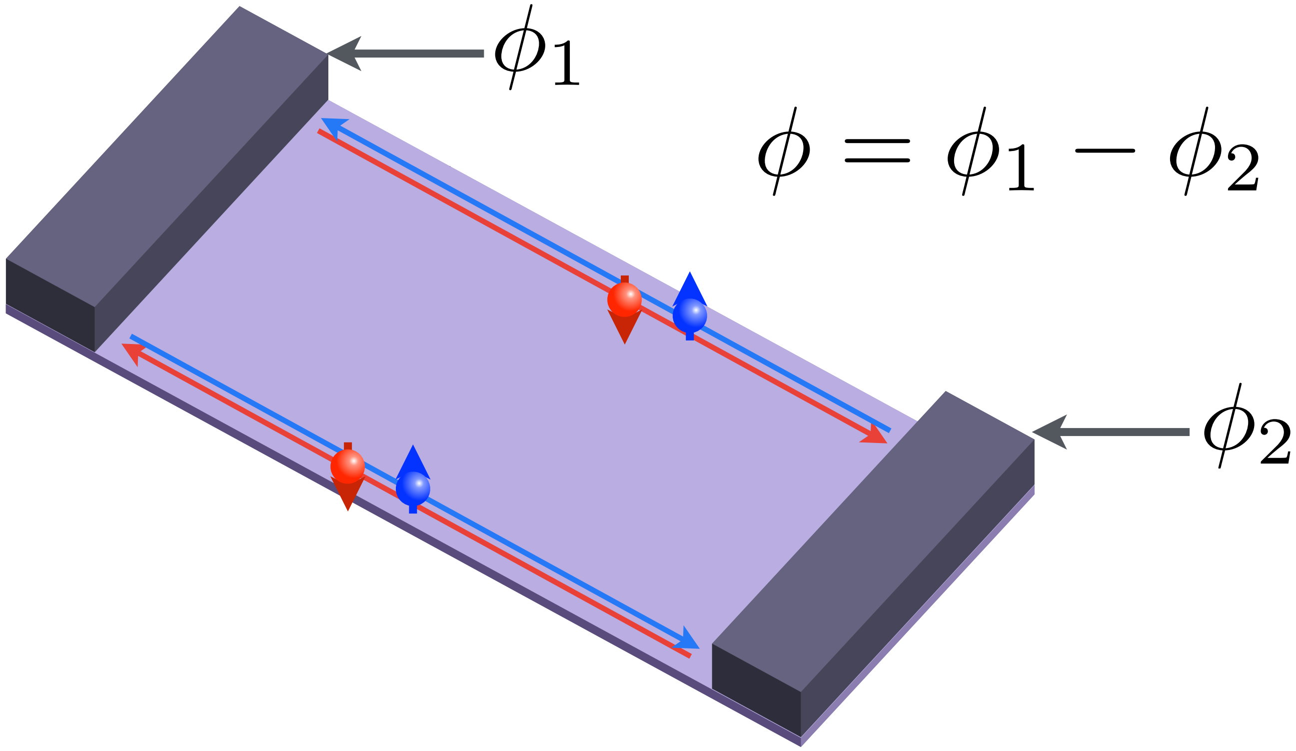

Fractional Josephson effect

$$\sin\phi \rightarrow \sin\phi/2$$

$$f_\mathrm{J} \rightarrow f_\mathrm{J}/2$$

(Fractional) Josephson effect

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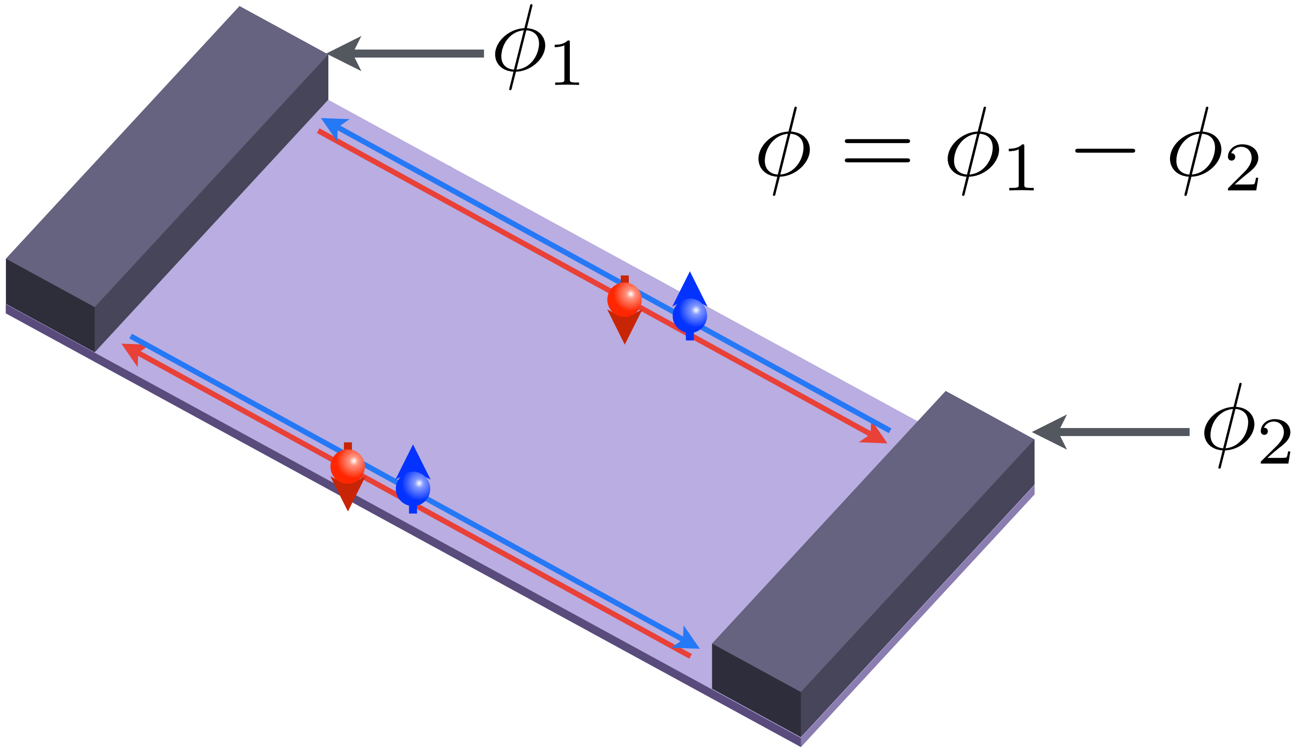


Fractional Josephson effect

$$\sin\phi \to \sin\phi/2$$

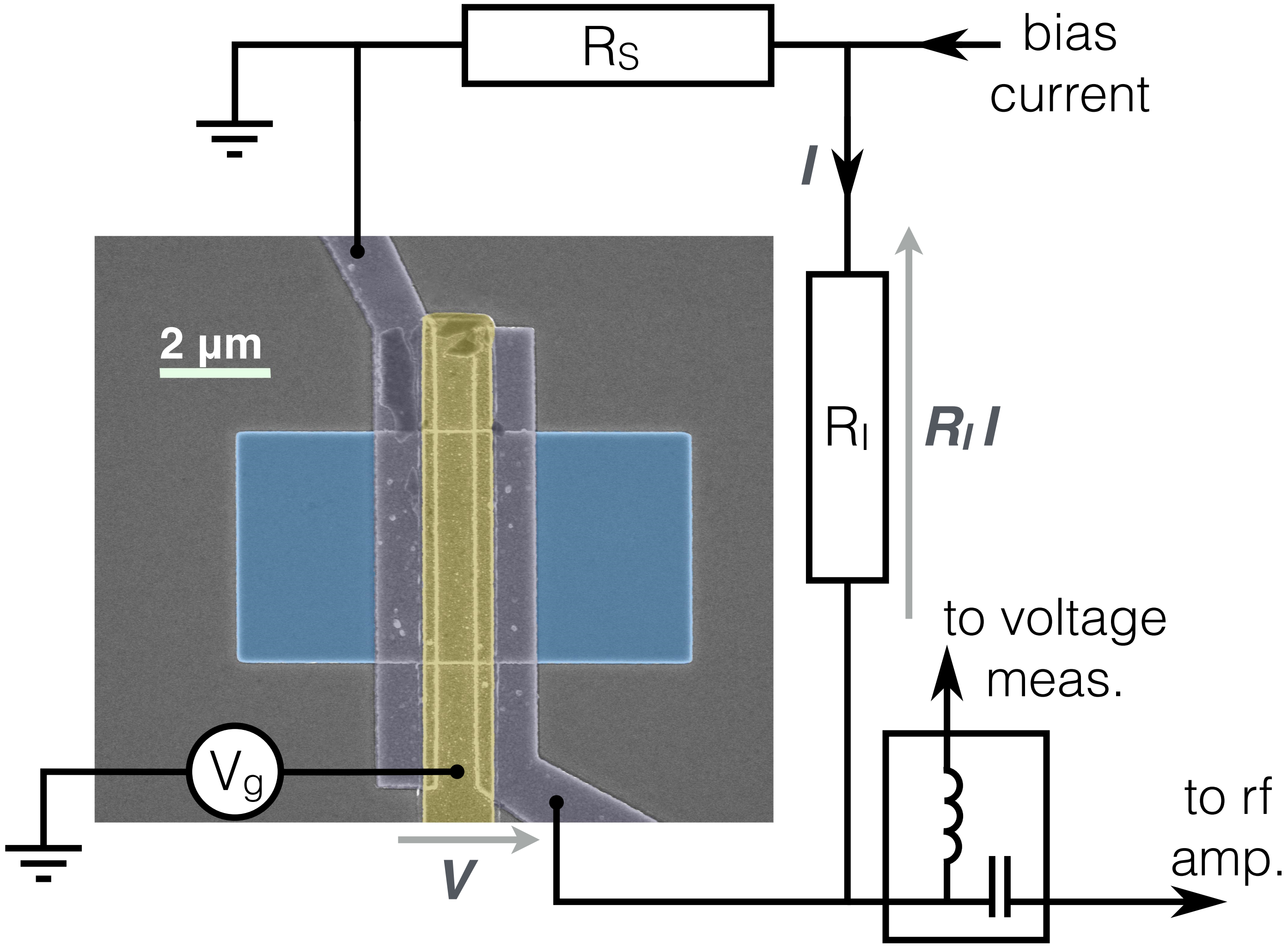
$$f_\mathrm{J} \to f_\mathrm{J}/2$$

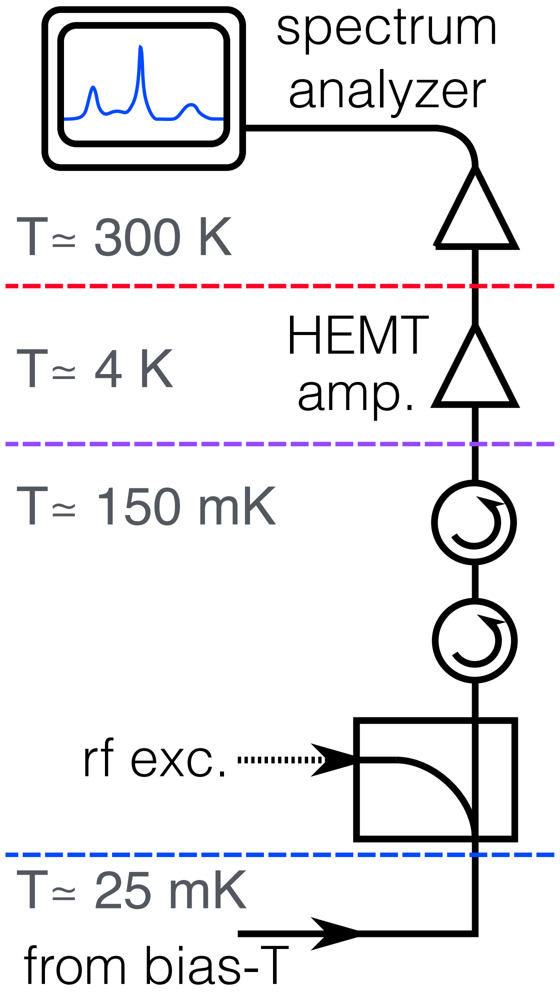
Detection

- ‘listening’ to Josephson emission
- beatings with ac excitation (Shapiro steps)

Detection setup

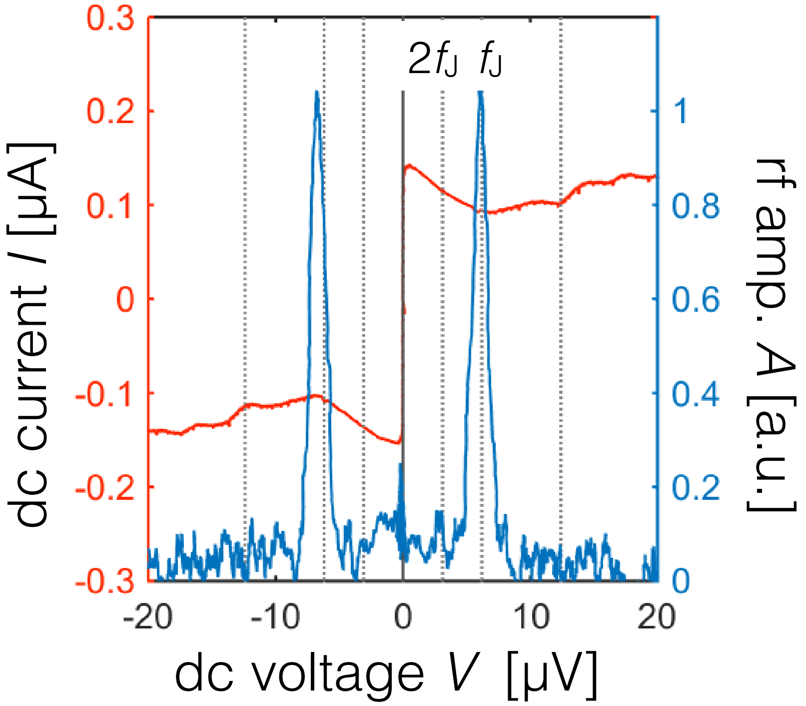
- ▹ voltage bias
  - shunt resistance R_S
  - current resistance R
- ▹ rf amplification setup
  - 1 cryo amp. (+ 2 amps at RT)
  - 0.1 fW (-130 dBm) in 8 MHz



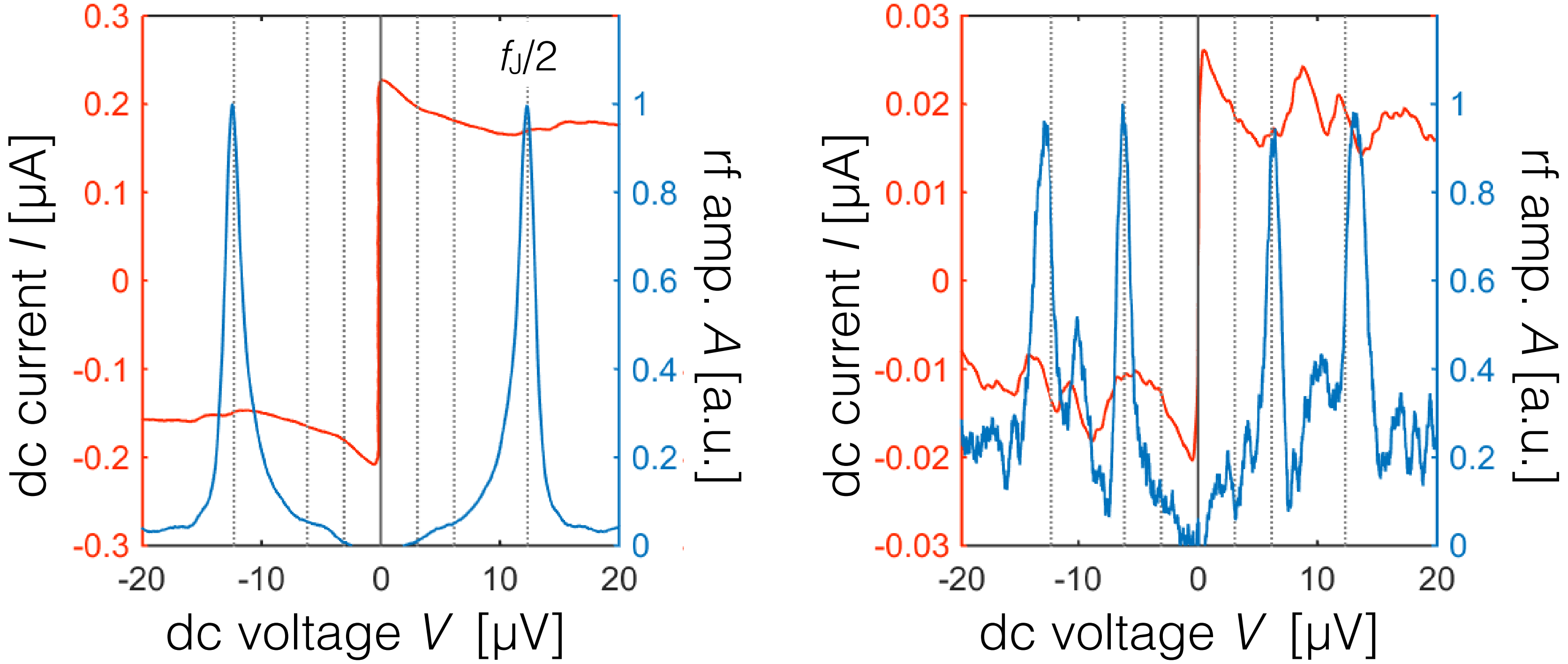


Emission spectra

Trivial QW



Topological QW



n- and QSH regime

p-regime

- voltage *V* swept
- integrated power at f_d=3 GHz (in 8 MHz bandwidth)

- trivial QW : signal at $f_d=f_J$
- topological QW : at $f_d=f_J$ and $f_J/2$

Deacon *et al.*, submitted, ArXiv 1603.09611 (2016)

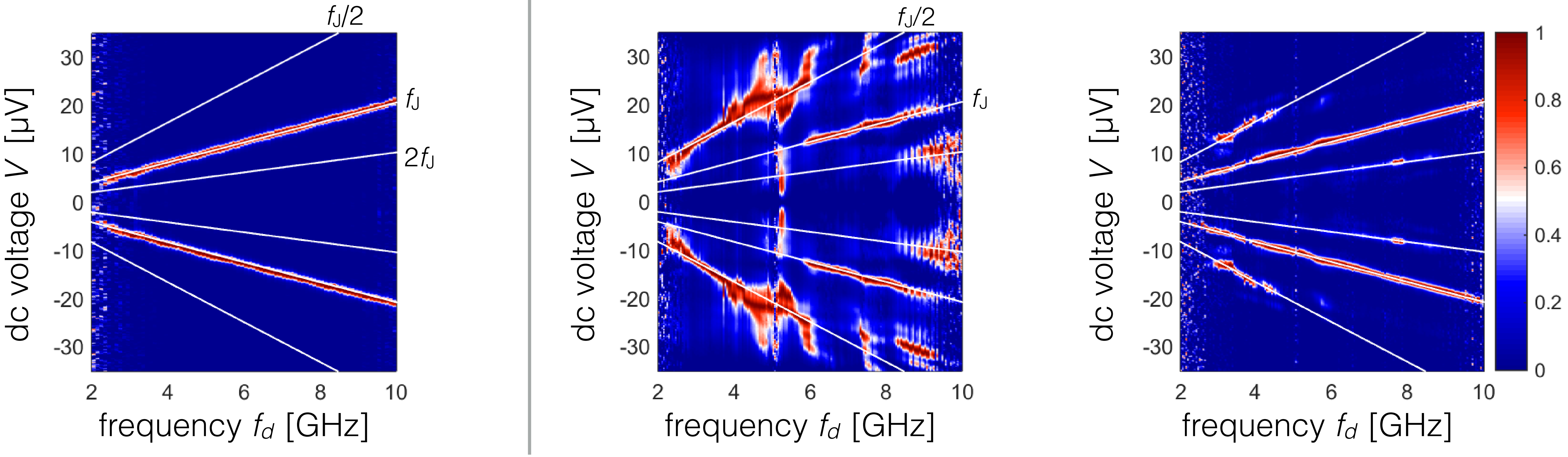
Frequency dependence

Trivial QW

Topological QW

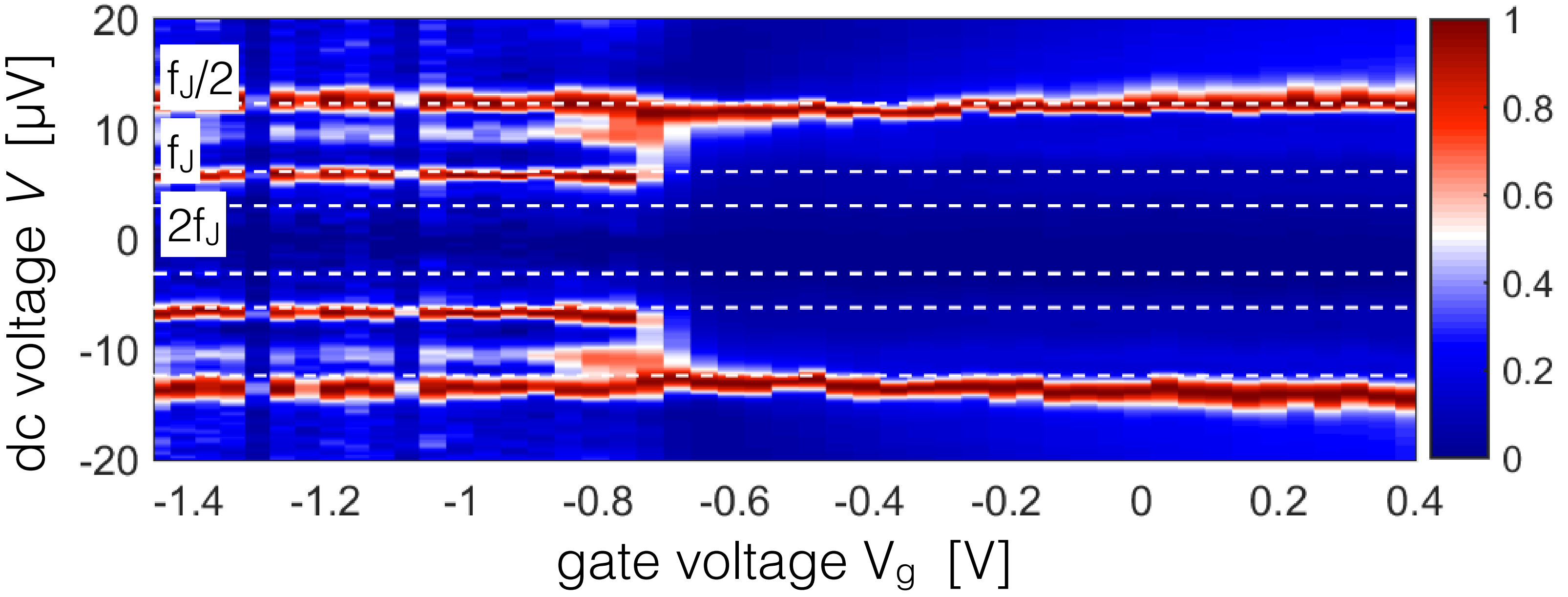
n- and QSH regime

p-regime



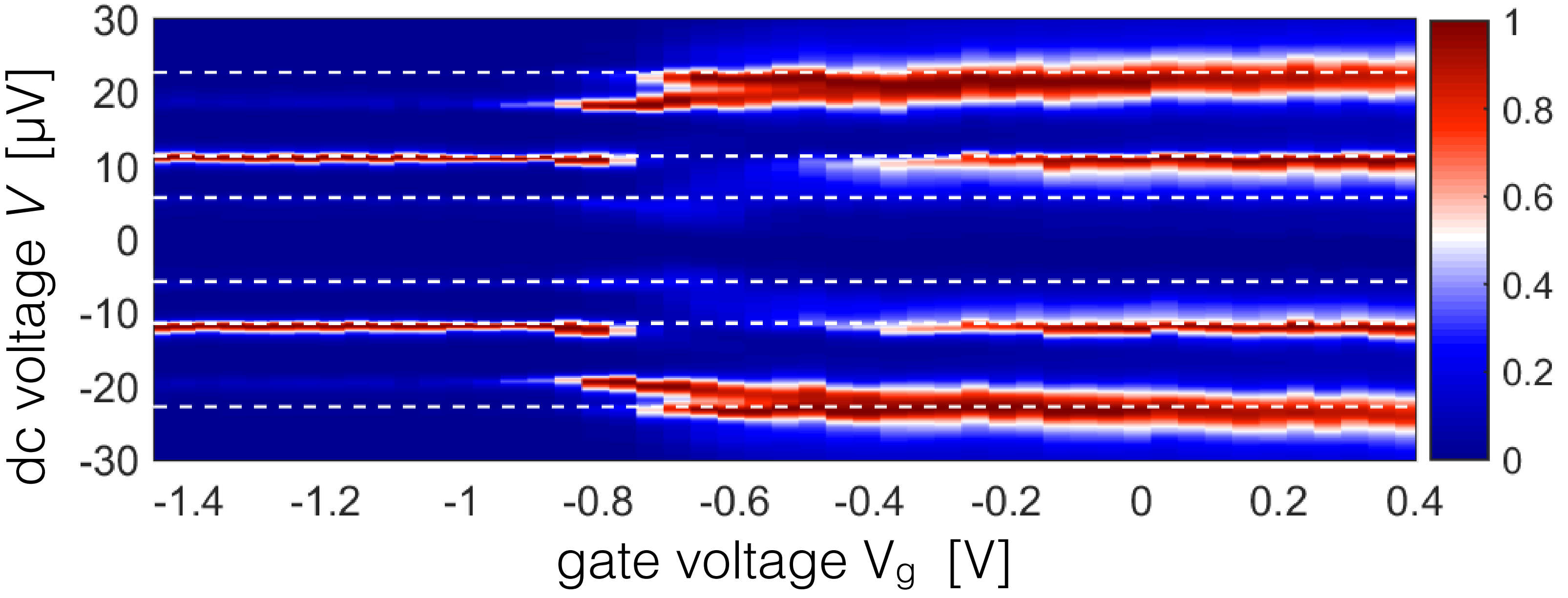
- Stronger $f_J/2$ signal at low frequencies
- Relative intensities of $f_J/2$ and f_J depending on V_g

Gate voltage dependence



Low frequency

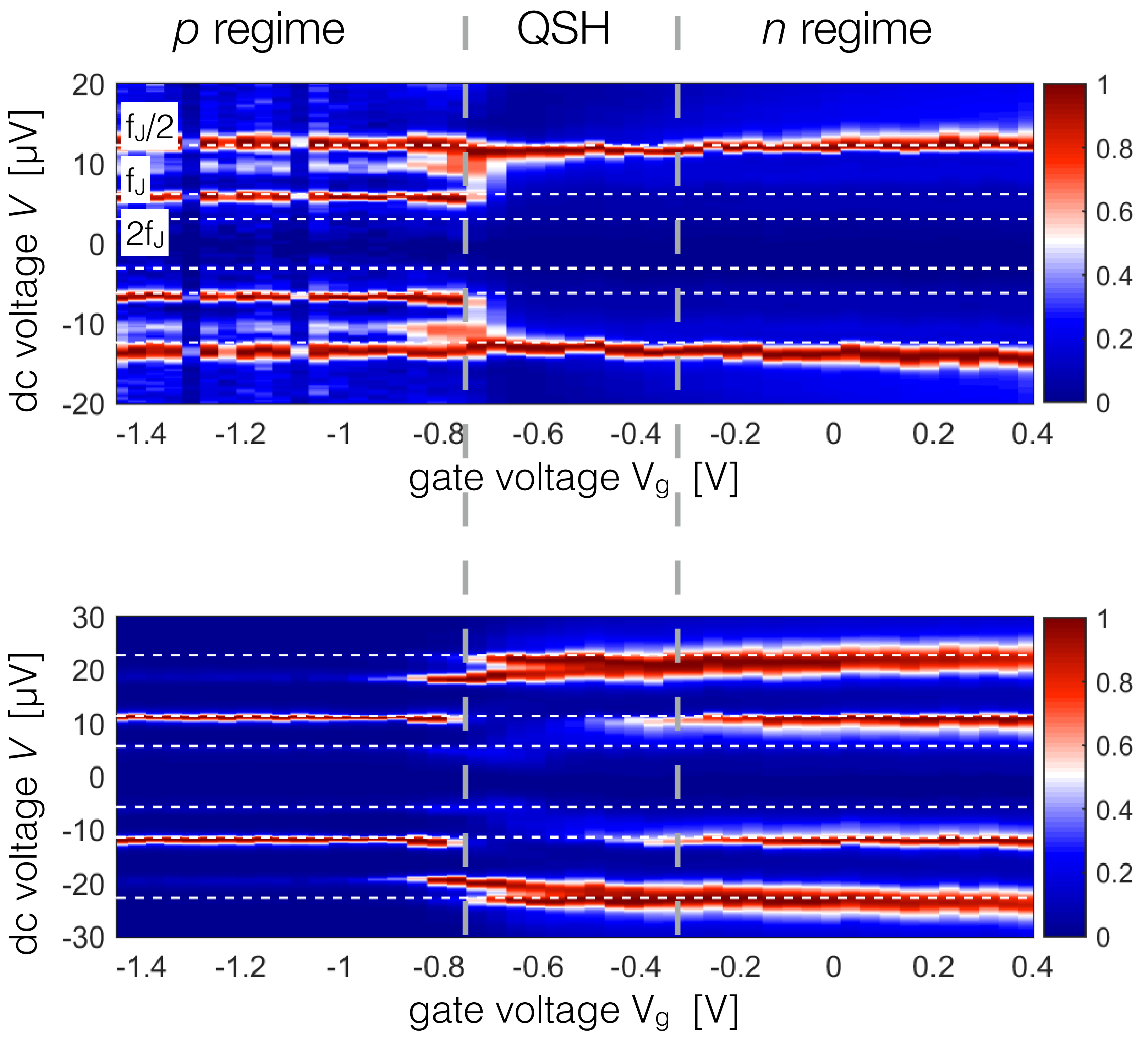
f_d= 3 GHz



High frequency

f_d= 5.5 GHz

Gate voltage dependence



Low frequency

f_d= 3 GHz

High frequency

f_d= 5.5 GHz

AC response : Shapiro steps

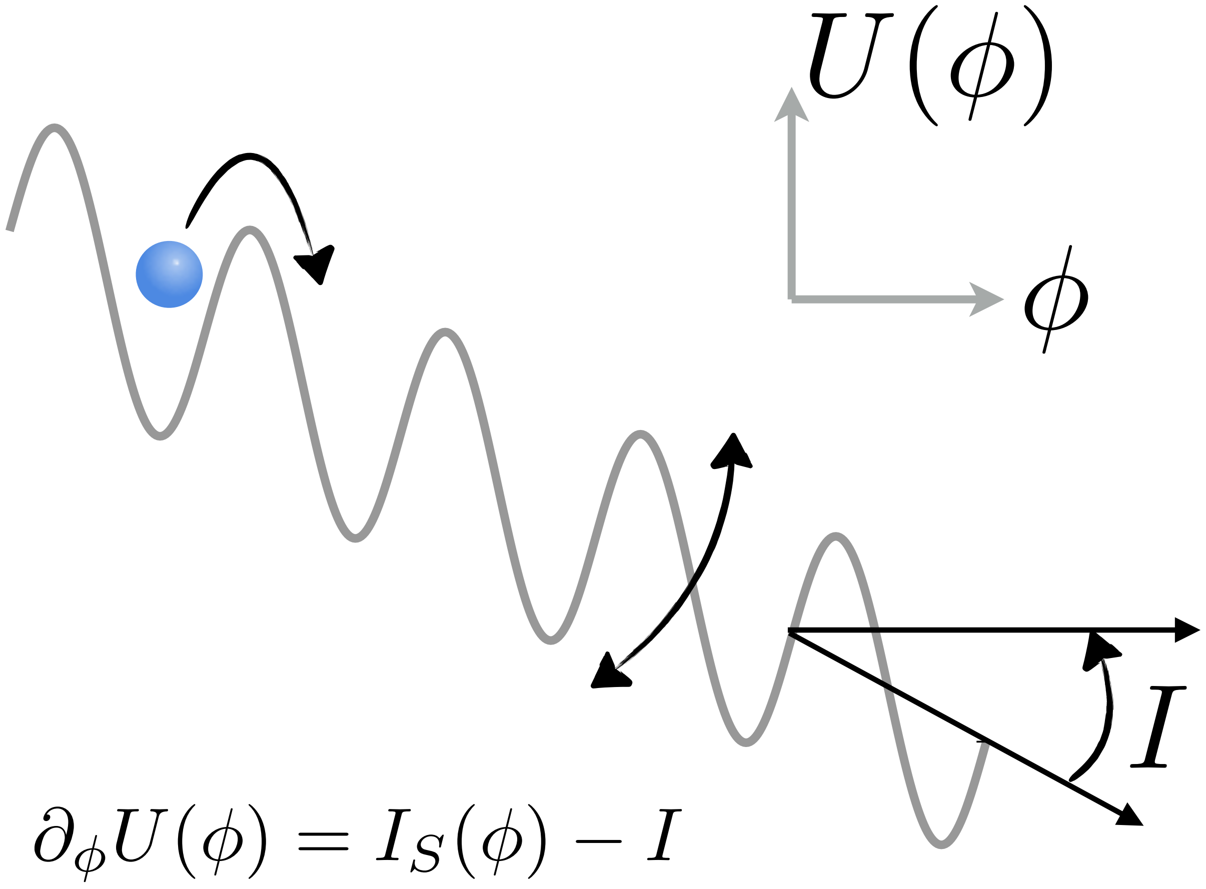
Phase-locked motion

- phase dynamics (RSJ model)

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar}$$
$$I = I_S(\phi) + \frac{\hbar}{2eR}\dot{\phi}$$

- motion locked to rf excitation

$$\frac{\Delta\phi}{\Delta t} = \frac{2\pi n}{1/f} \Rightarrow V_n = n\frac{hf}{2e}$$



Shapiro, PRL **11**, 80 (1963)
Russer, J. App. Phys. **43**, 2008 (1972)

AC response : Shapiro steps

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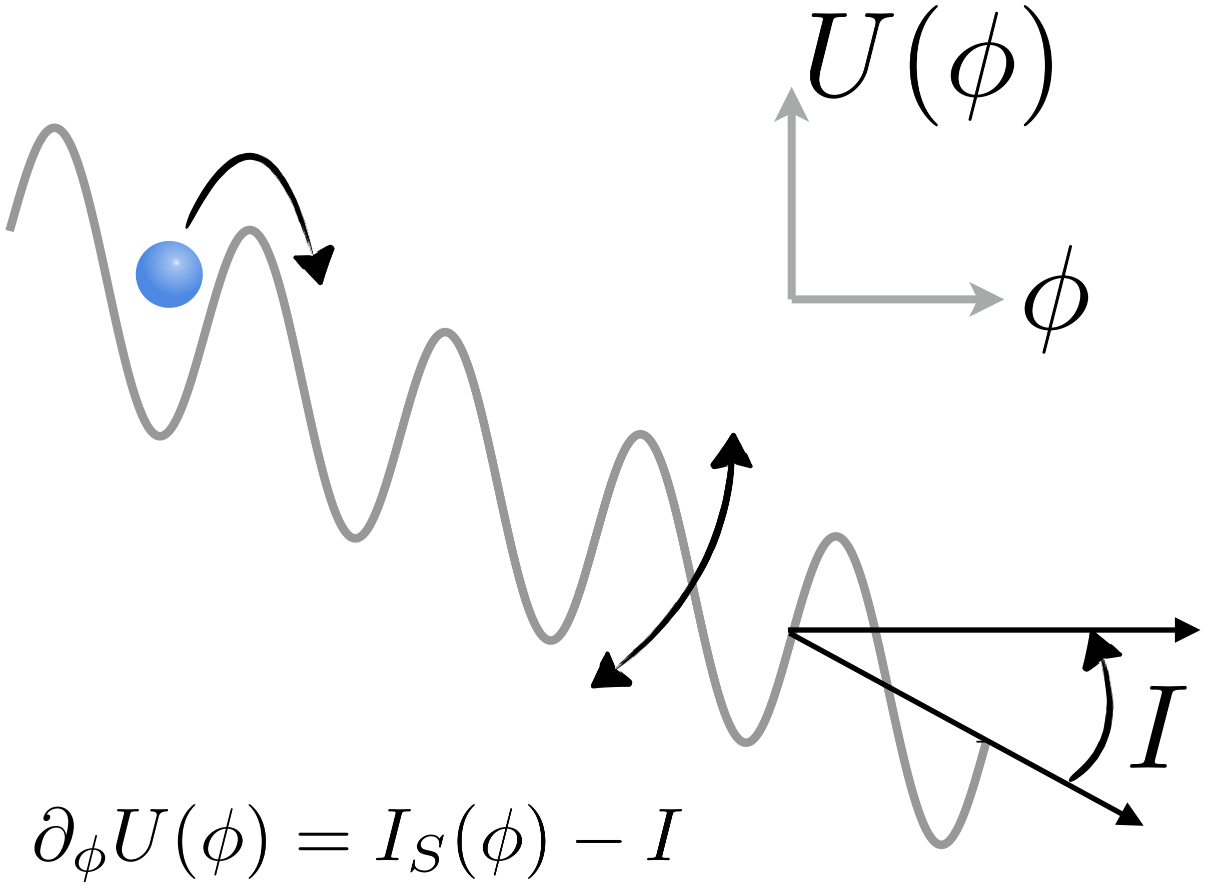

$$\partial_\phi U(\phi) = I_S(\phi) - I$$

Shapiro, PRL **11**, 80 (1963)
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AC response : Shapiro steps

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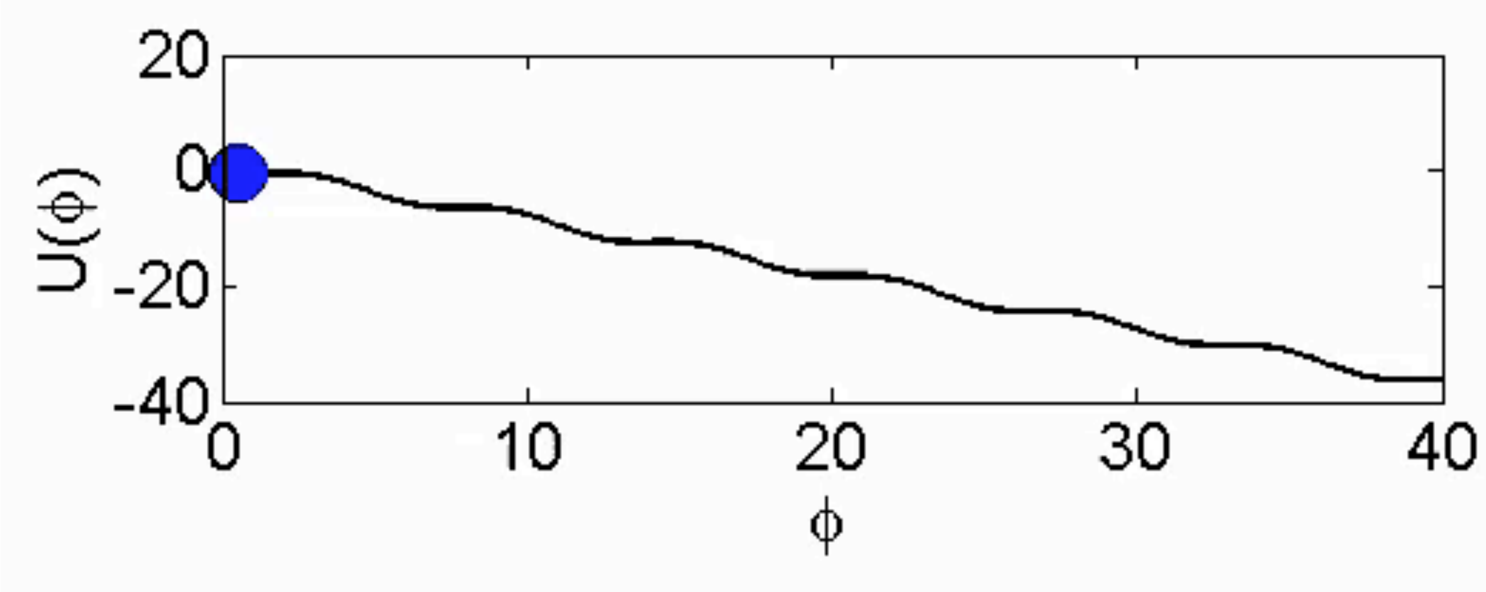

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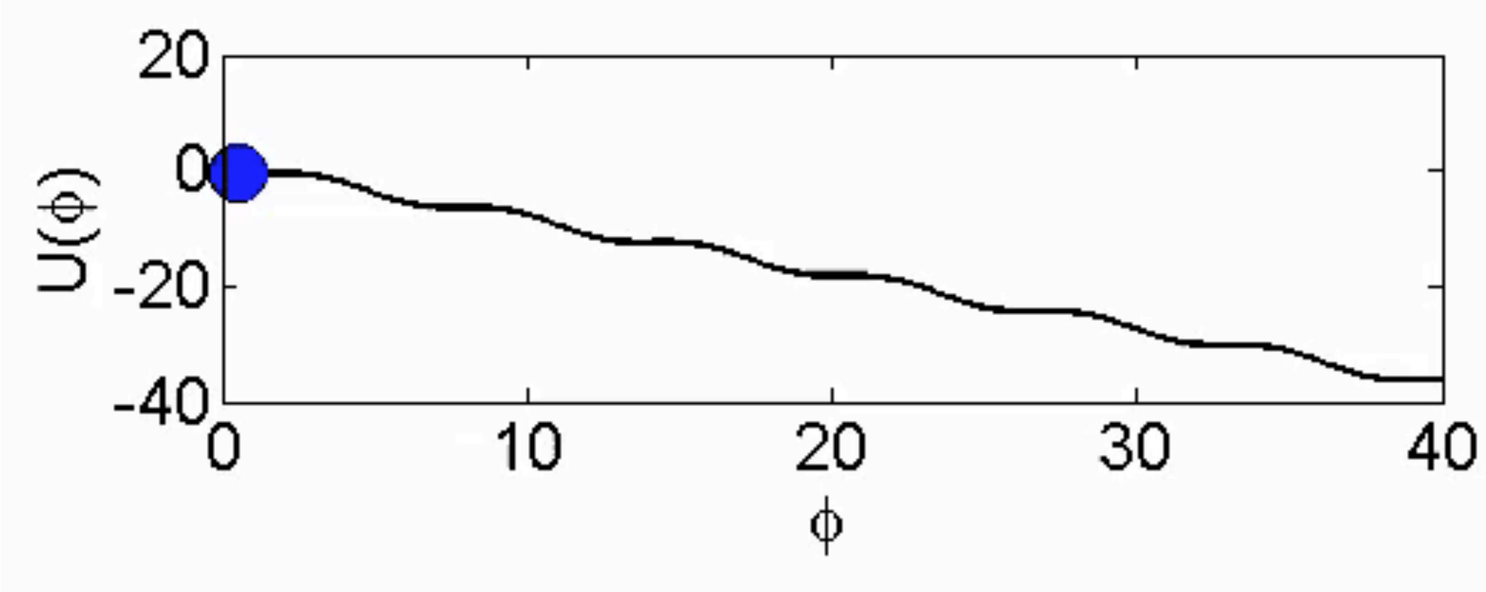


4π-periodic supercurrent

- doubled steps

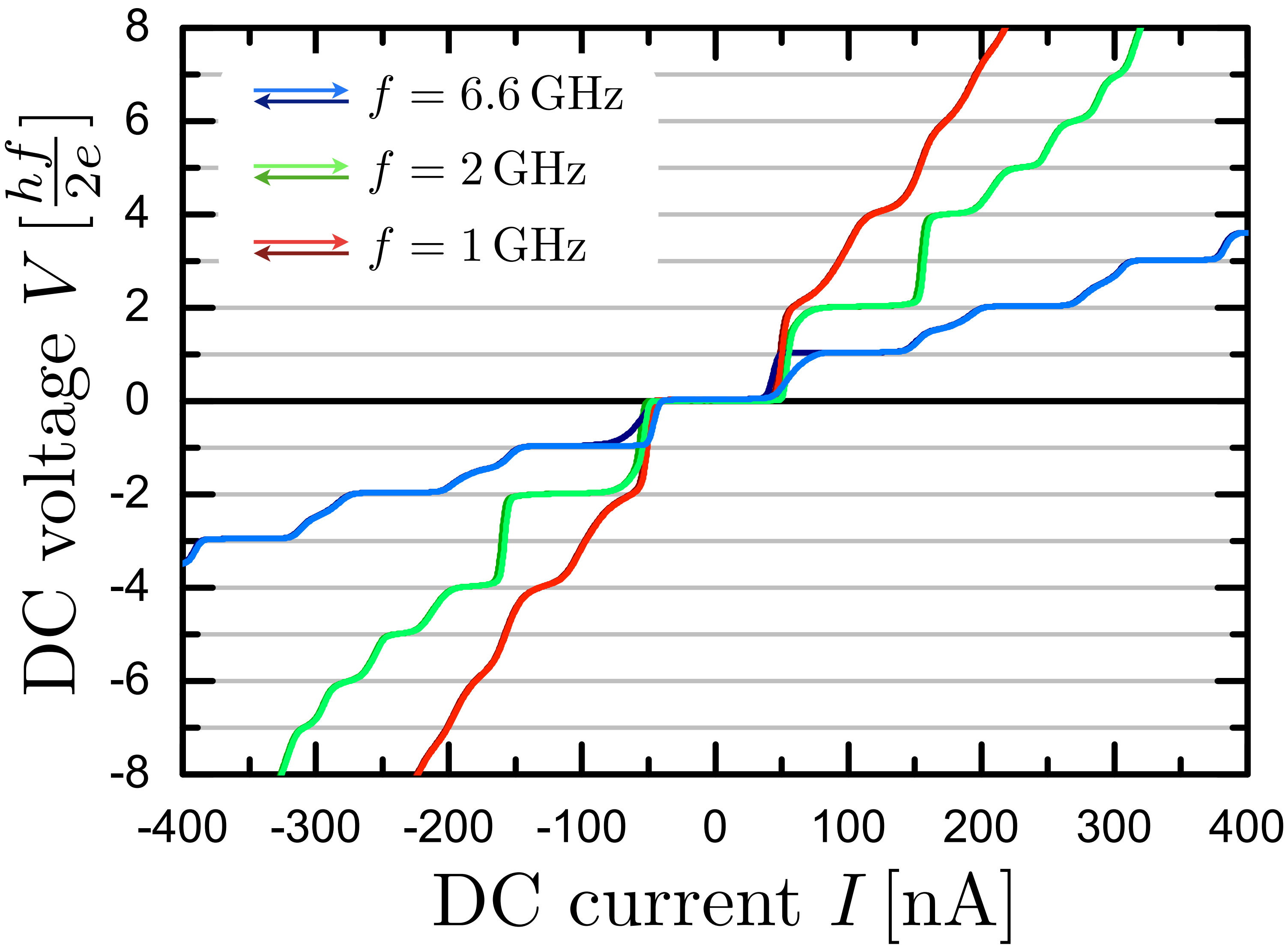
$$\sin\phi \rightarrow \sin\phi/2$$

$$V_n \rightarrow V_{2n}$$

- mixture 2π/4π ?

Shapiro, PRL **11**, 80 (1963)
Russer, J. App. Phys. **43**, 2008 (1972)

Shapiro response : frequency



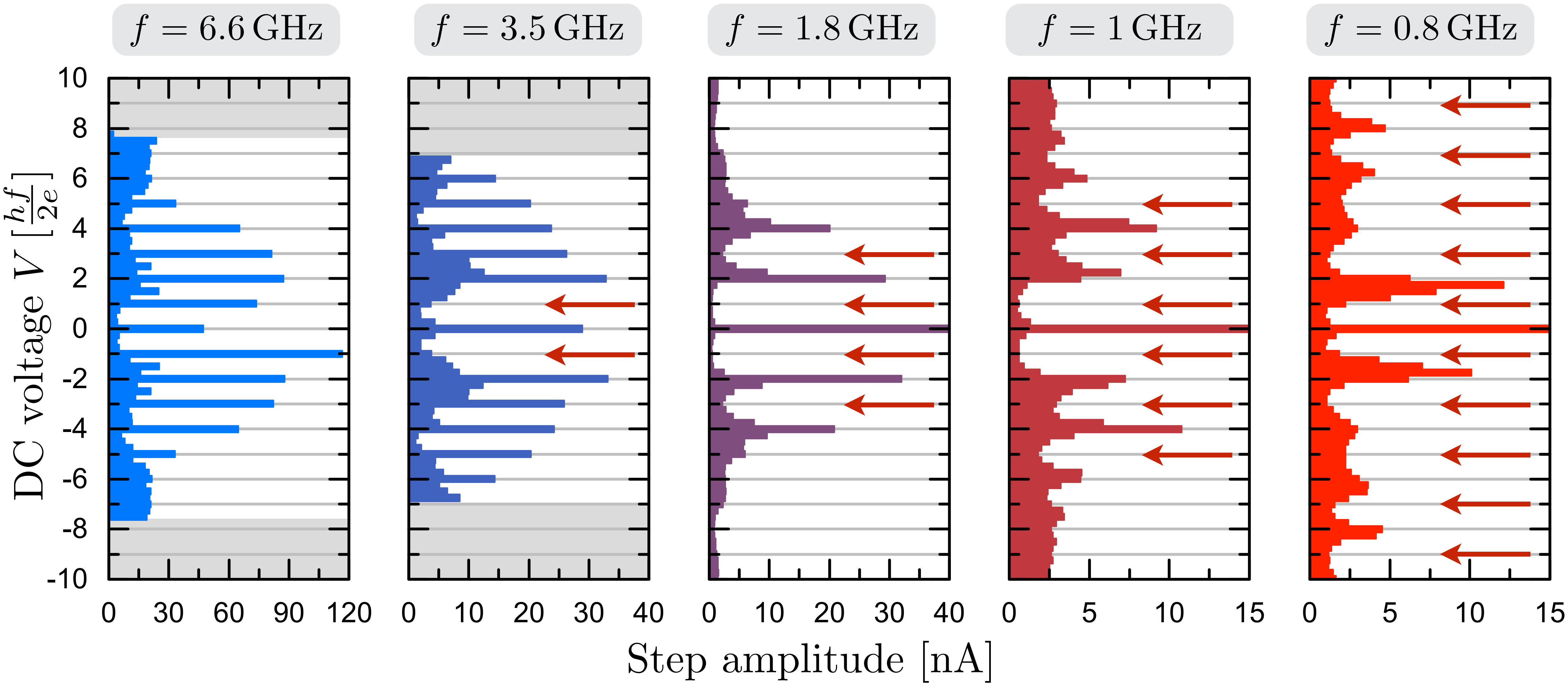
Shapiro response

- >12 steps visible
- weak hysteresis on 1st step

- multiple odd steps missing
- at low frequency f ≲ 4 GHz

Rokhinson *et al.*, Nat. Phys. **86**, 146503 (2012)
Wiedenmann *et al.*, Nat. Comms **7**, 10303 (2016)
Bocquillon *et al.*, Nat. Nano, DOI: 10.1038/NNANO.2016.159

Shapiro response : frequency



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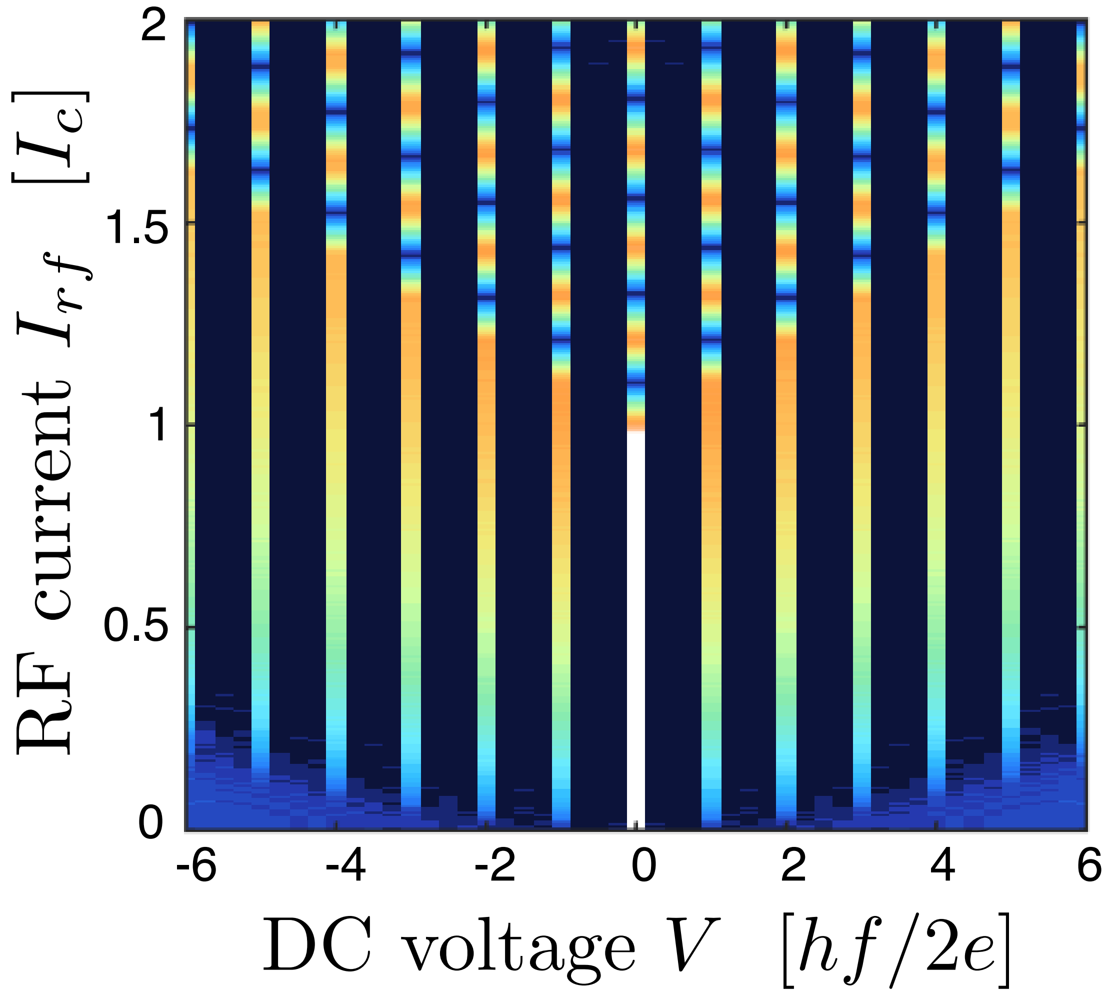
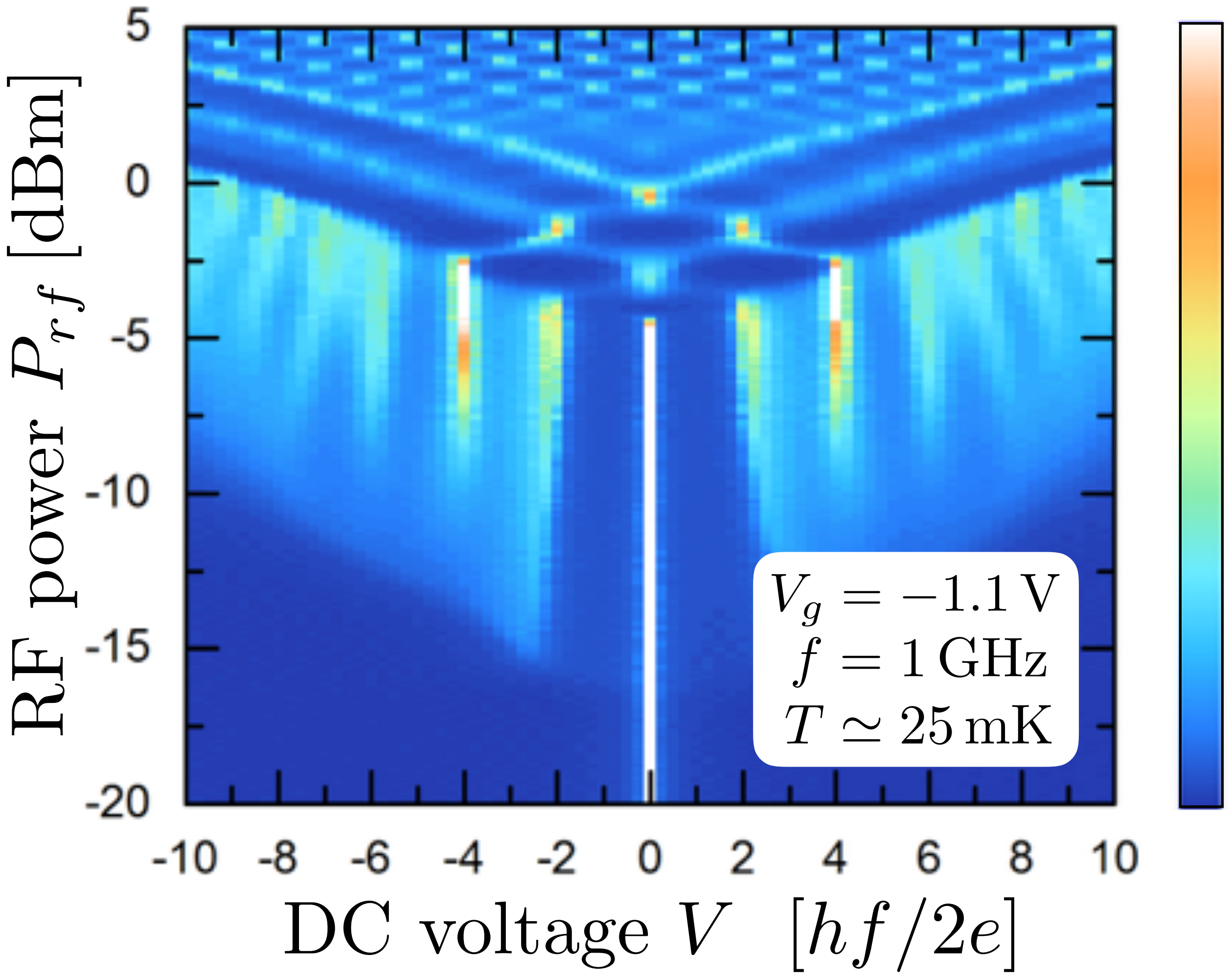
Shapiro response : power





Simulated response

- at low power : steps forming
- at high power : oscillatory pattern

Our device

- missing n=1,3,5
- « dark fringes »

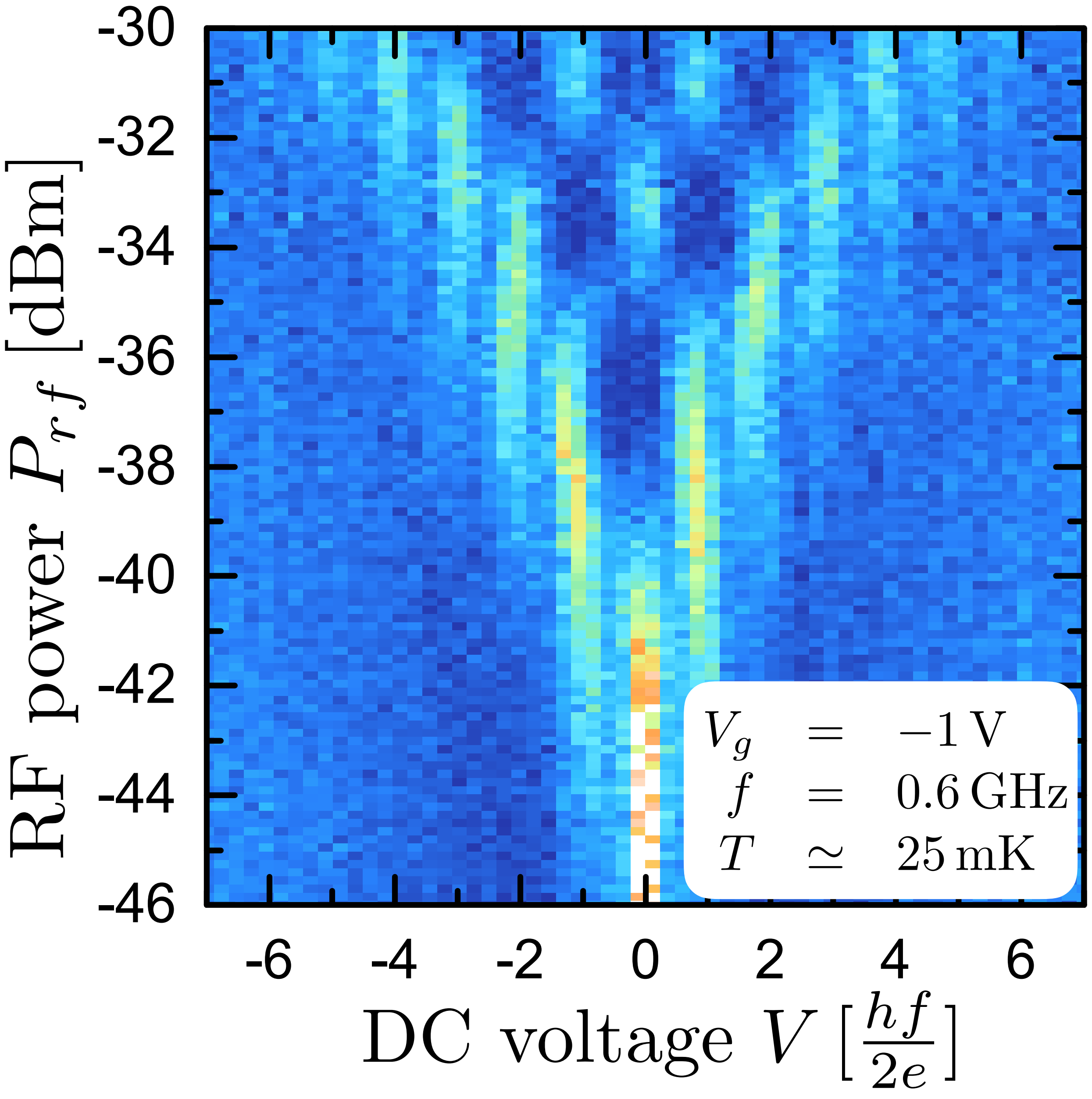
Non-topological HgTe quantum well

Non-topological QW

- narrow well (5 nm)
- no band inversion
- similar mobility 1.5 10^5 cm^{2}V^{-1}s^{-1}

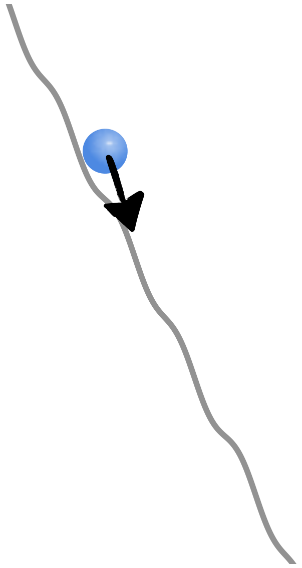
Shapiro steps

- no missing steps
- *n*-, *p*- regimes and gap verified
- down to f= 0.6 GHz



RSJ frequency dependence

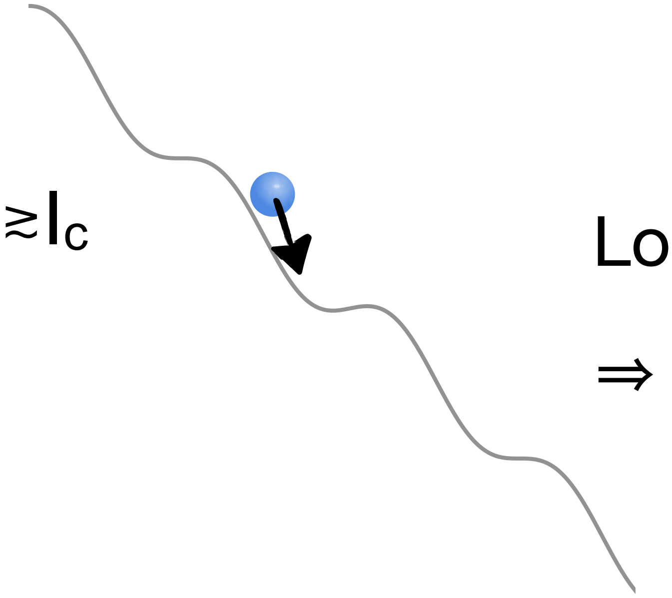
$I \gg I_c$



High frequency

⇒ almost sinusoidal

$I \gtrsim I_c$



Low frequency

⇒ non-sinusoidal

Domínguez *et al.*, PRB **86**, 146503 (2012)

RSJ frequency dependence

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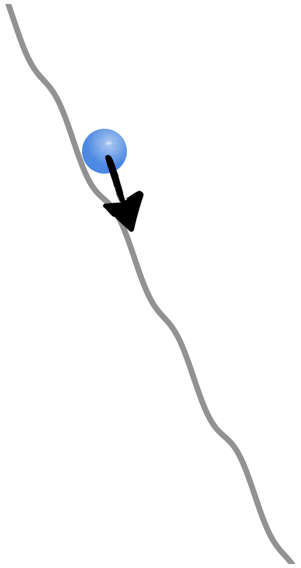

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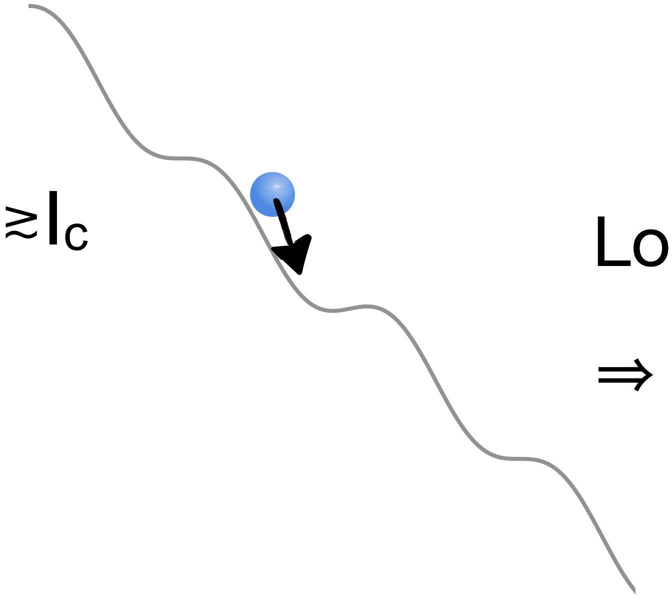
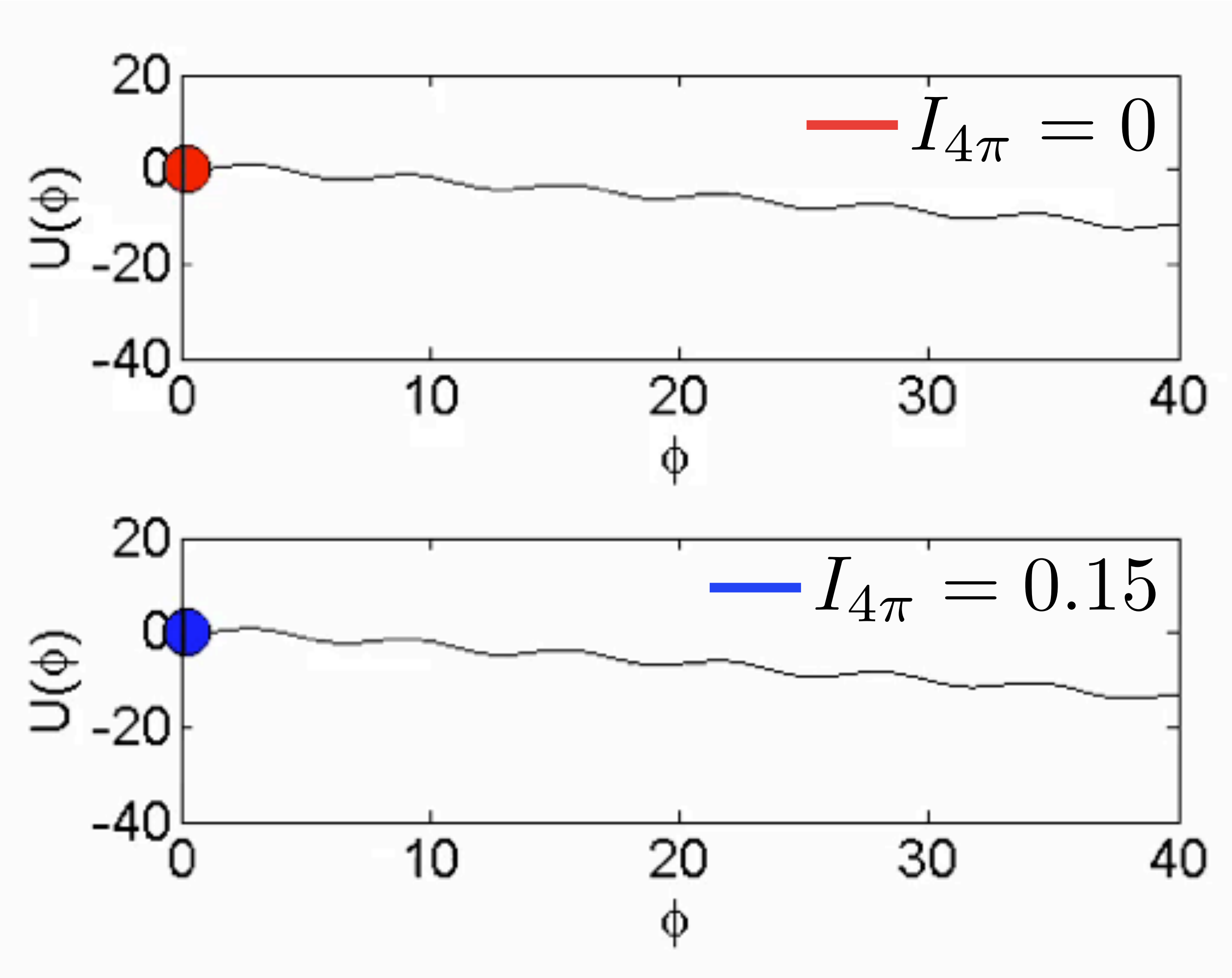


Low frequency

⇒ non-sinusoidal



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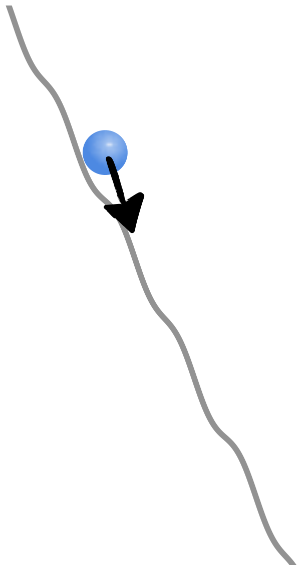

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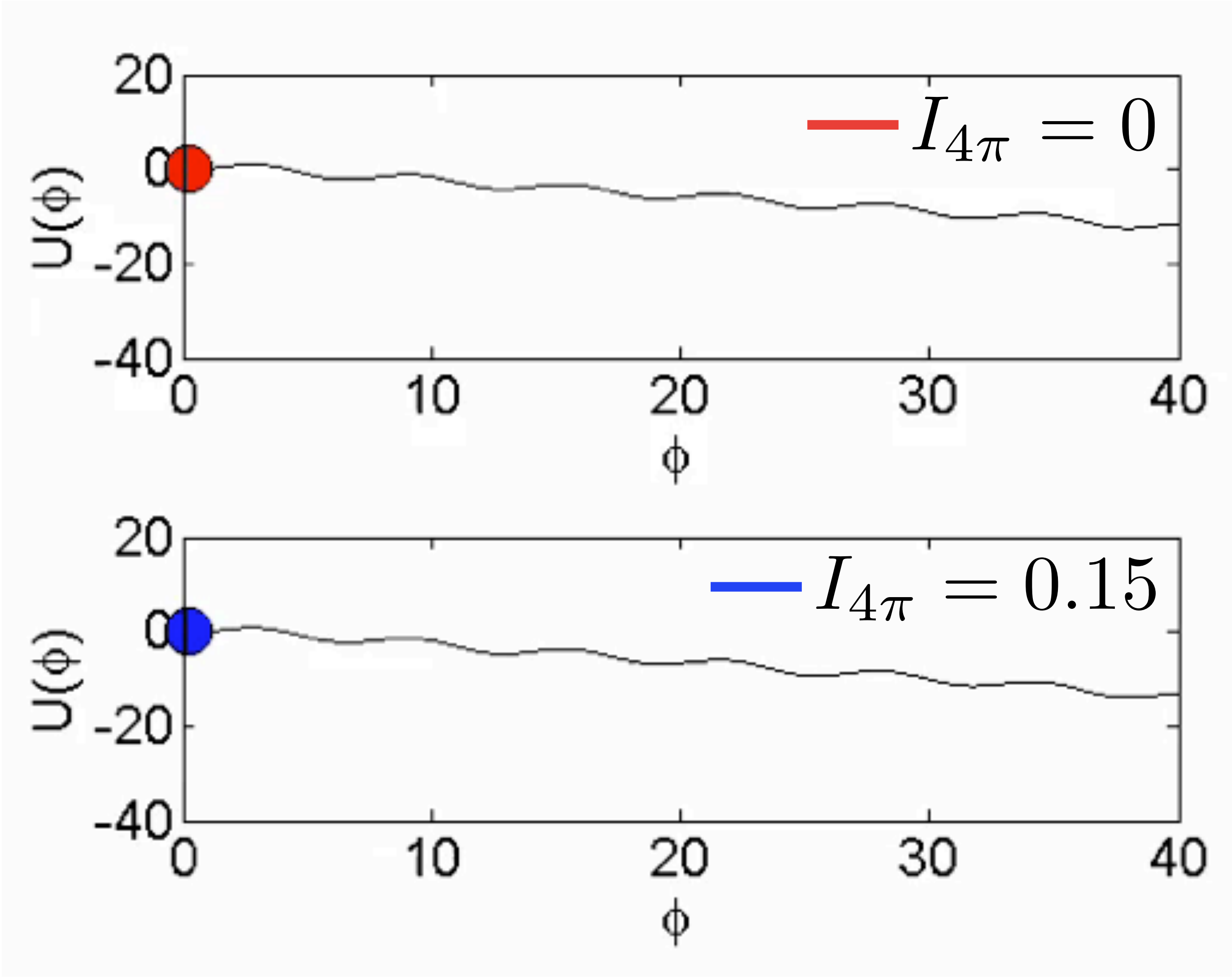

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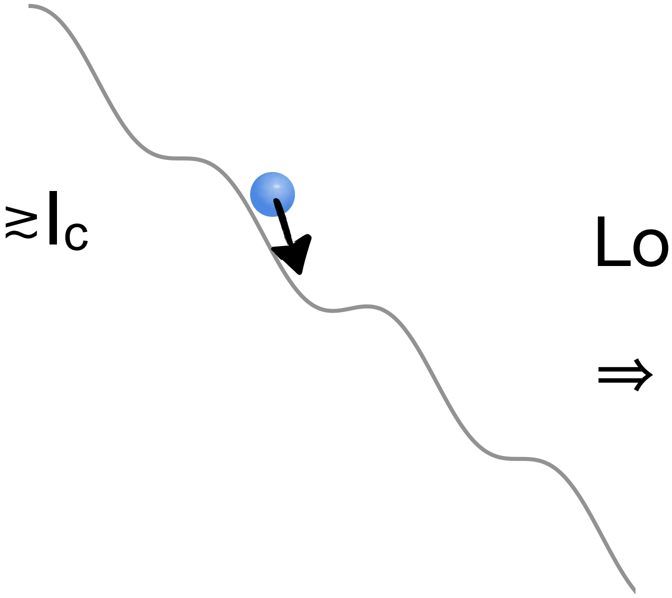


Low frequency

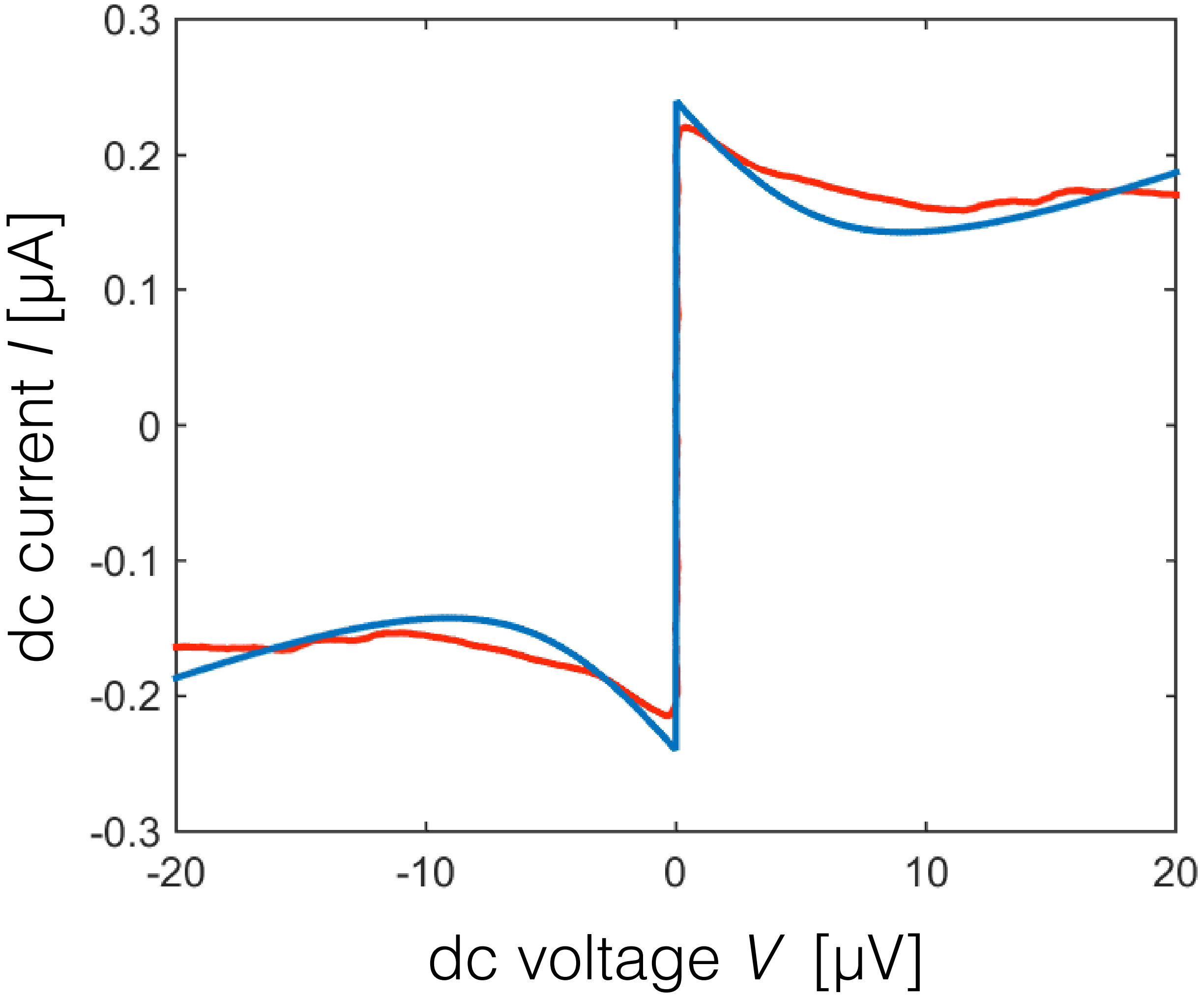
⇒ non-sinusoidal

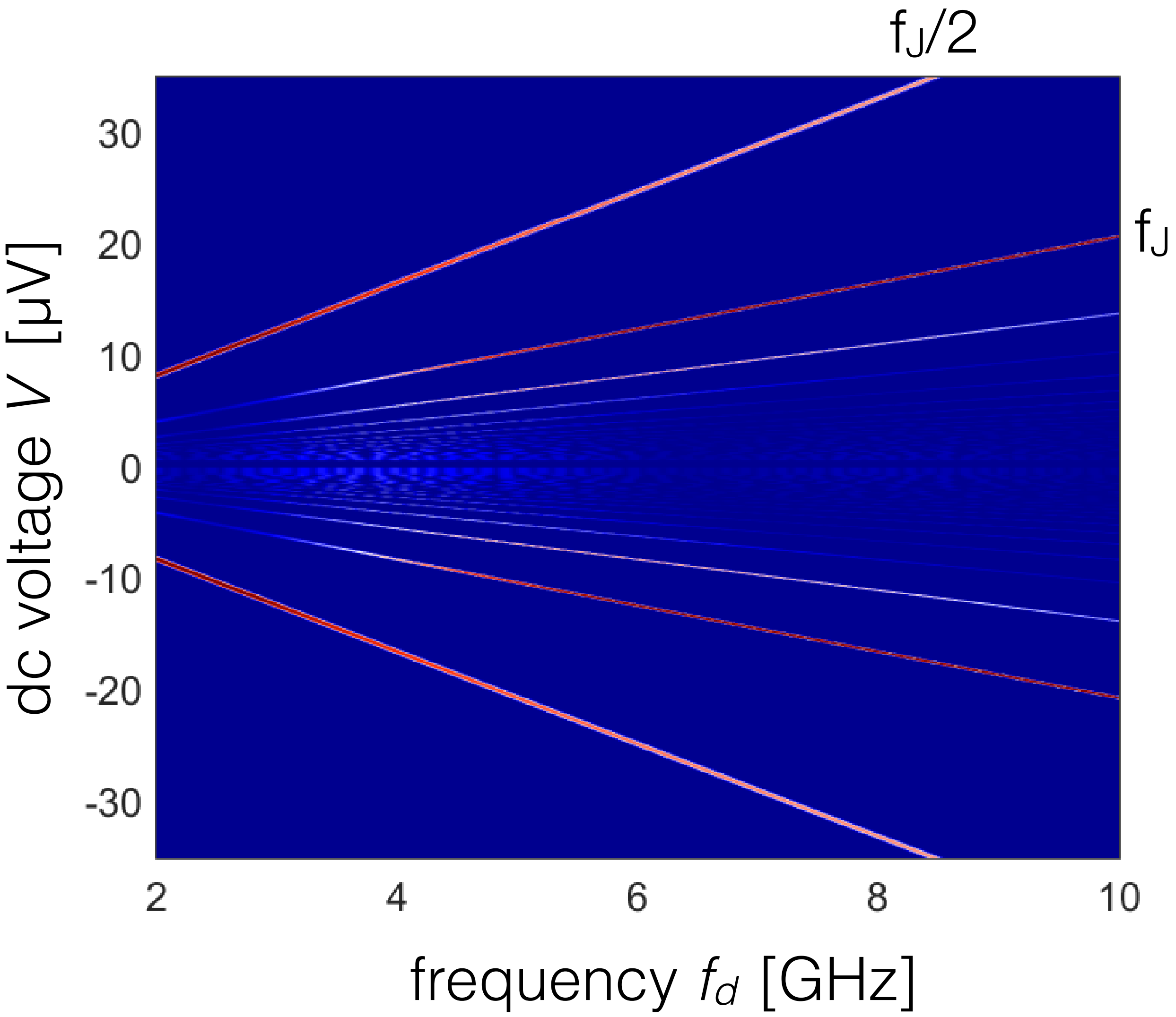
⇒ crossover $f_{4\pi}$ yields : 1-3 modes

⇒ no Landau-Zener transitions ?

Domínguez *et al.*, PRB **86**, 146503 (2012)

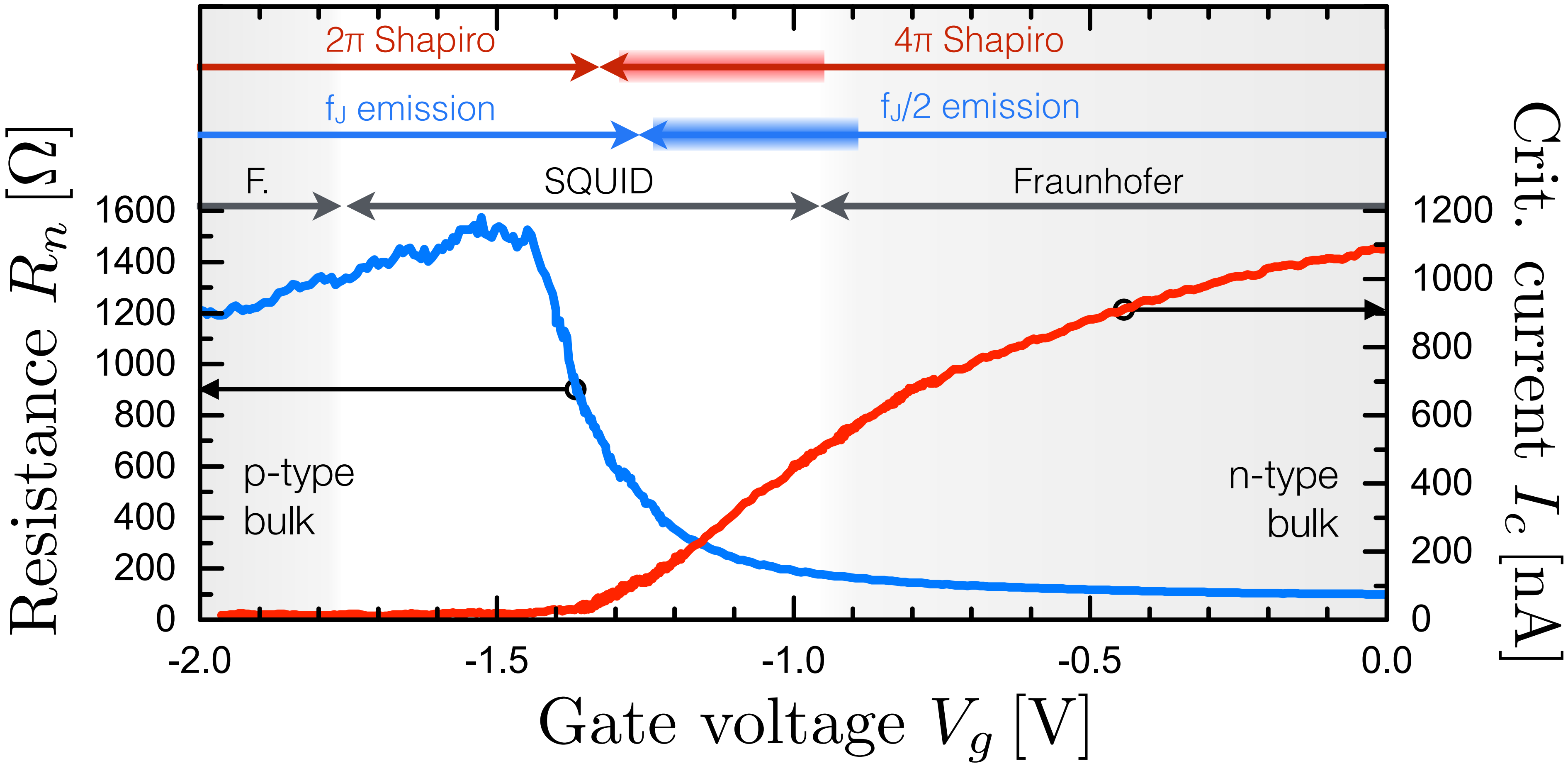
RSJ simulations





Theory : F. Domínguez & E. M. Hankiewicz

Summary

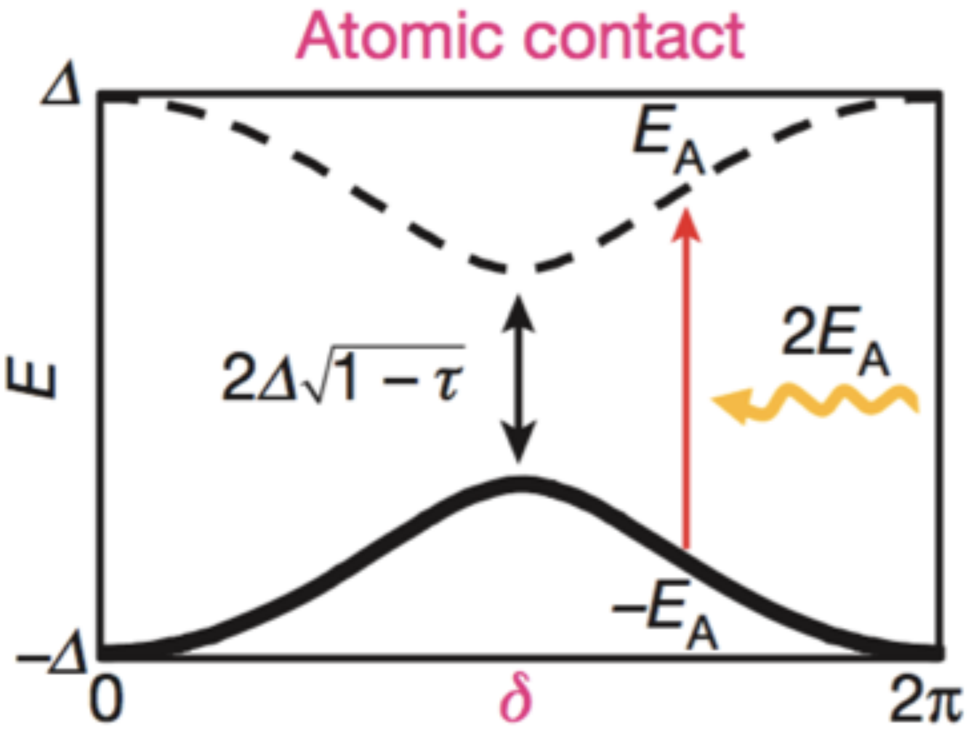


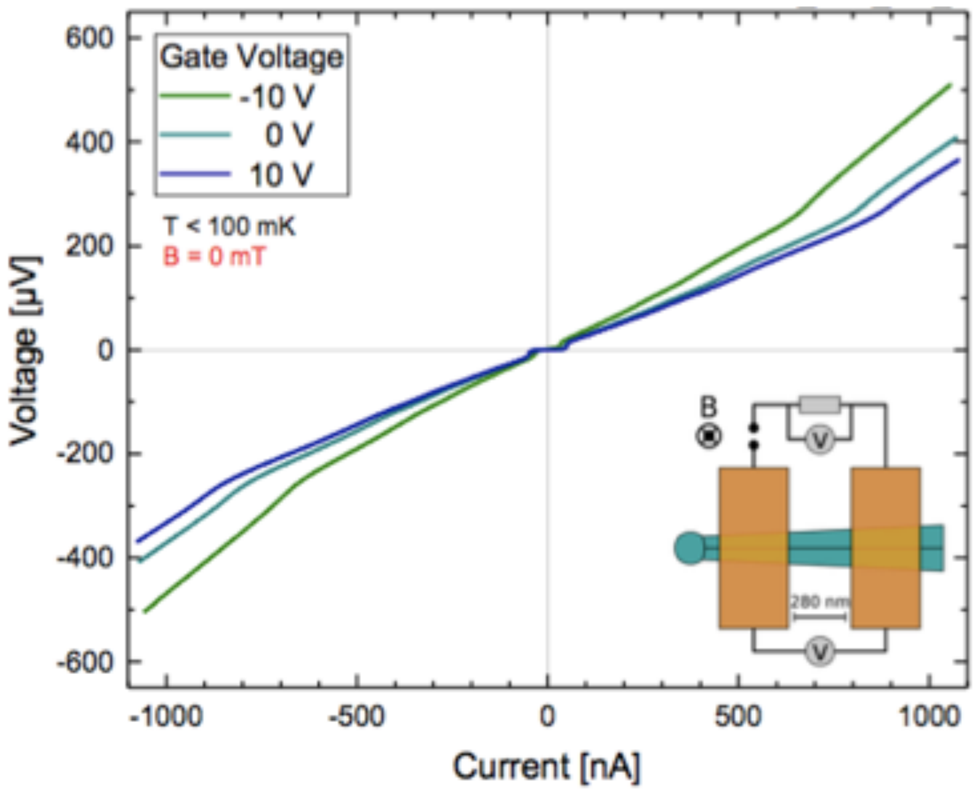
Fractional Josephson effect

- even sequence of Shapiro steps
- emission at $f_J/2$
- Landau-Zener transitions unlikely
- edge currents (SQUID pattern)
- contribution: 1-3 modes
- coexistence with conduction band ?
- discrepancy with R_n ?

Dai et al., PRB **77**, 125319 (2008)
Hart *et al.*, Nat. Phys. **10**, 638 (2014)

Outlook





Spectroscopy of Andreev bound states

- ▹ tunneling DOS
- ▹ absorption spectroscopy
- ▹ SN junctions

⇒ towards Majorana qu-bits

Pillet *et al.*, Nature Phys. **6**, 965 (2010)
Bretheau *et al.*, Nature **499**, 312 (2013)
Astafiev *et al.*, Science **327** 840 (2010)
Peng *et al.*, arXiv 1604.04287 (2016)

Other HgTe systems

- ▹ HgTe nanowires
- ▹ QSH, QH, QAH, Weyl

Open positions !

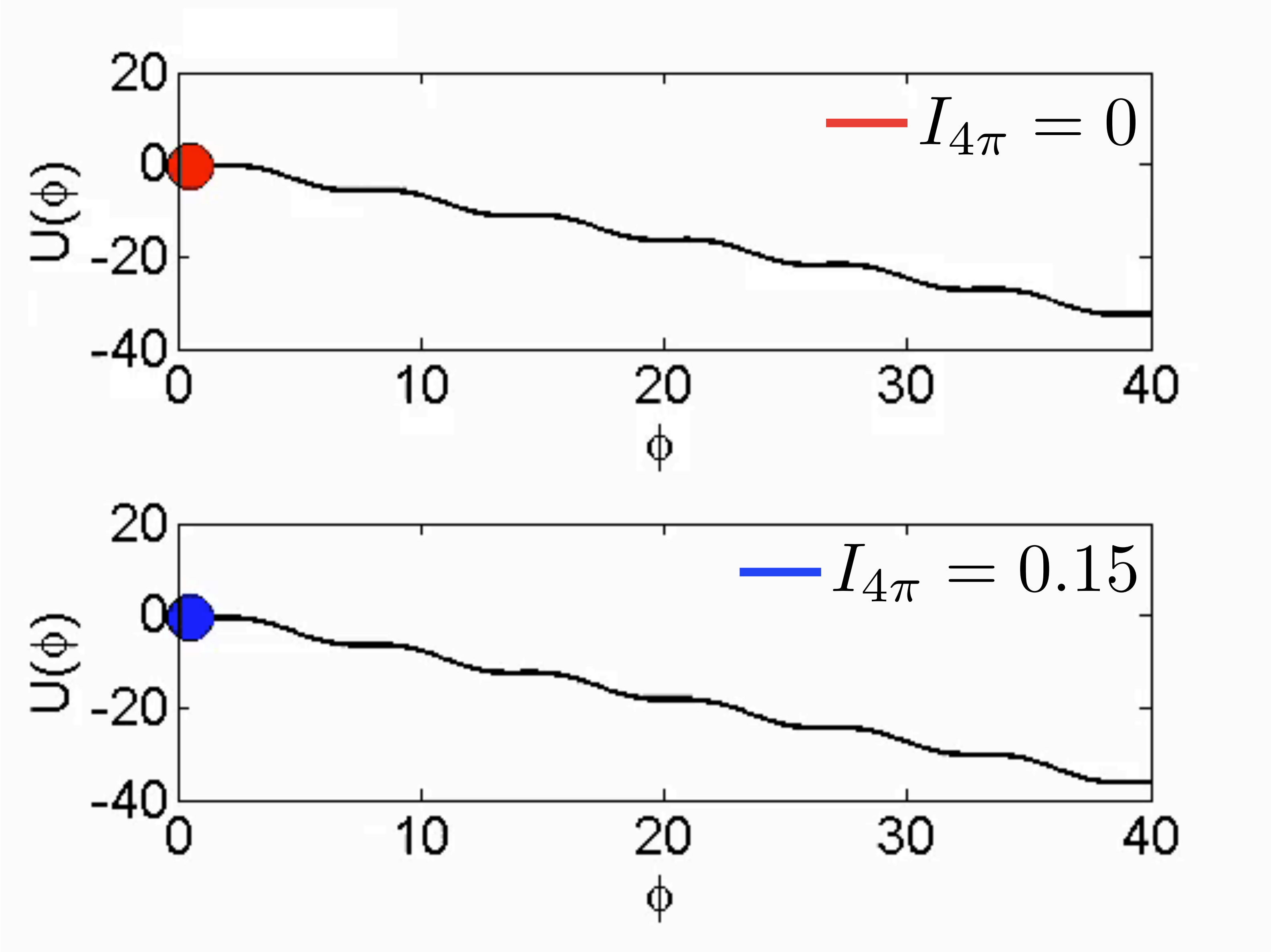
Thank you for your attention !

Open positions!

Post-docs

PhD students

Shapiro response : RSJ model



RSJ simulations

- anharmonic motion
- doubled step due to 4π
- transition at $f \sim f_{4\pi} = \frac{2eR_nI_{4\pi}}{h}$

4π-supercurrent

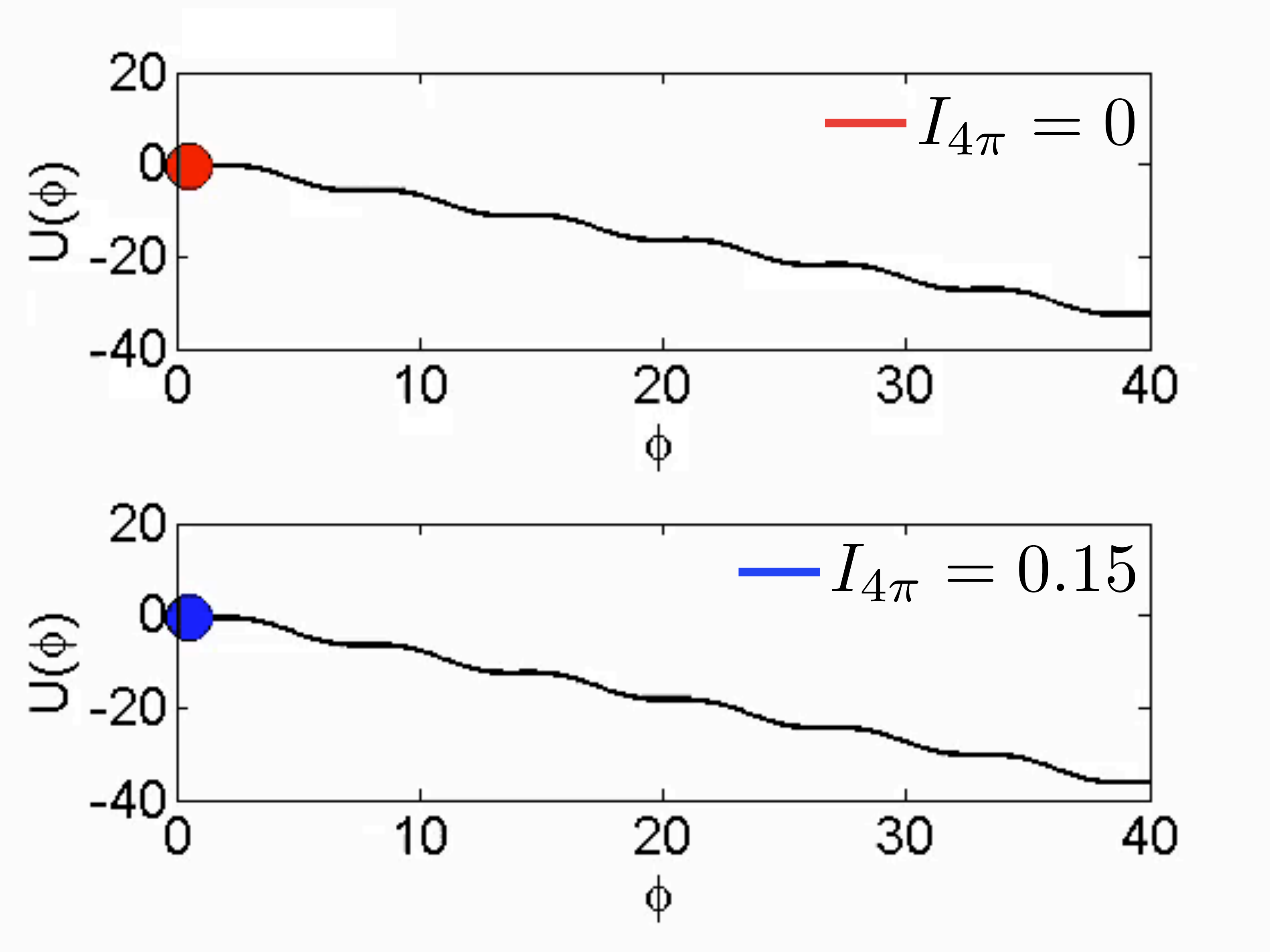
- ≃ 20 nA at V_g=-1.1 V
- 1-2 modes

Landau-Zener transitions

- stronger at high frequency

Dominguez *et al.*, PRB **86**, 146503 (2012)
Wiedenmann *et al.*, Nat. Comms **7**, 10303 (2016)

Shapiro response : RSJ model



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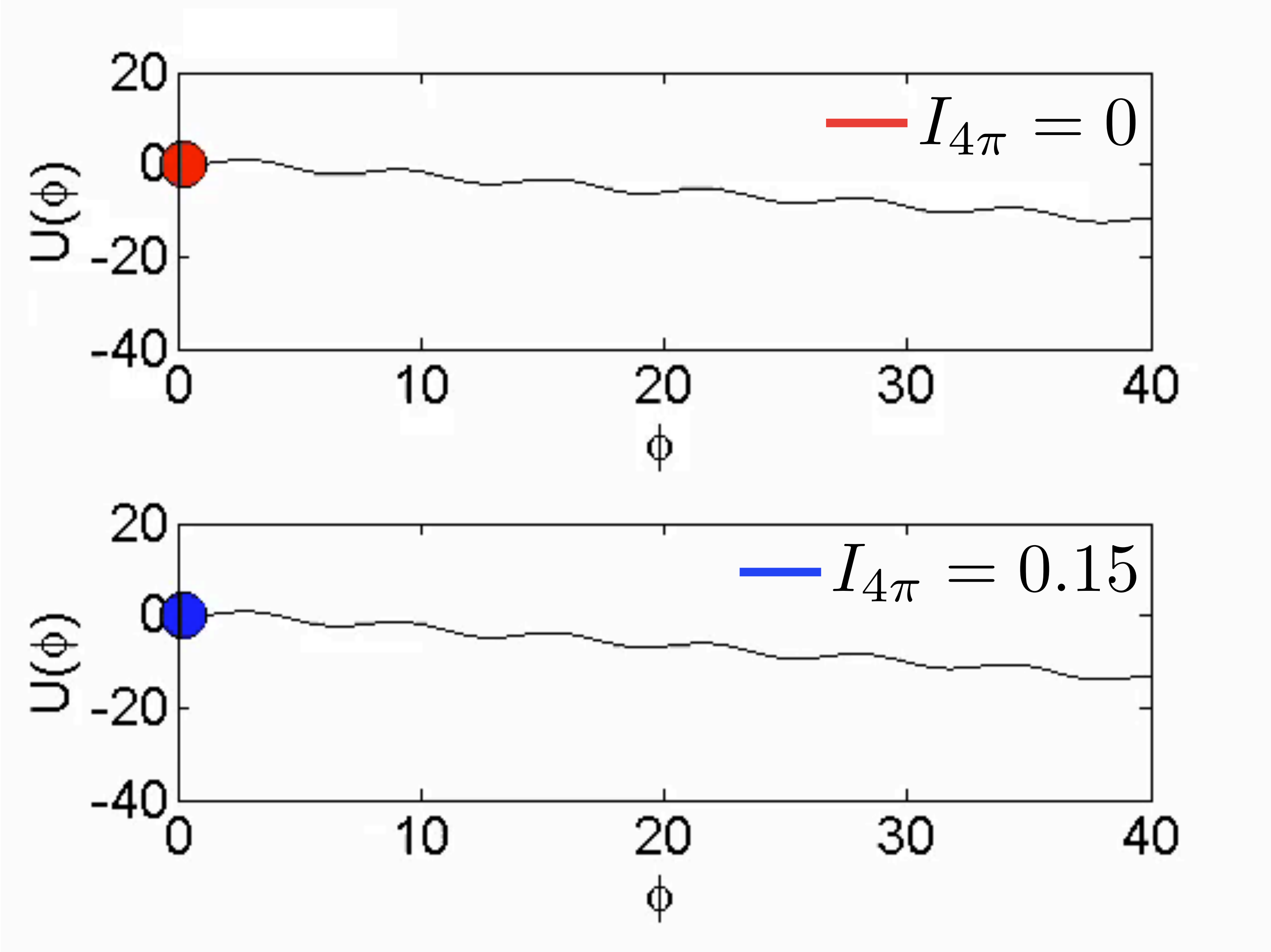

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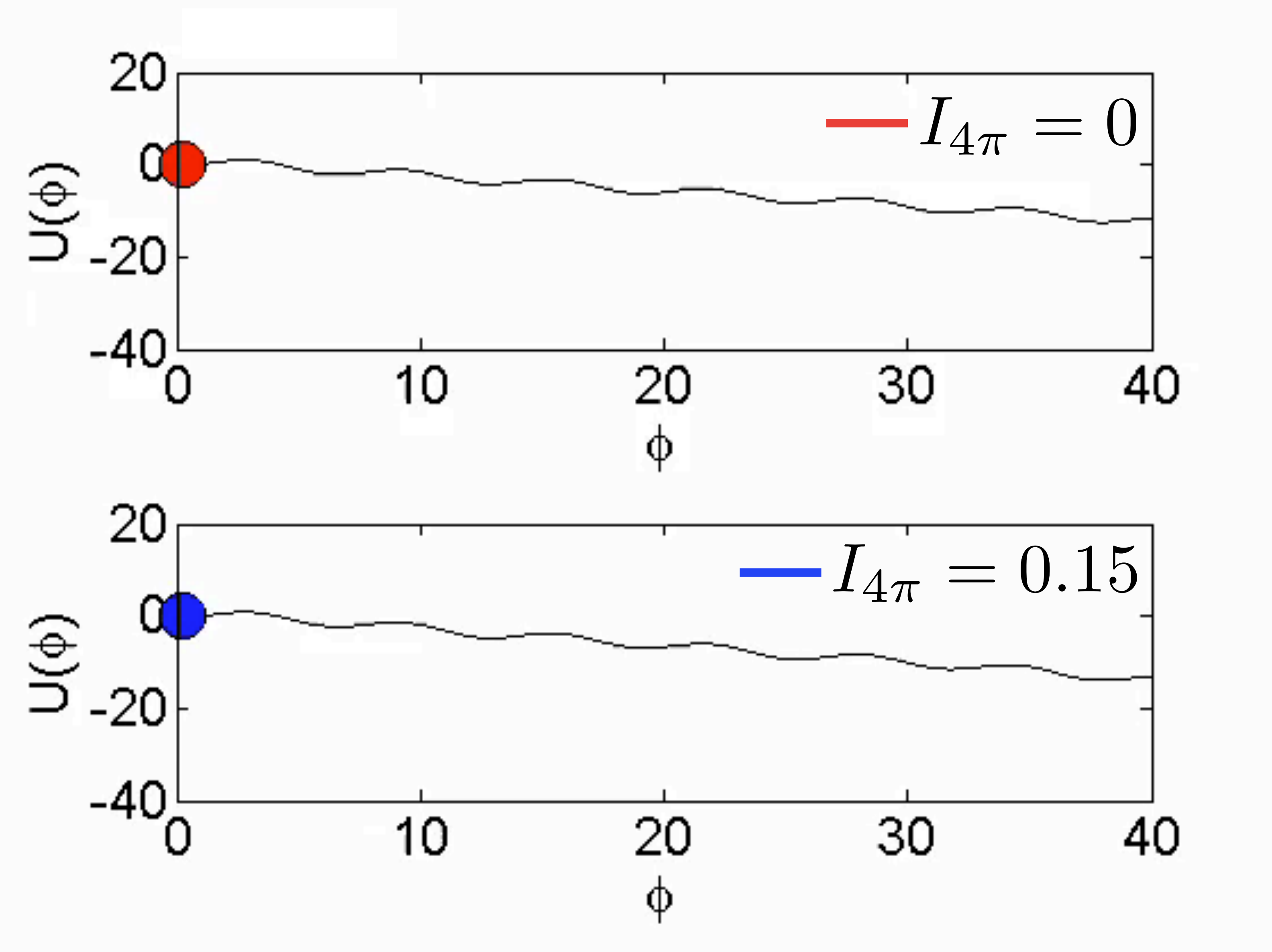

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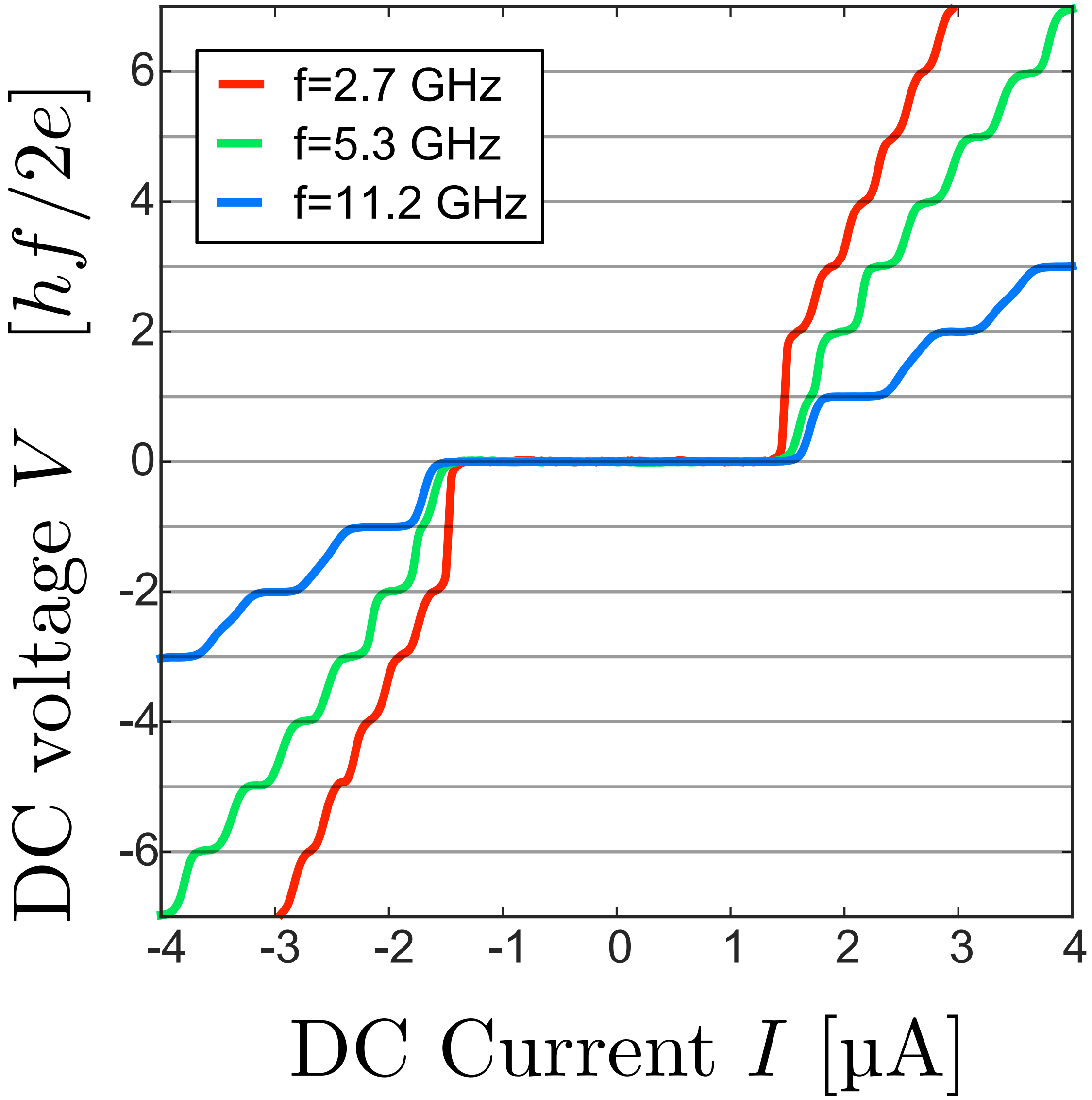
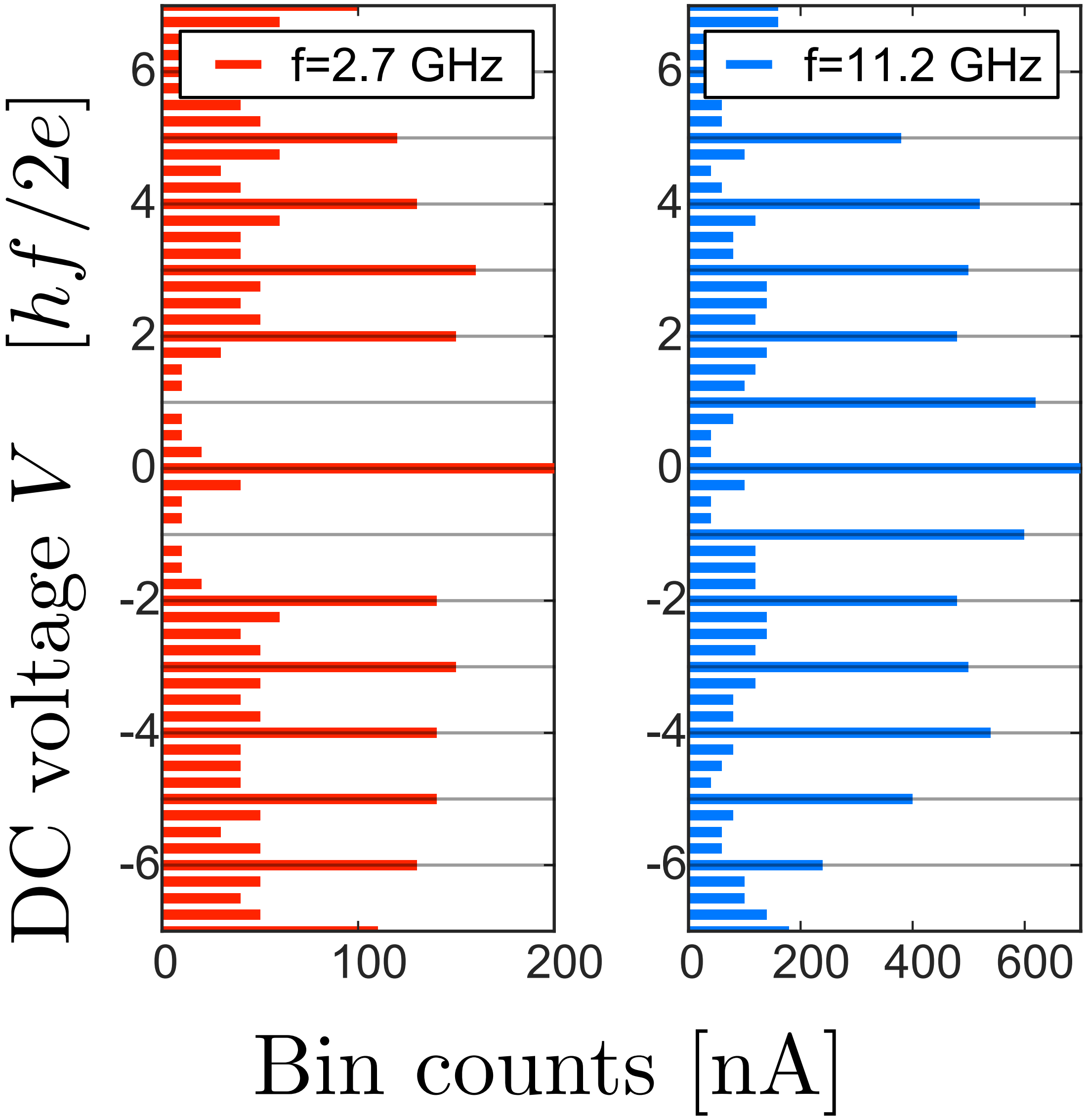
Shapiro steps





High frequency

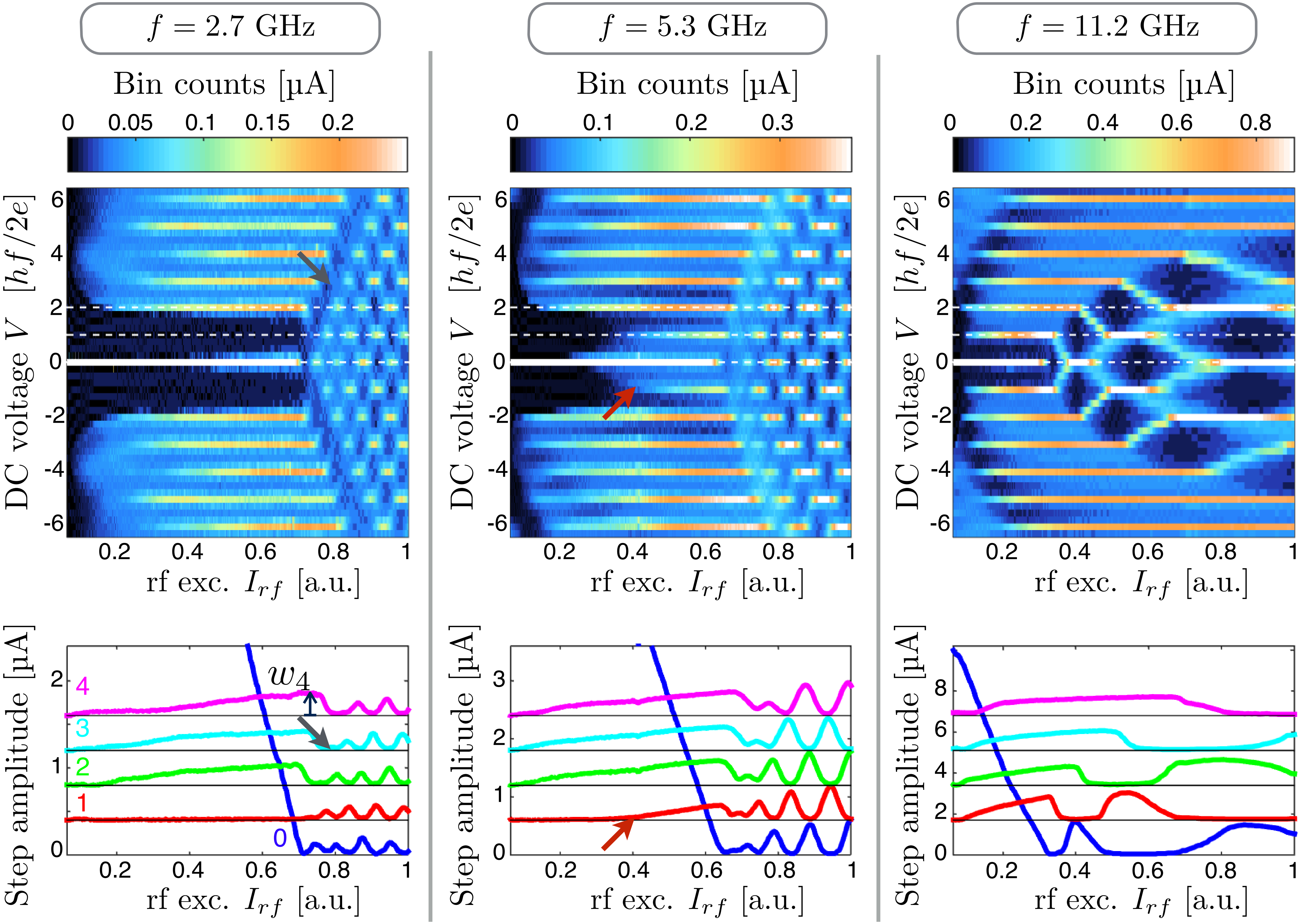
- >12 steps visible
- all steps seen: $n = 0, \pm 1, \pm 2, \ldots$

Low frequency

- first step missing: $n = 0, \pm 2, \pm 3 \ldots$
- 4π supercurrent?

Rokhinson *et al.*, Nat. Phys. **86**, 146503 (2012)

Frequency dependence



Mechanism for a missing step?

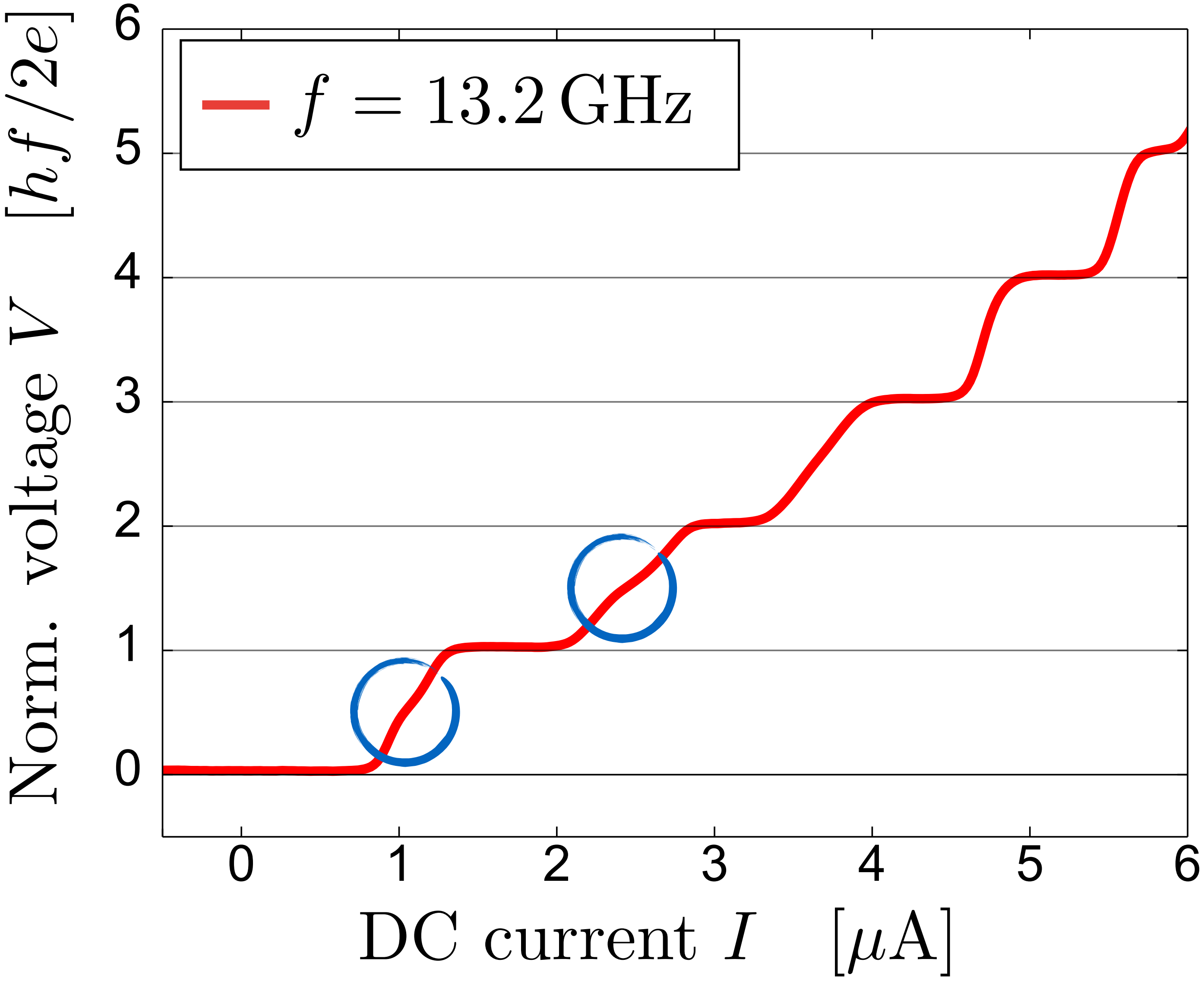
Additional subharmonic steps

$n = 1/2, 3/2, 1/3, \ldots$

- ▹ capacitive effects
- ▹ non-linearities
- ▹ higher harmonics in the CPR

Missing steps

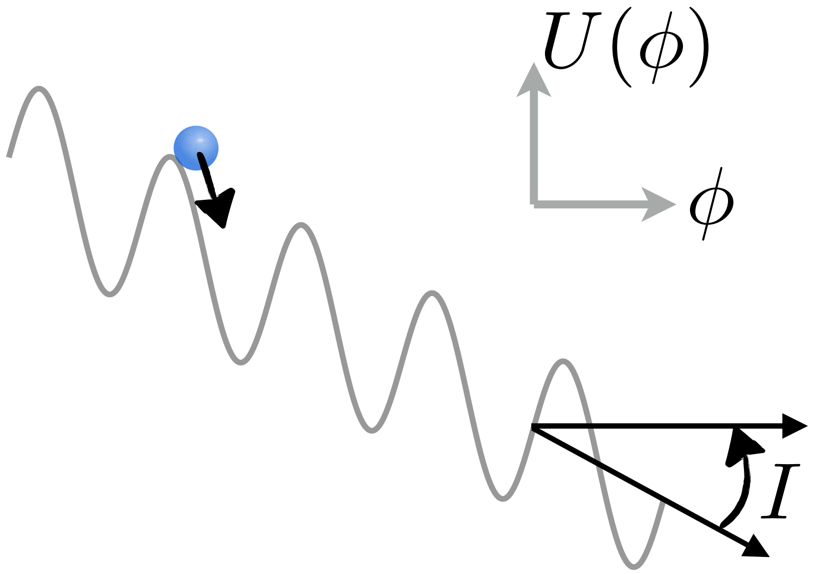
- ▹ 4π supercurrent $I_{4\pi} \sin \phi/2$
- ▹ dominates at low frequency
  $f \sim f_{4\pi} = \frac{2eR_n I_{4\pi}}{h}$
- ▹ confirmed by RSJ simulations



Renne *et al.*, R. Phys. App. **9**, 25 (1974)
Valizadeh *et al.*, JNMP **15**, 407 (2008)
Sochnikov *et al.*, PRL **114** 066801 (2014)

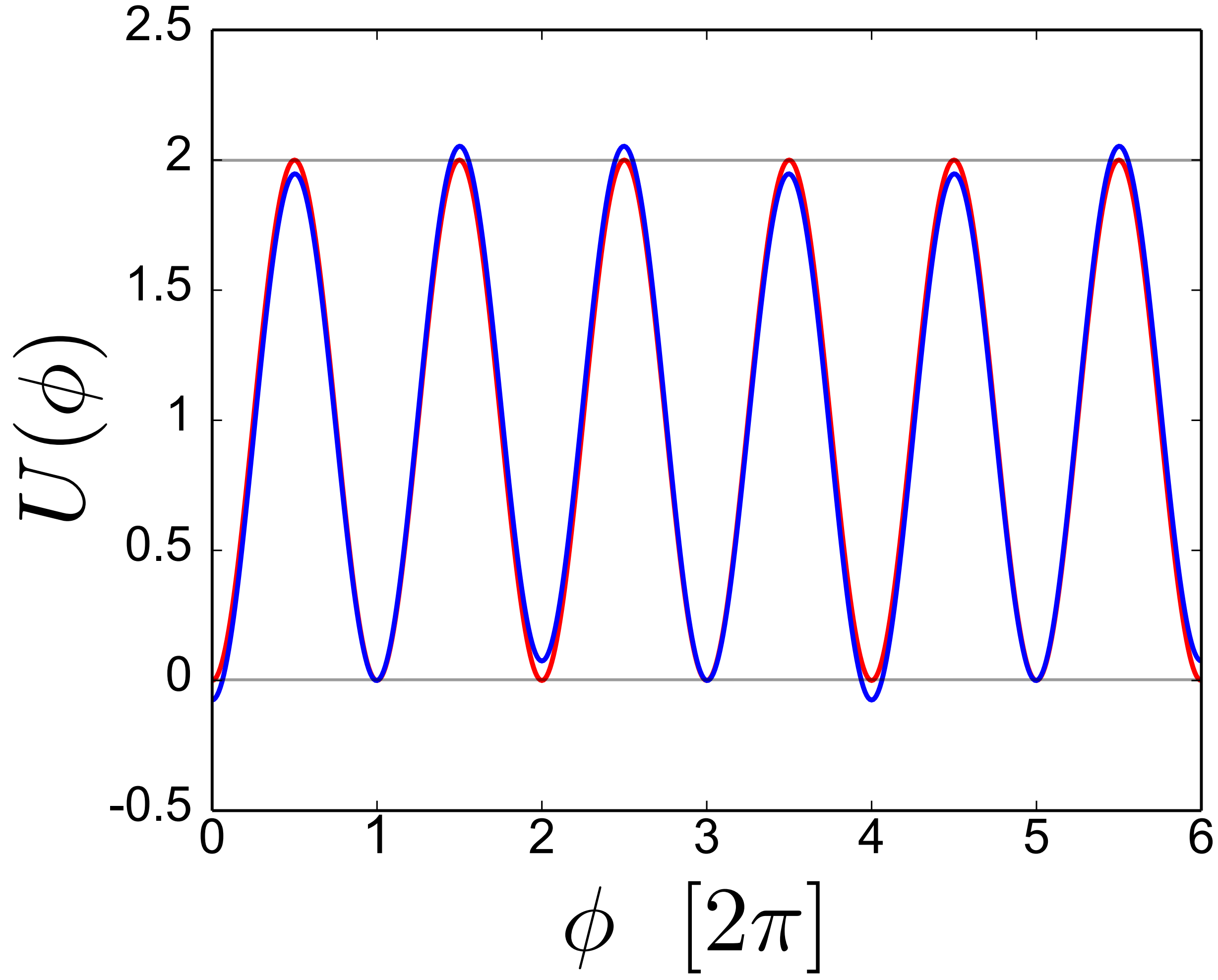
Dominguez *et al.*, PRB **86**, 146503 (2012)
Badiane *et al.*, PRL **107**, 17702 (2011)

RSJ dynamics



Motion of fictitious particle

- ▹ $\partial_\phi U(\phi) = I_S(\phi) - I$
- ▹ $I_S(\phi) = I_{4\pi} \sin\frac{\phi}{2} + I_{2\pi} \sin\phi + \dots$



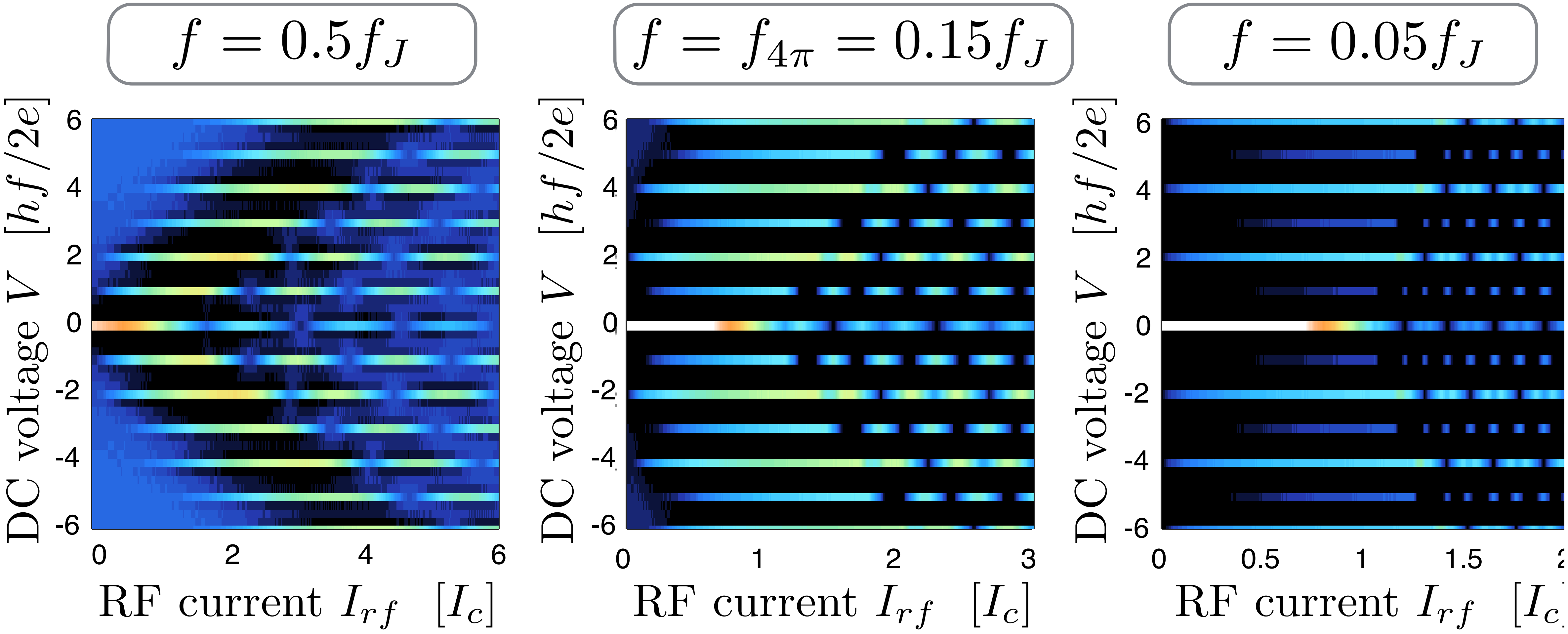
Energy potentials

🟥 $I_{2\pi} = 1,\ I_{4\pi} = 0$

🟦 $I_{2\pi} = 1,\ I_{4\pi} = 0.15$

- ▹ 13% of 4π supercurrent
- ▹ 11% of increase of I_c

RSJ simulations



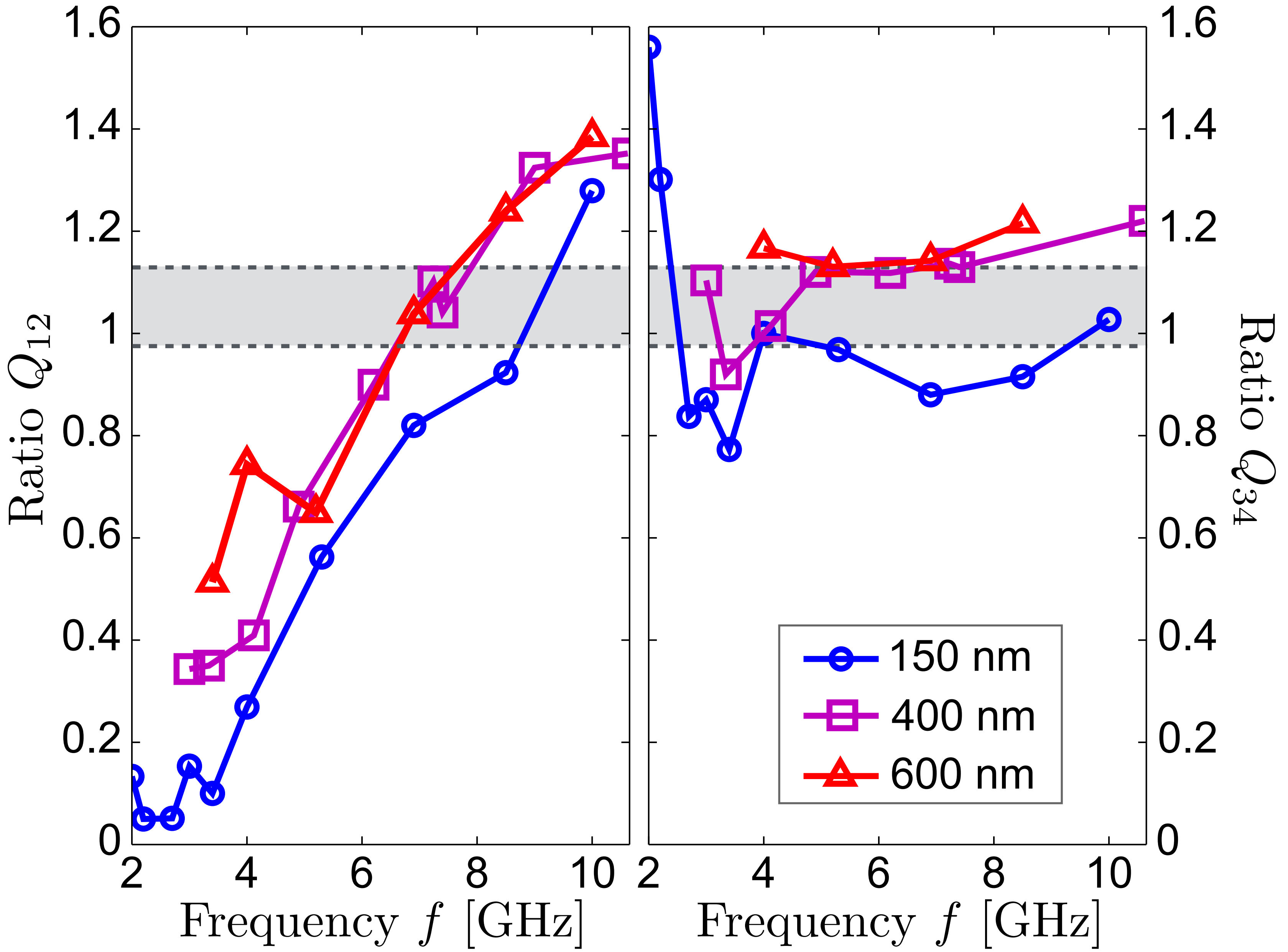
$f = 0.5 f_J$

$f = f_{4\pi} = 0.15 f_J$

$f = 0.05 f_J$

- ▹ progressive disappearance of all odd steps
- ▹ crossover at $f \sim f_{4\pi} = \frac{2eR_nI_{4\pi}}{h}$
- ▹ 4π-periodicity required
- ▹ no effect of 2π-periodic CPR

Evaluation of the 4π contribution



Ratios

- $Q_{12} = \frac{w_1}{w_2} \to 0$
- $Q_{34} = \frac{w_3}{w_4} \simeq 1$

w_n max. amplitude of nth step

Contribution

- crossover $f \sim f_{4\pi}$ ⇒ current $I_{4\pi}$
- number of 4π modes $I_0 = \frac{e\Delta_i}{\hbar}$ ⇒ 1-3 modes

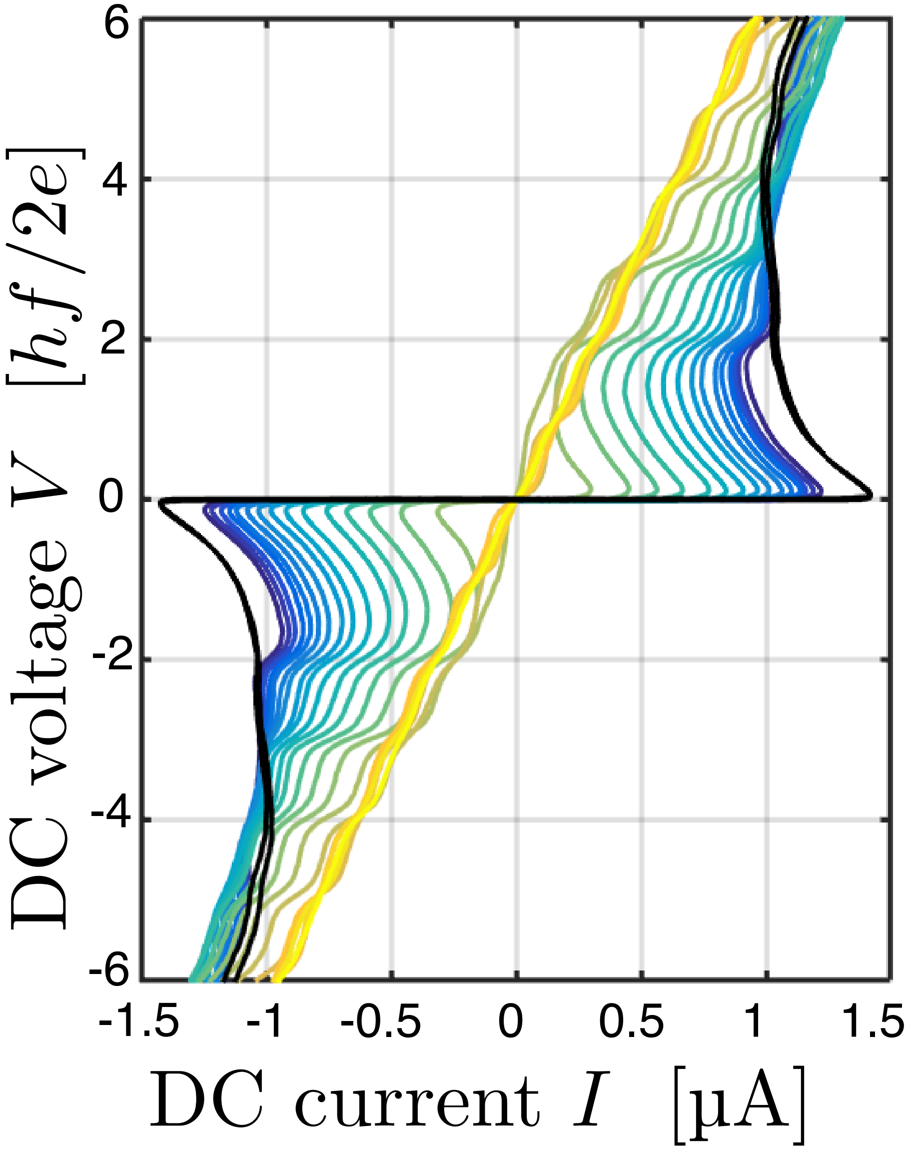
Other mechanisms

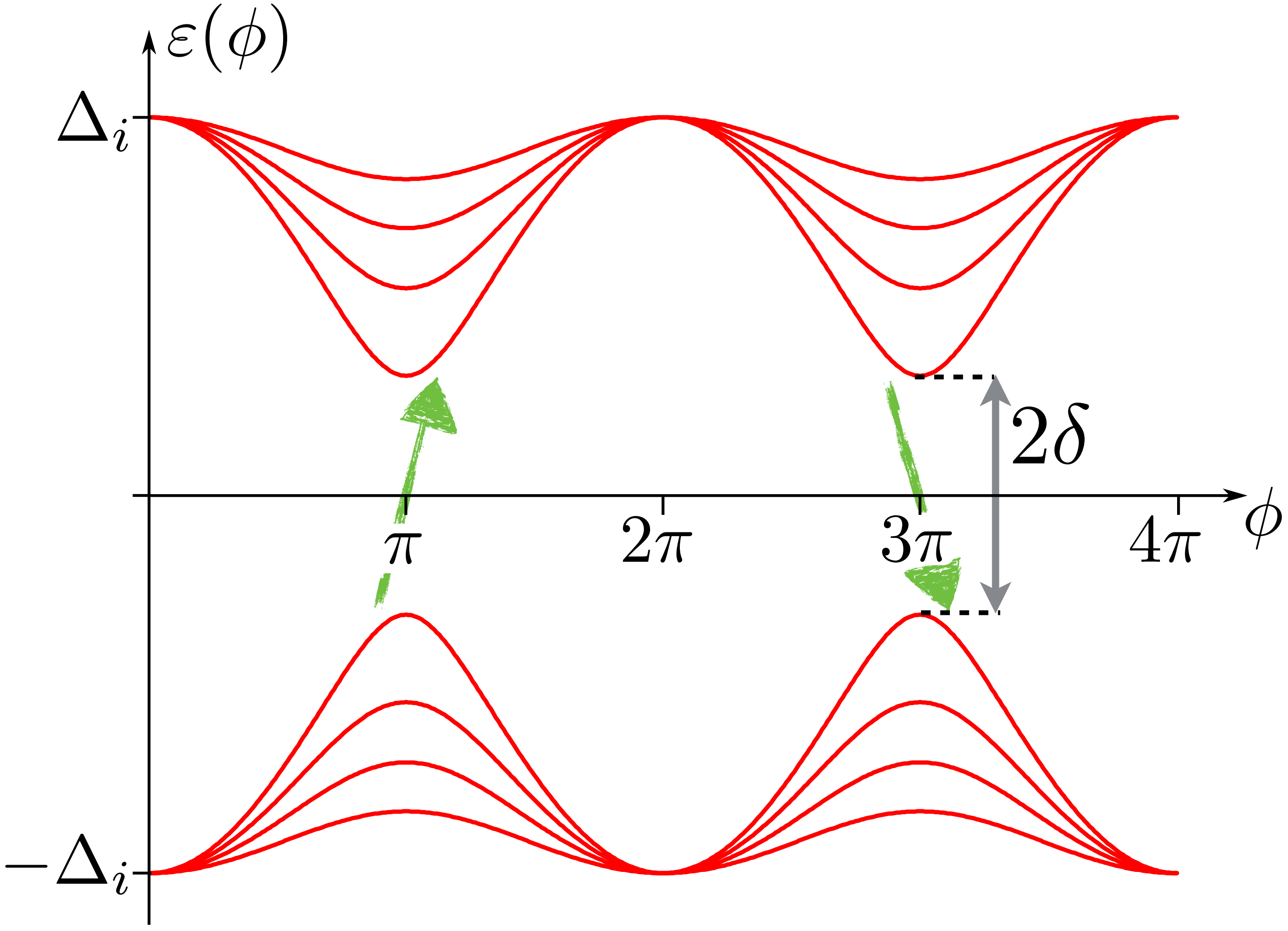
Bias instability/switch

- instability suppressed by shunt
- n=1 step still missing

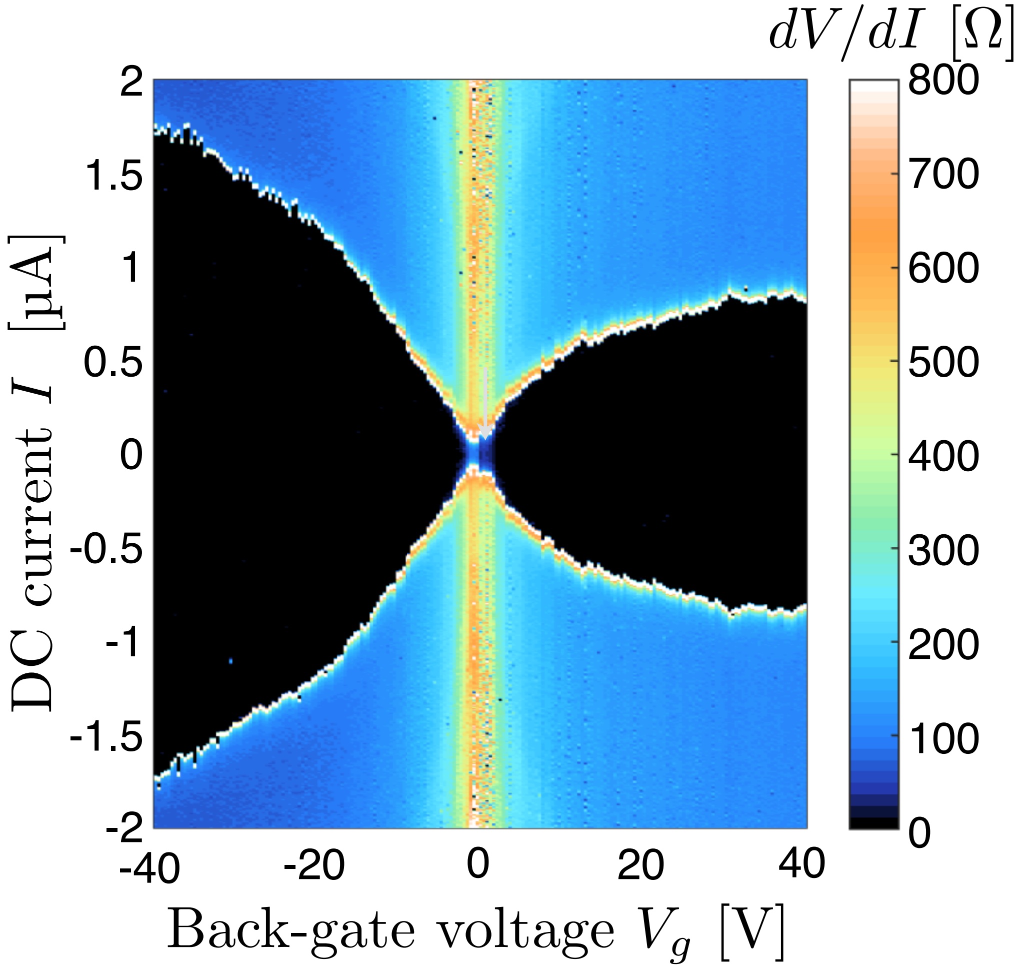
Landau-Zener transitions

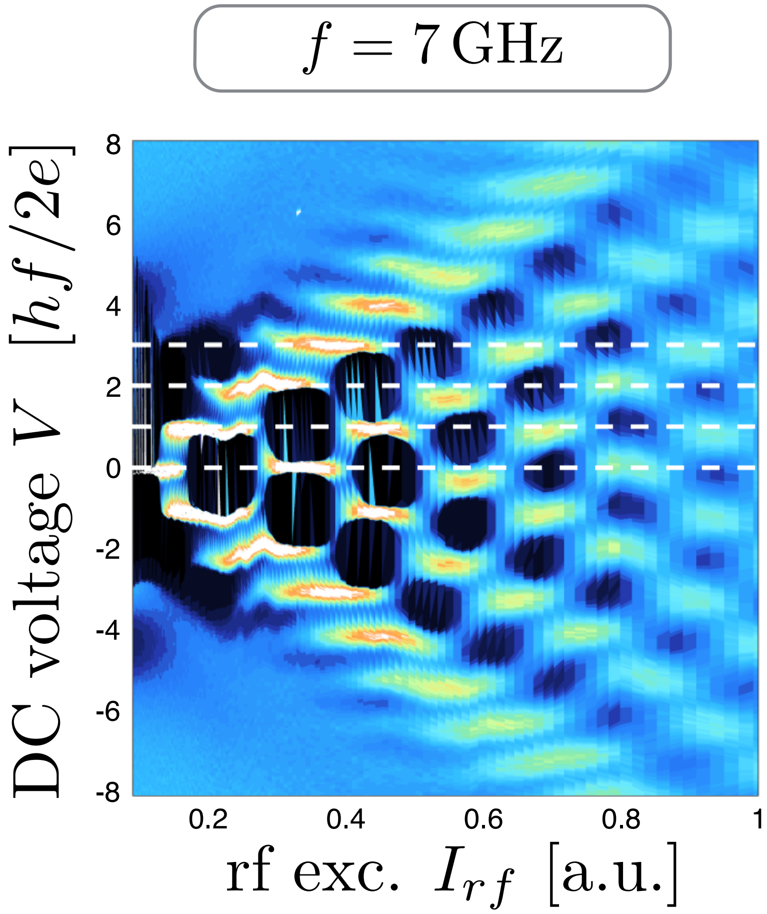
- enhanced at high frequency
- evaluated minimal transmission $D > 0.994$

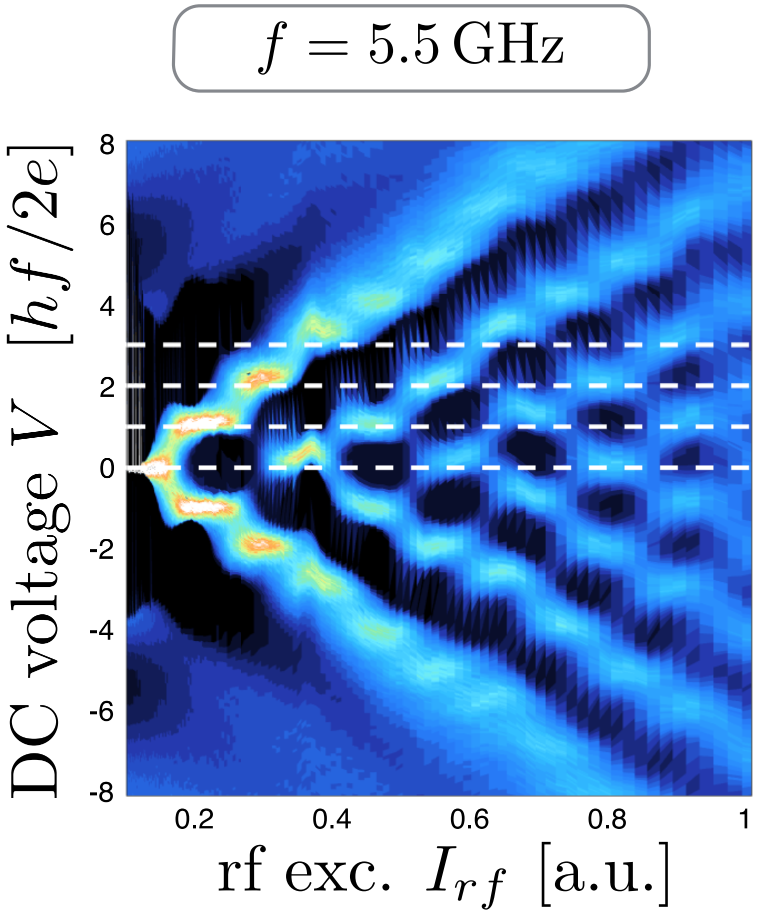




Graphene-based junctions







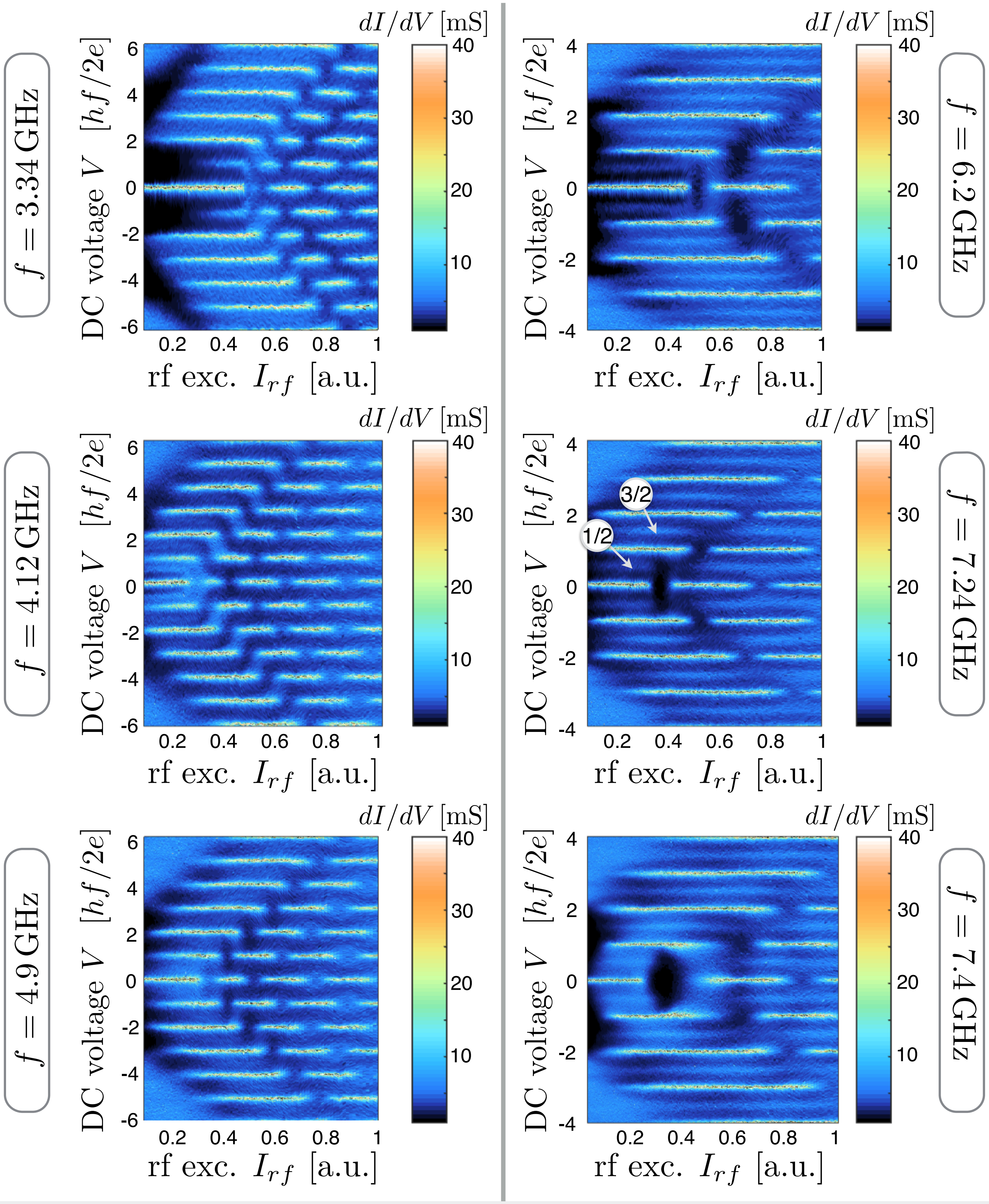
Graphene junctions

- ▹ density : $-2.5 \times 10^{12}\,\mathrm{cm}^{-2}$ to $2.5 \times 10^{12}\,\mathrm{cm}^{-2}$
- ▹ mobility : ~5000 $\mathrm{cm}^2\mathrm{V}^{-1}\mathrm{s}^{-1}$

Shapiro steps

- ▹ n=1 step always visible
- ▹ different densities (*n*, *p*, DP) and frequencies (4-14 GHz)

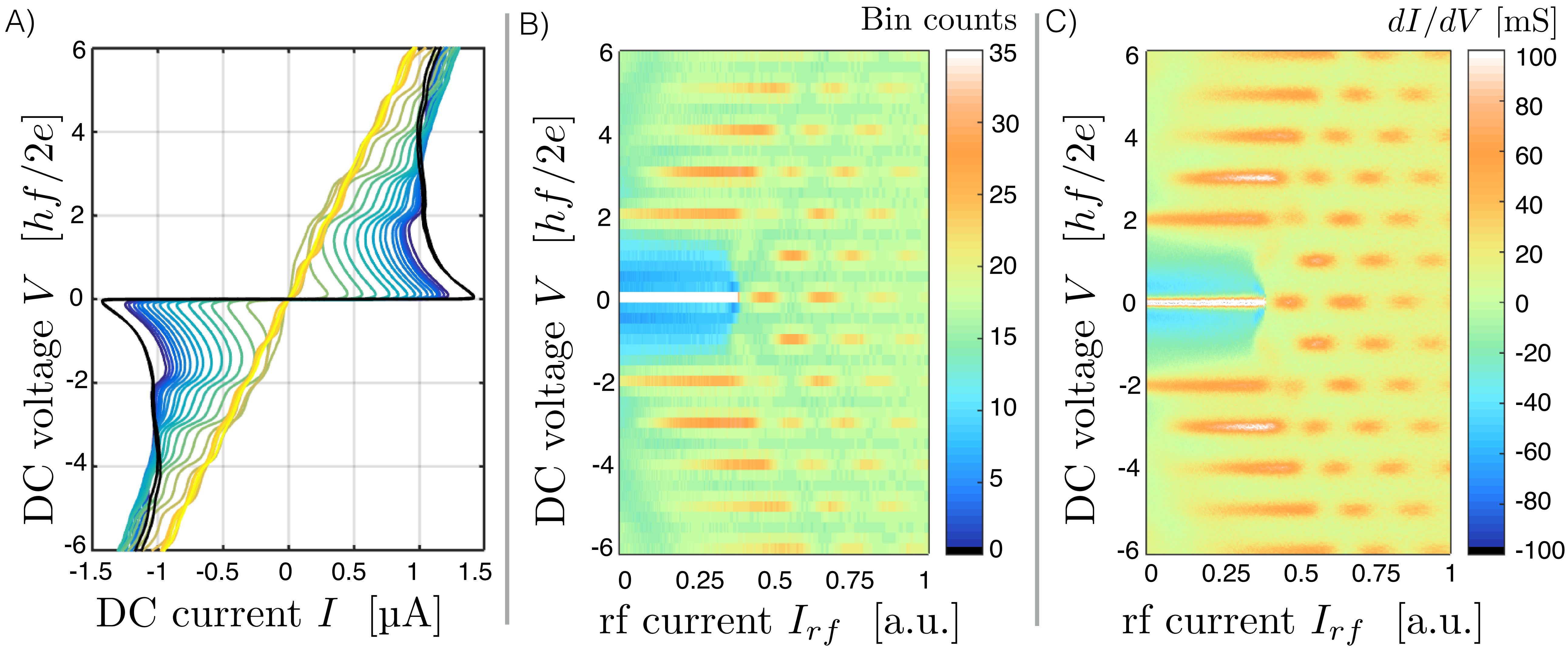
Frequency dependence



Frequency dependence

- missing n=1 step at low frequency
- subharmonic steps at high frequency

Shapiro steps : voltage bias



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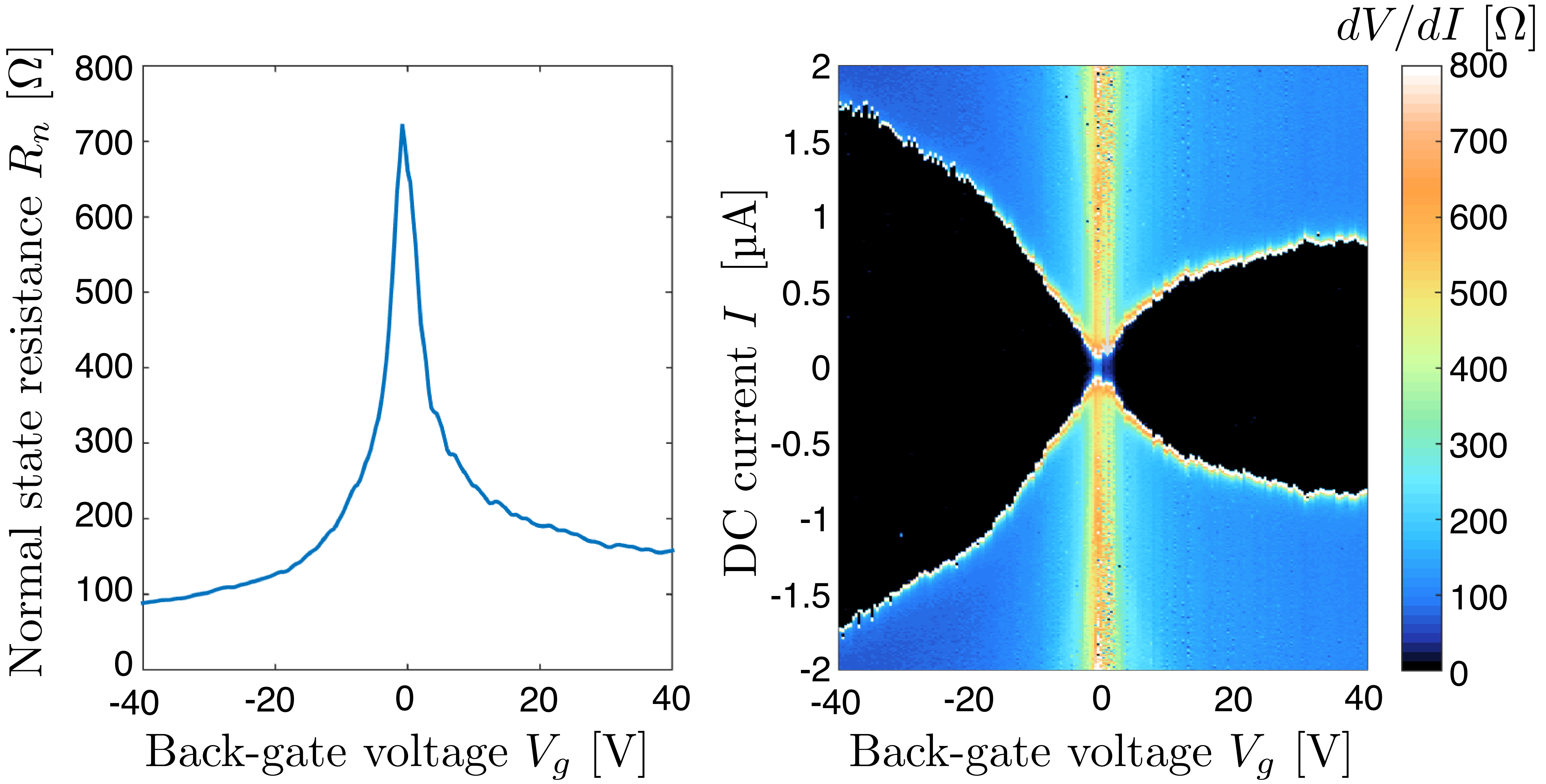
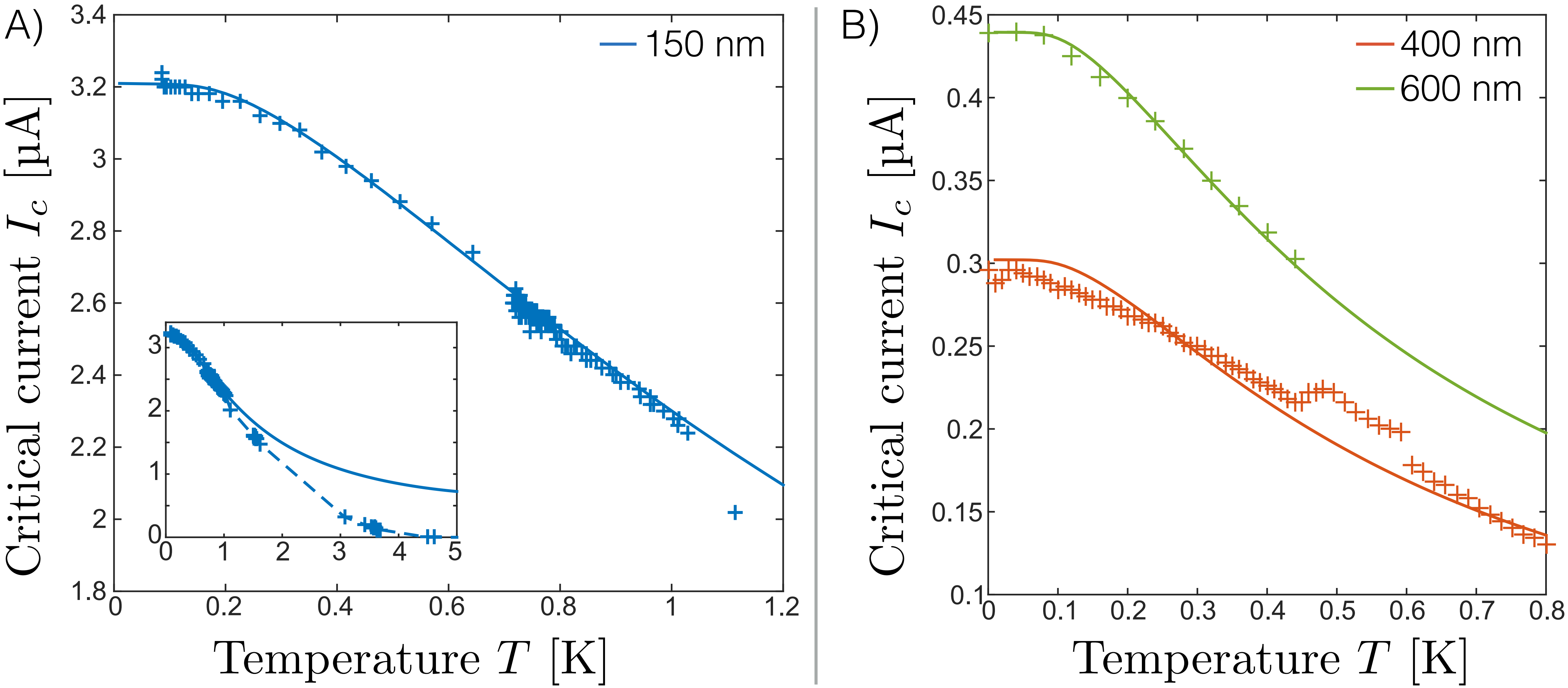


Temperature dependence of I_c

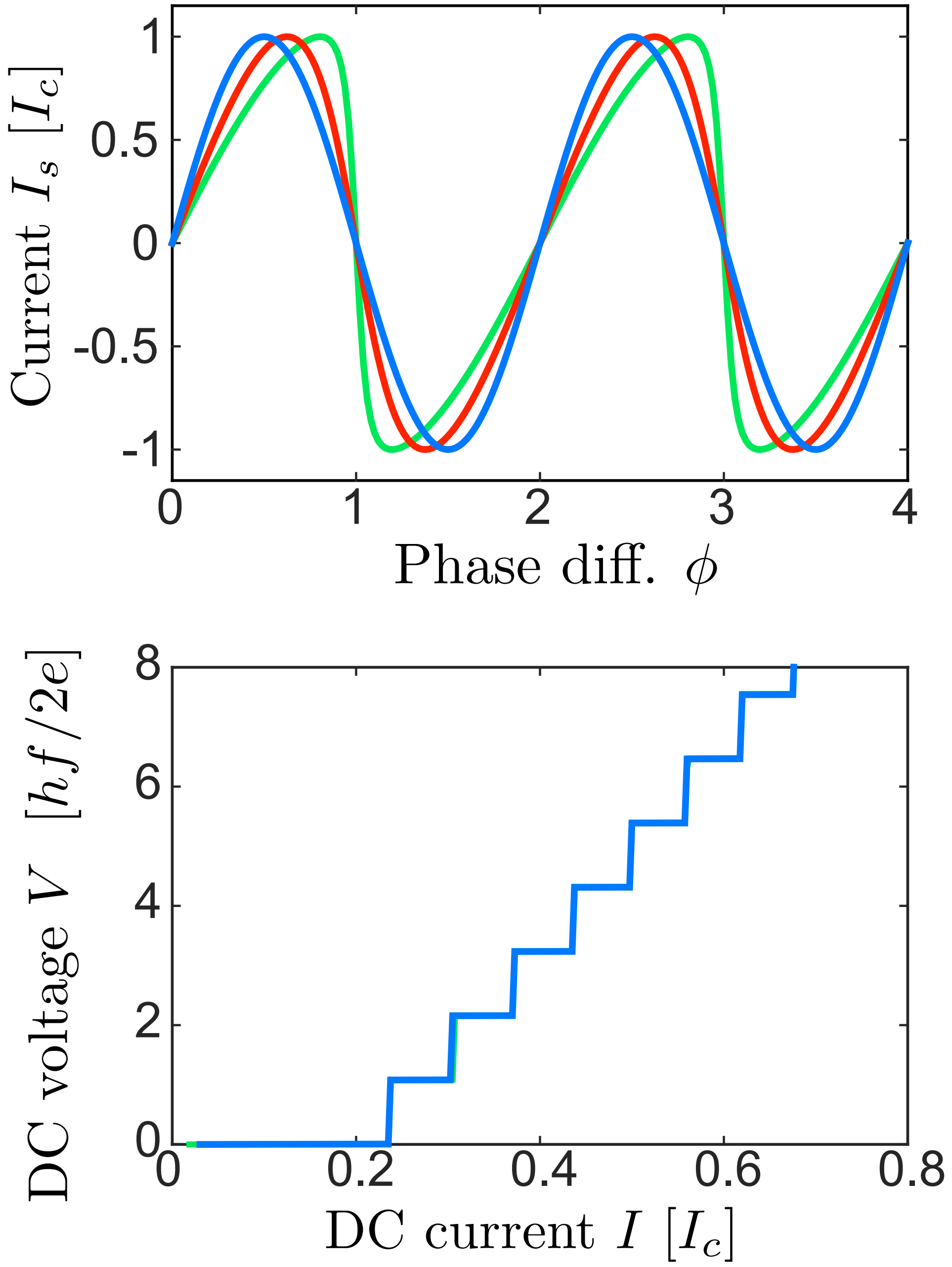


- fits for in 0-1K region to access induced gap
- perturbation theory diverges at high temperature

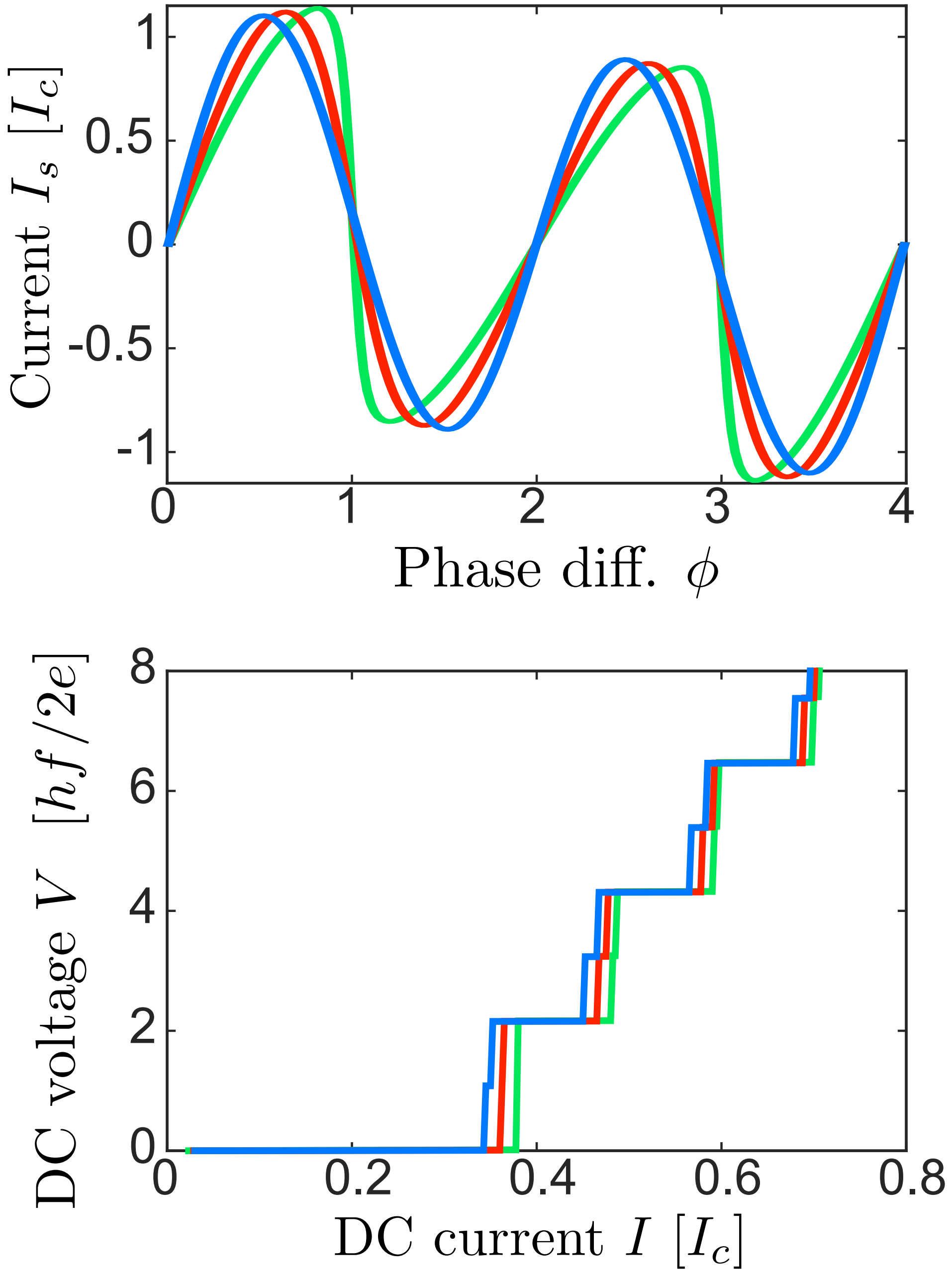
Tkachov *et al.*, PRB **88**, 075401 (2013)

Effect of CPR in RSJ simulations

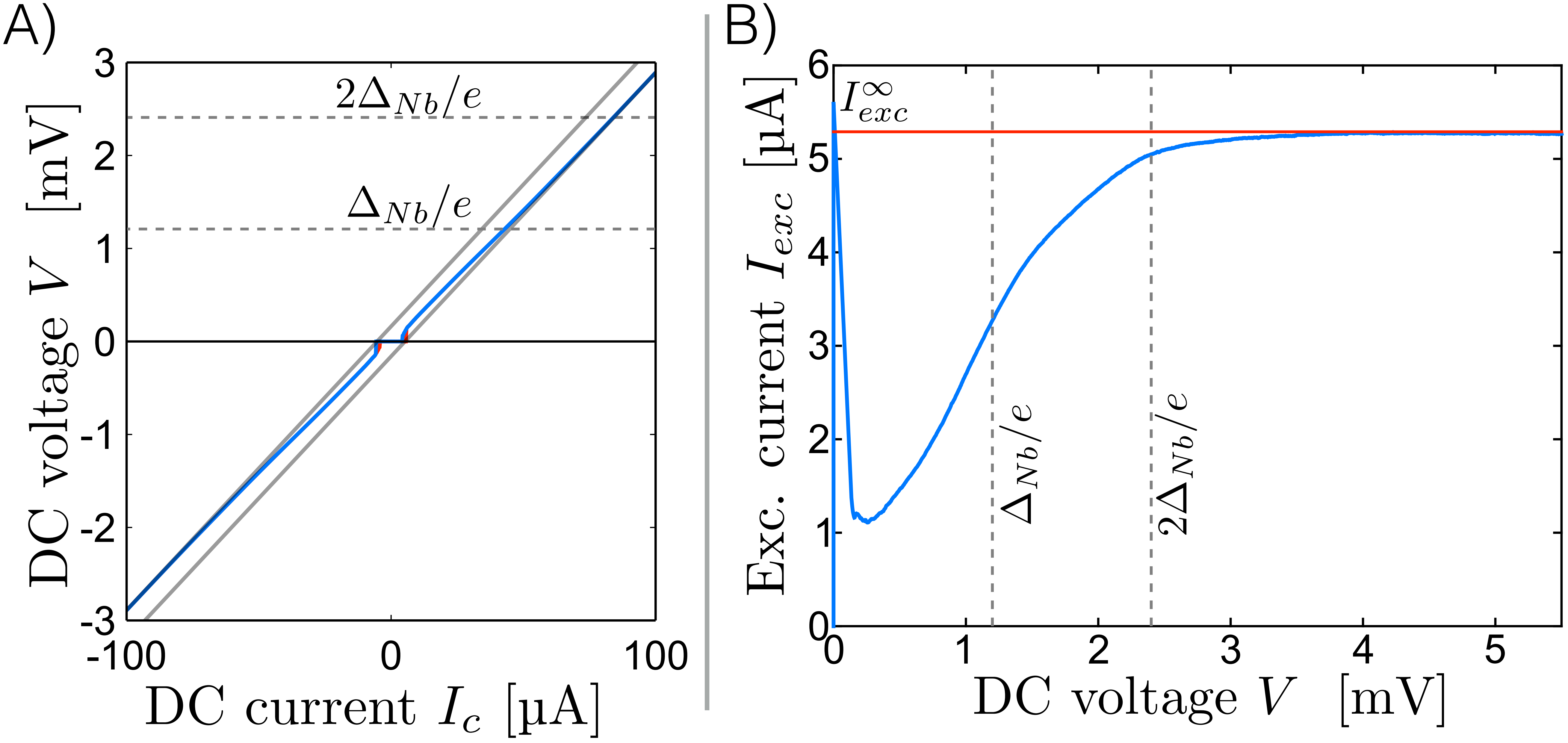
2π modes



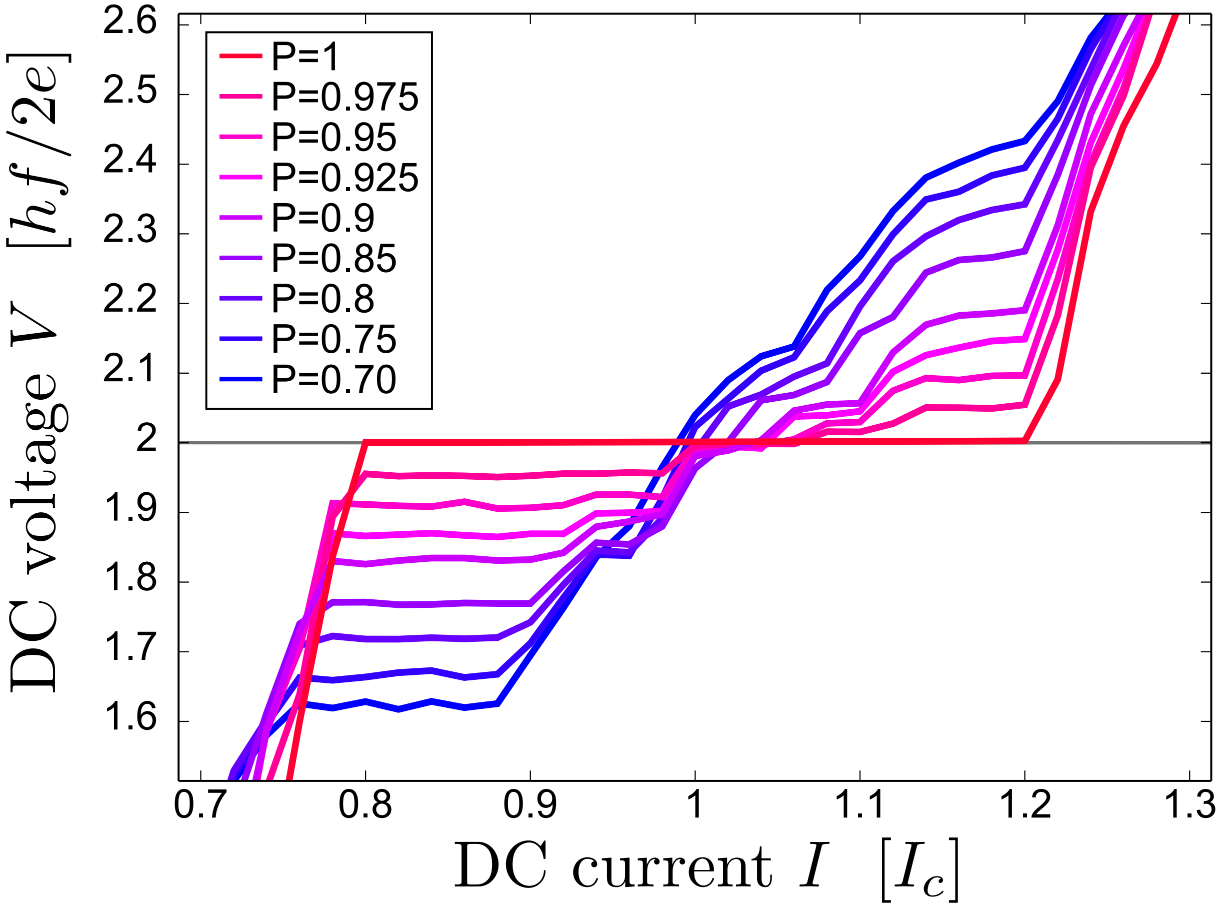
$2\pi + 4\pi$ modes

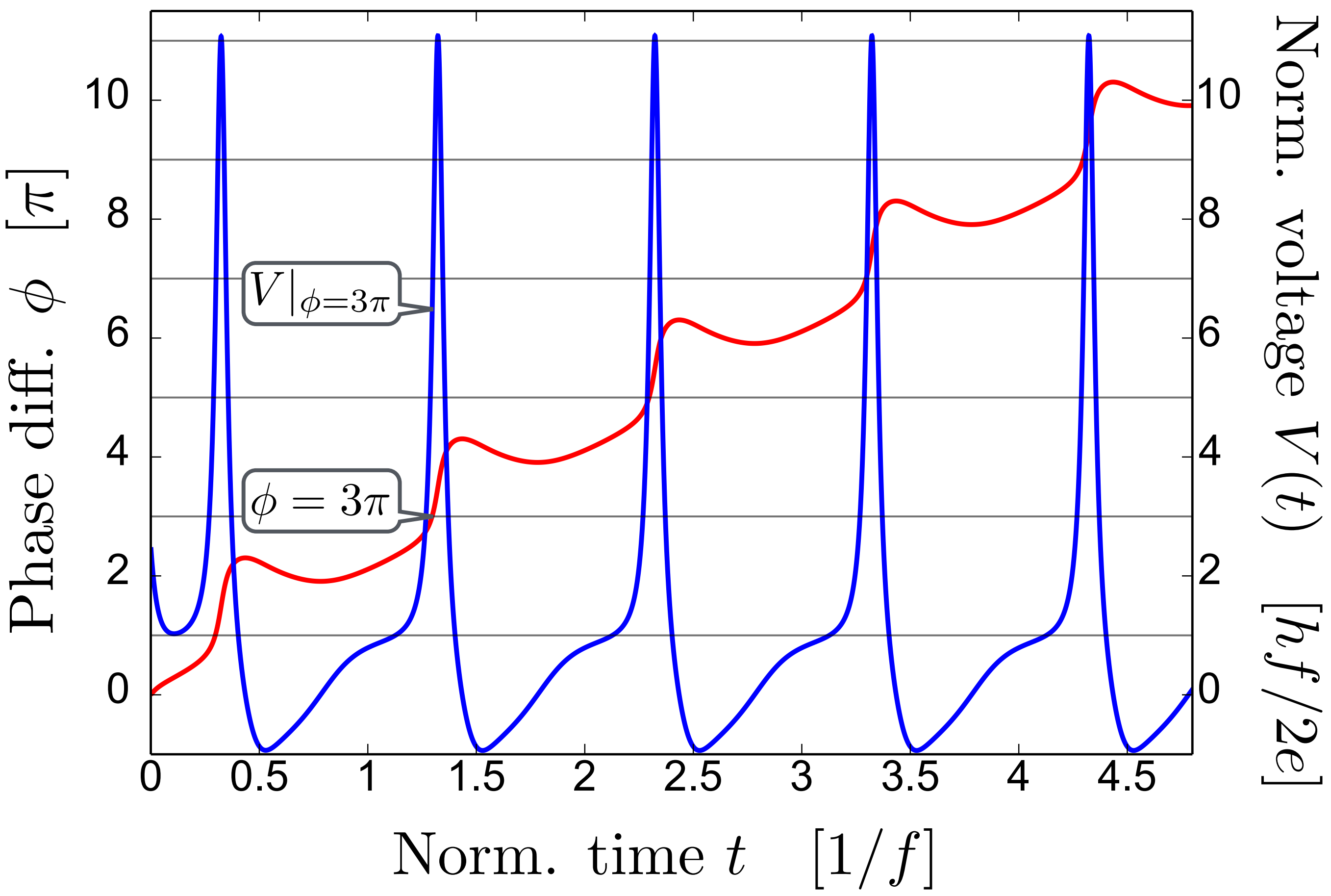


Excess current



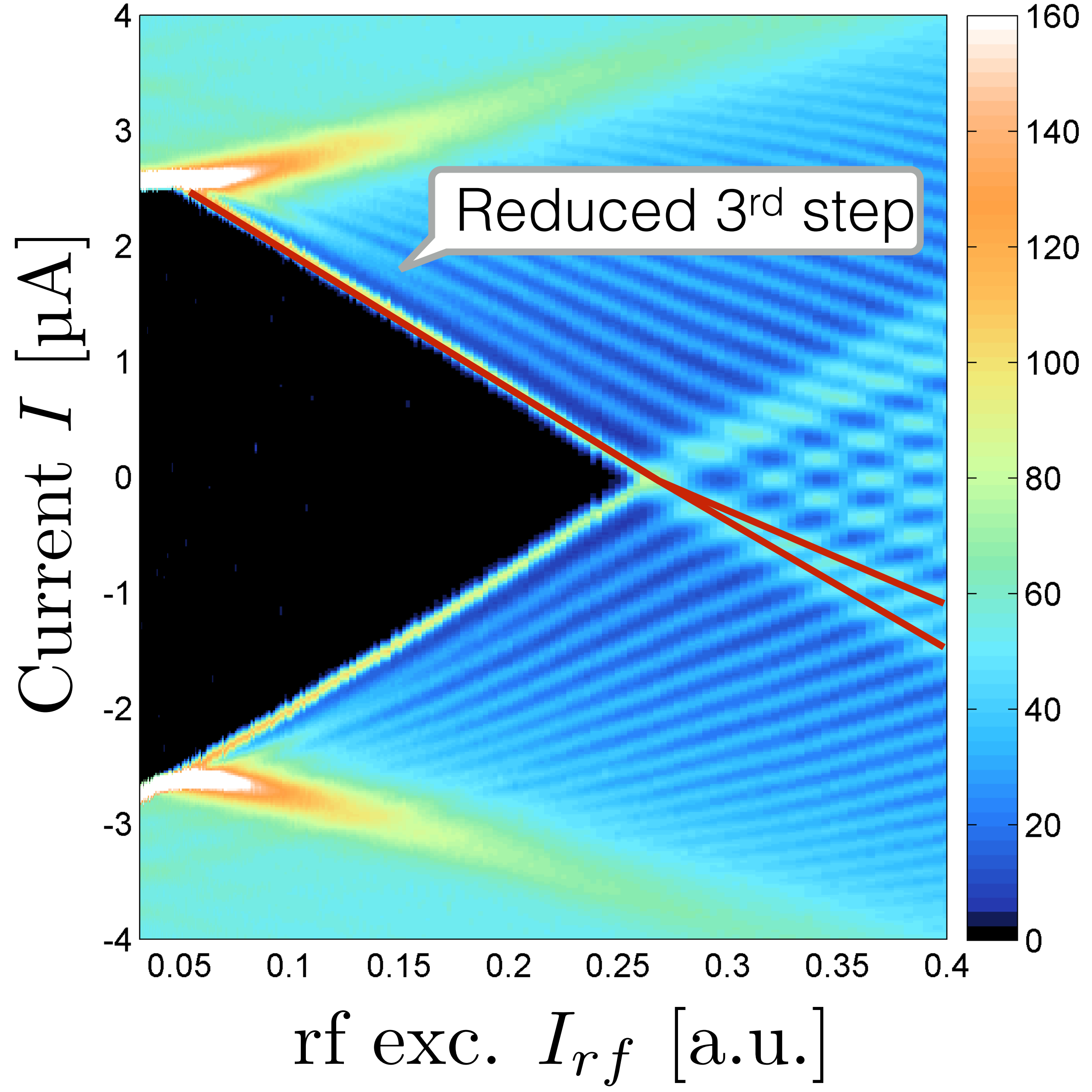
Landau-Zener transitions



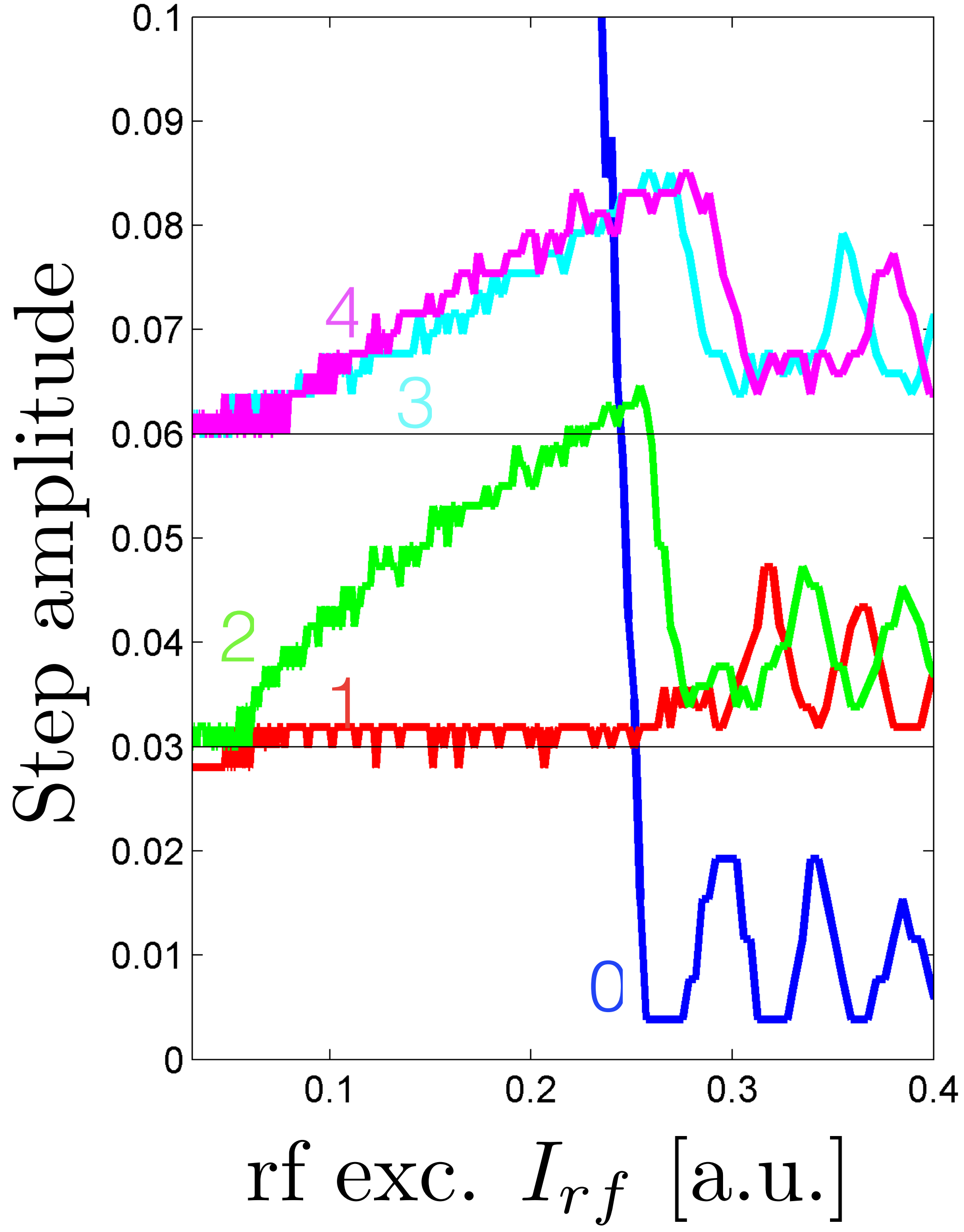


- Andreev bound states $\varepsilon_{\pm}(\phi) = \pm\sqrt{4\delta^2 + \Delta_i^2 \cos^2 \phi/2}$
- Landau-Zener transition probability $P_{LZ} = \exp\left(-2\pi \frac{4\delta^2}{\Delta_i \hbar \dot{\phi}}\right)$

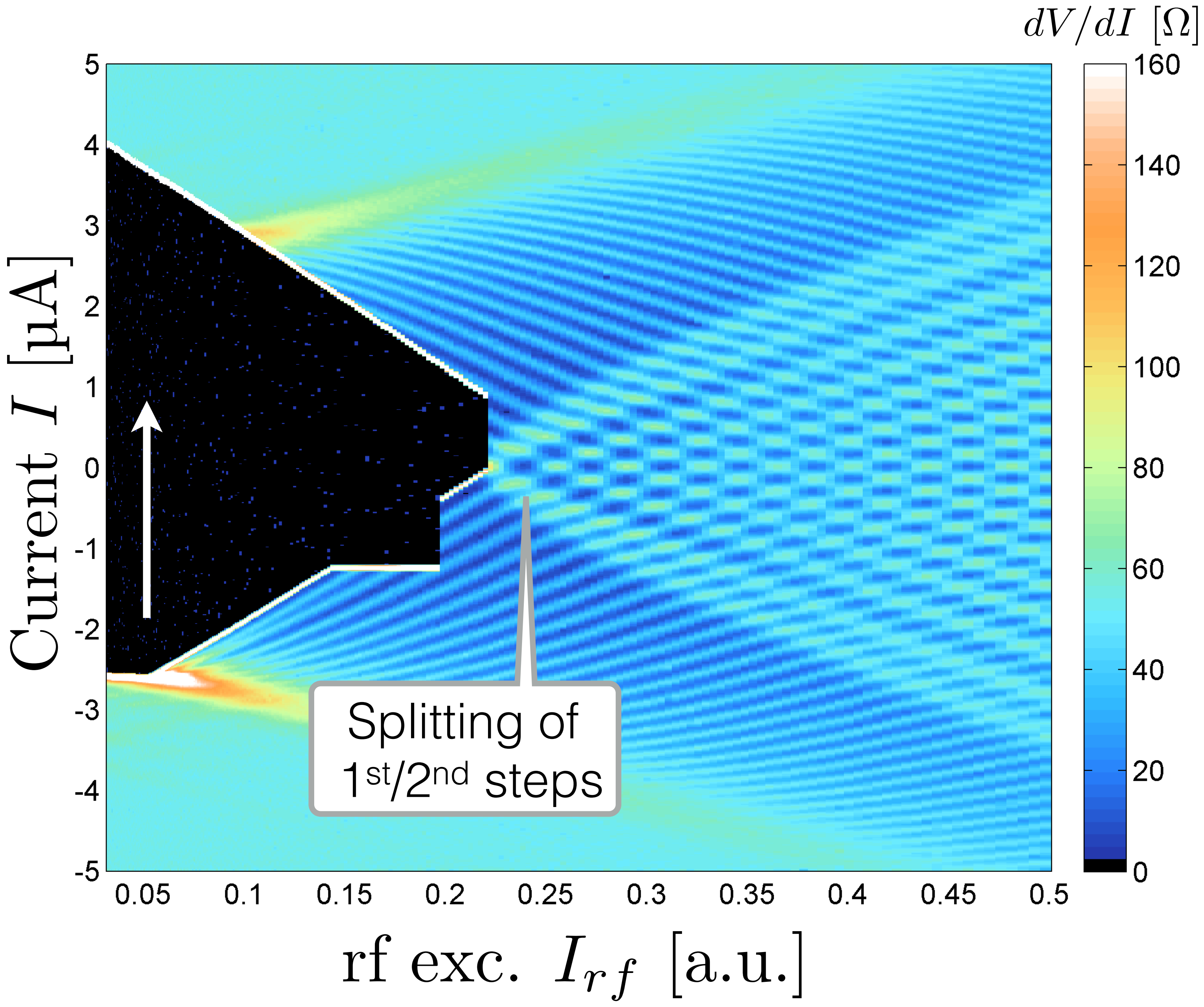
Weak suppression of third step



Reduced 3rd step



Hysteresis & Shapiro steps



Splitting of 1st/2nd steps

Hysteresis

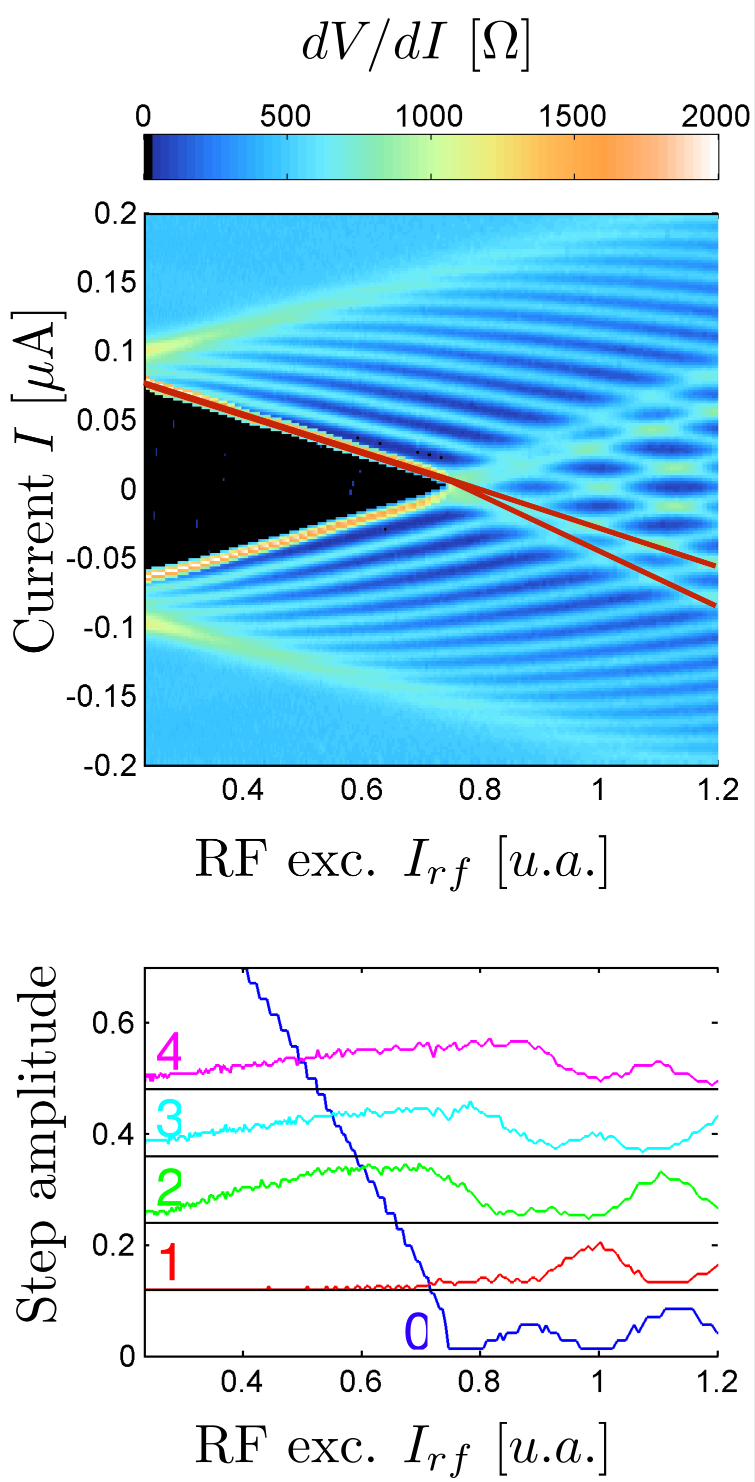
- ▹ unknown origin
- ▹ common in similar systems (graphene, TIs)

Shapiro steps

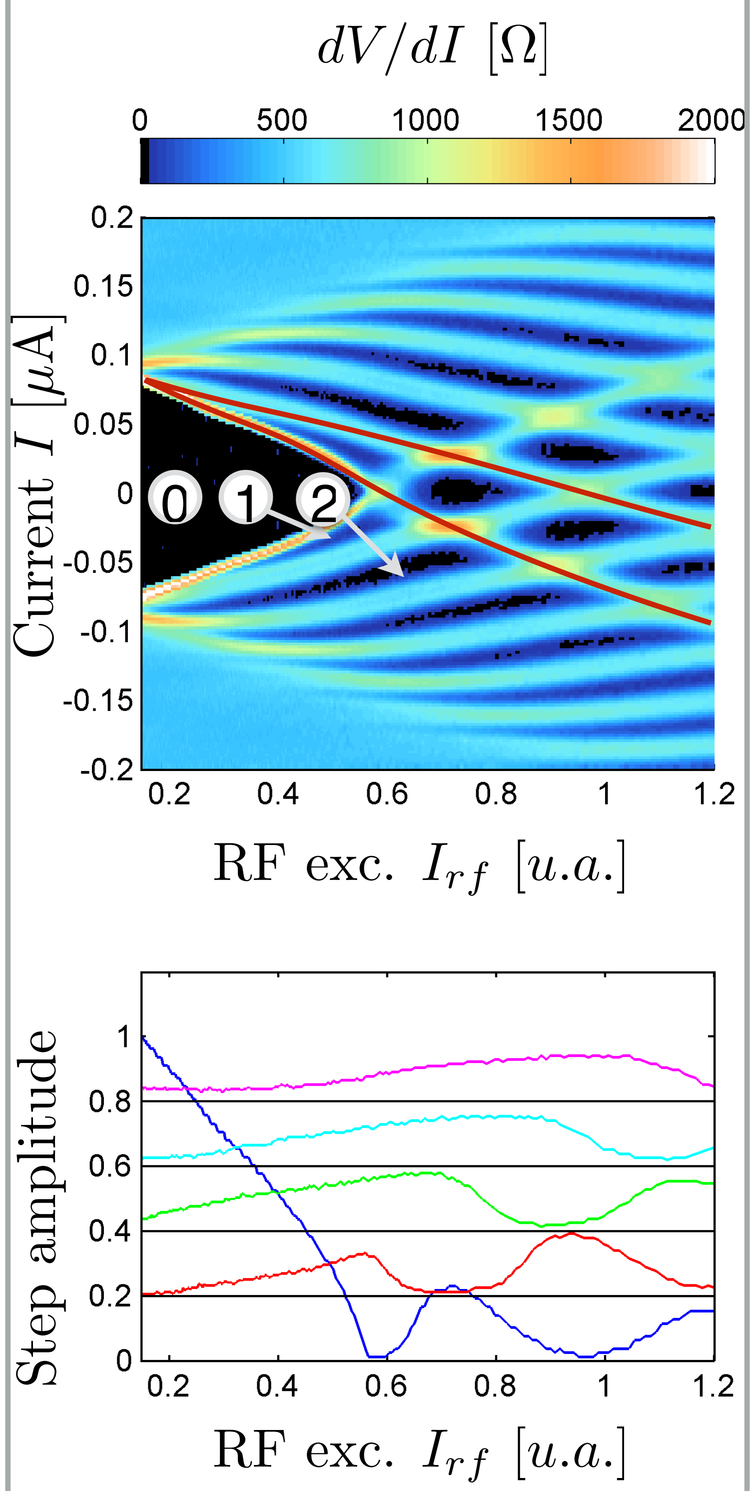
- ▹ bistability
  ⇒ no phase locking
- ▹ Shapiro steps invisible

Frequency dependence (2)

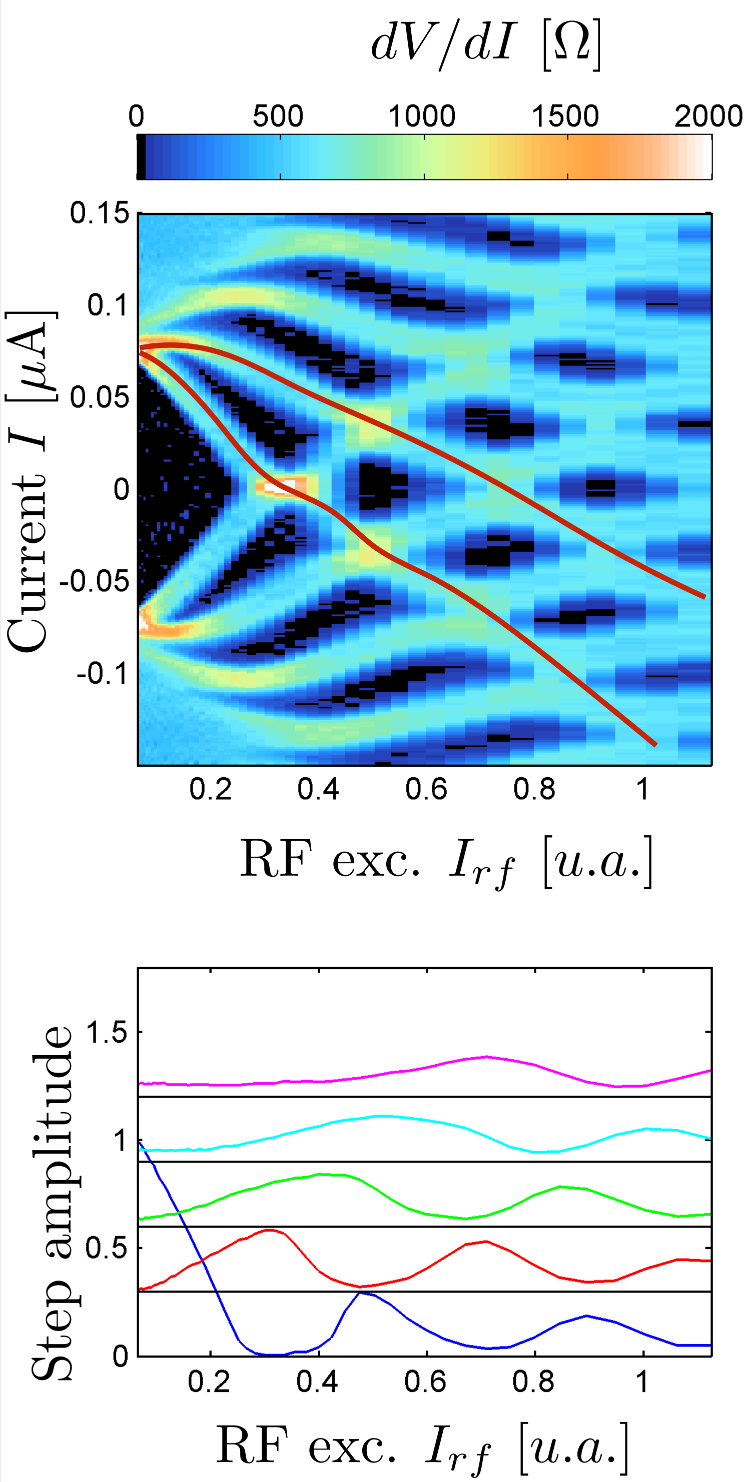
$f = 3.04$ GHz



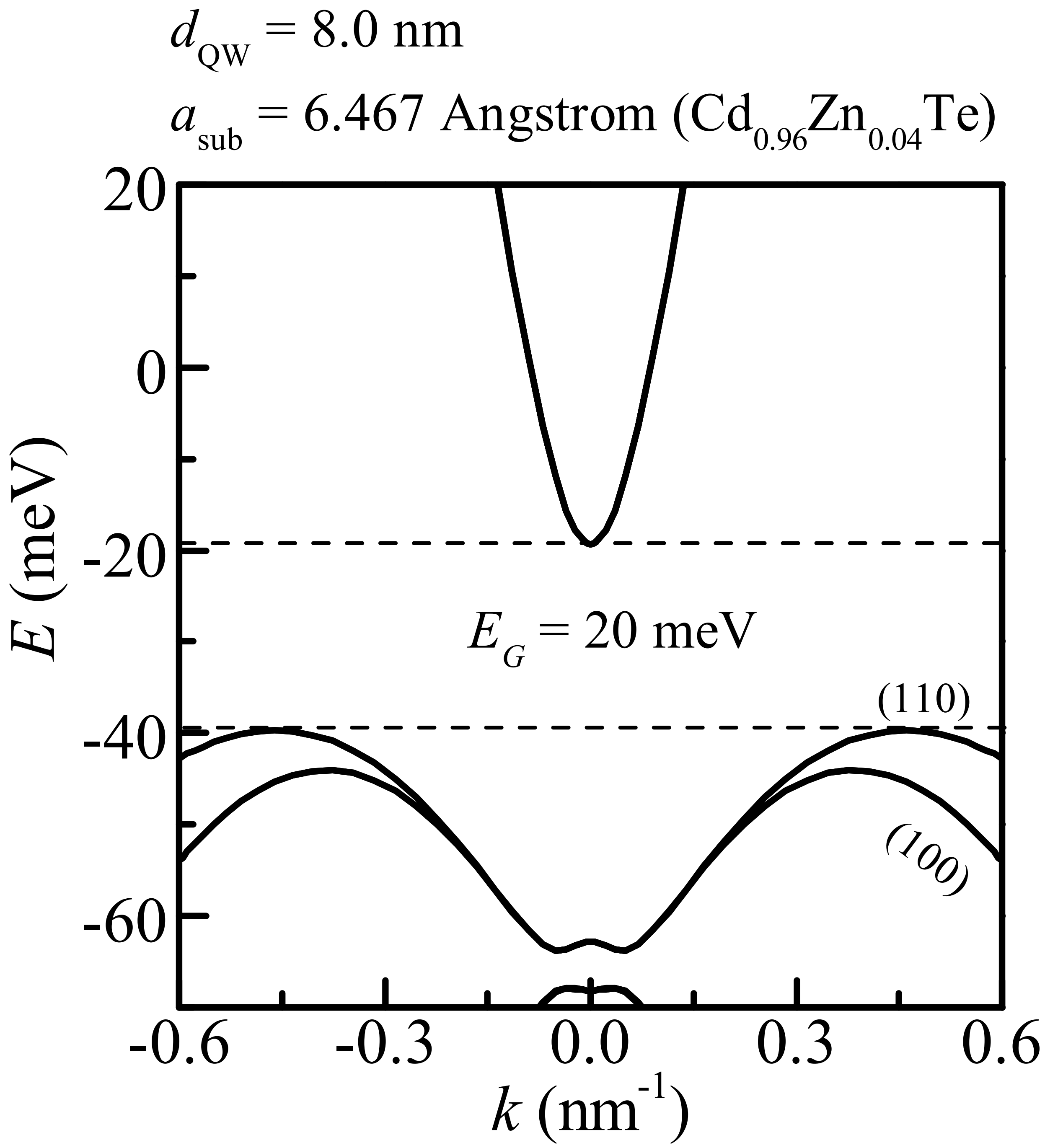
$f = 6.04$ GHz



$f = 7.5$ GHz

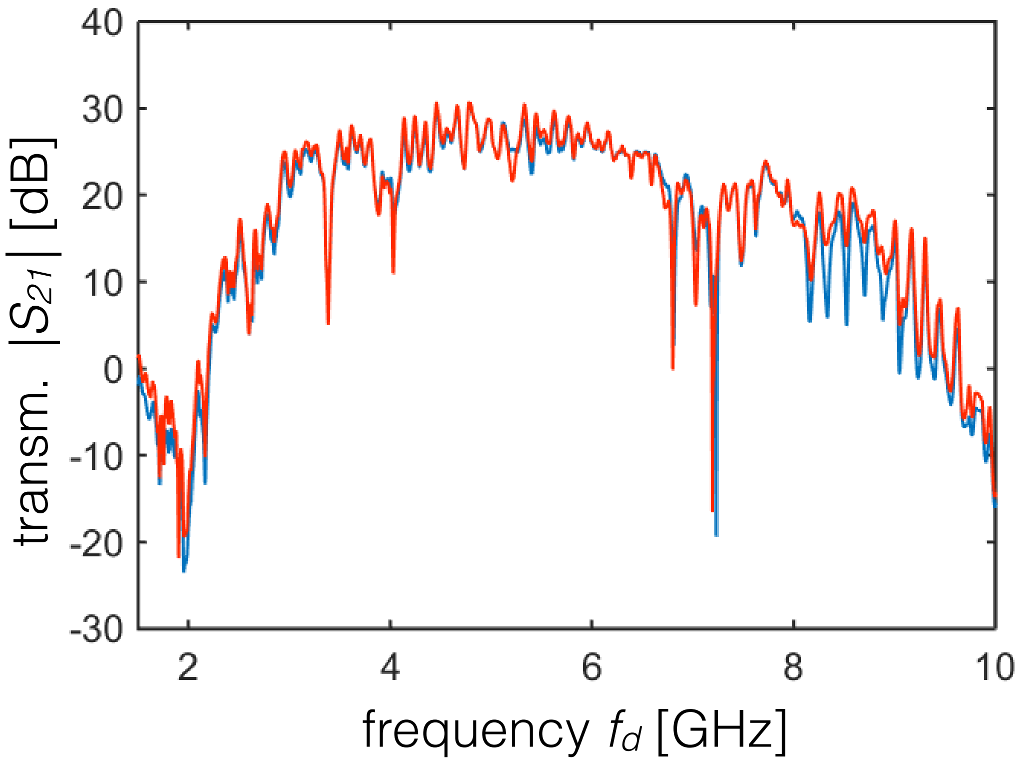


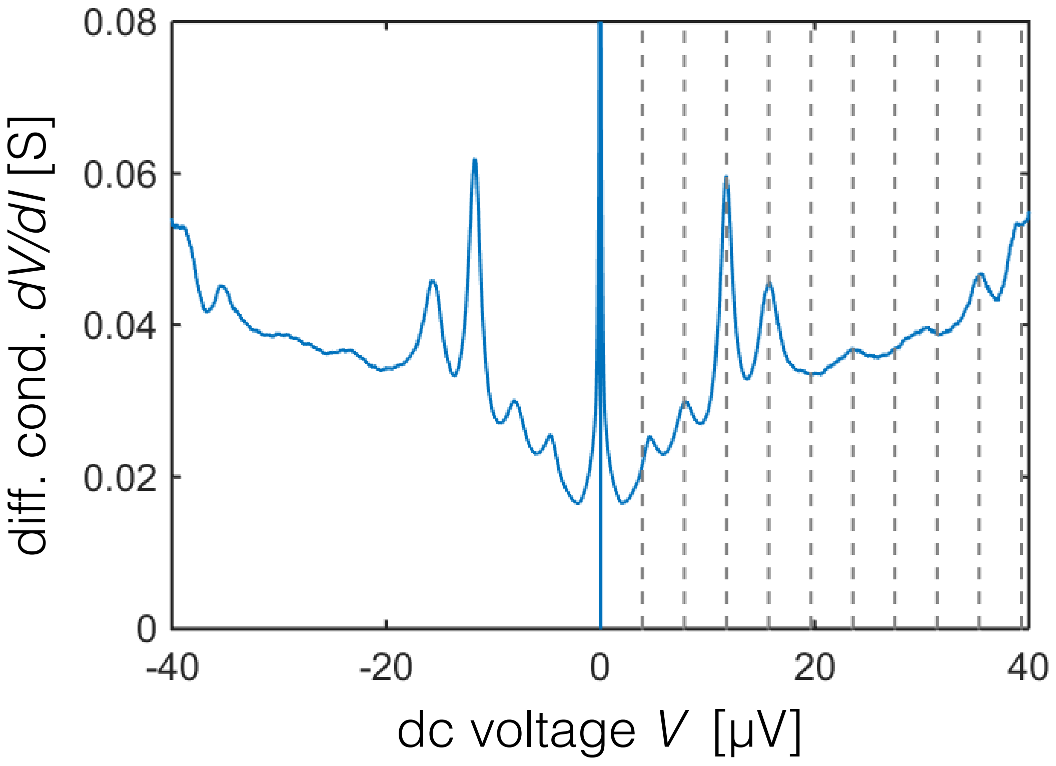
Band structure of quantum wells

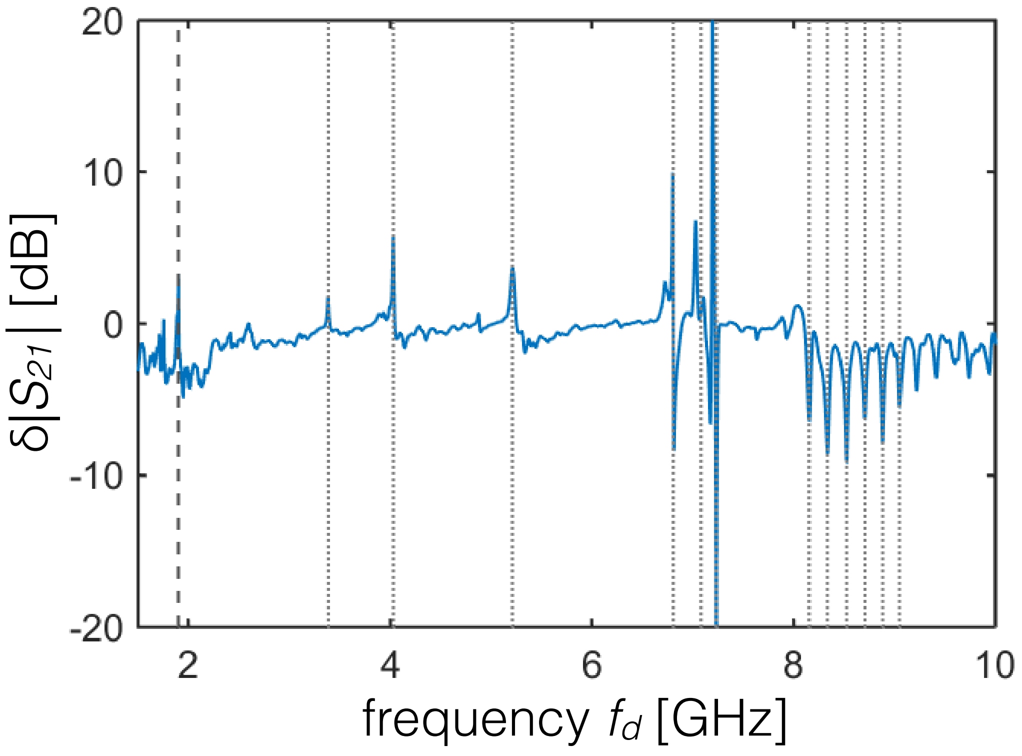


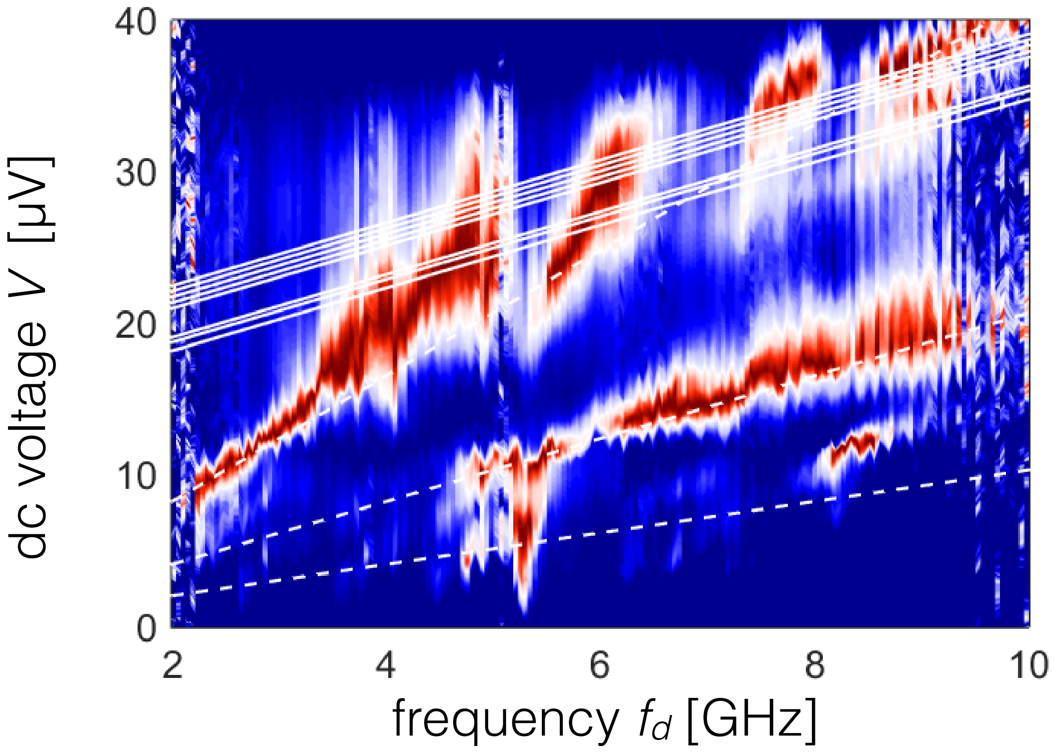
- ▹ « camelback » structure
  ⇒ low mobility in p-regime
- ▹ gap around 20 meV
- ▹ edge states not calculated

Resonances at high frequencies









Emission in 3D TI