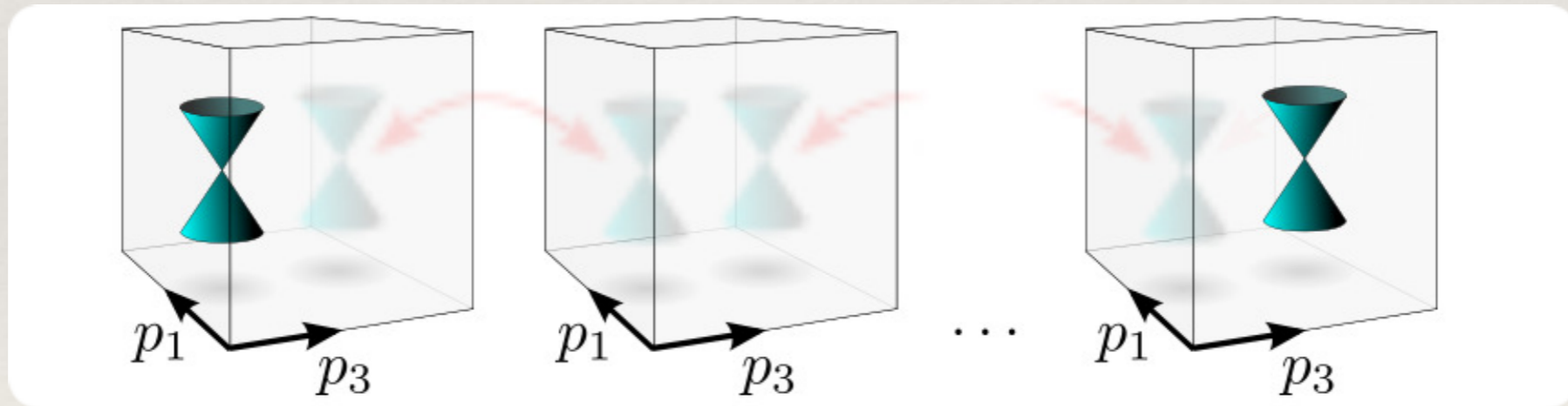
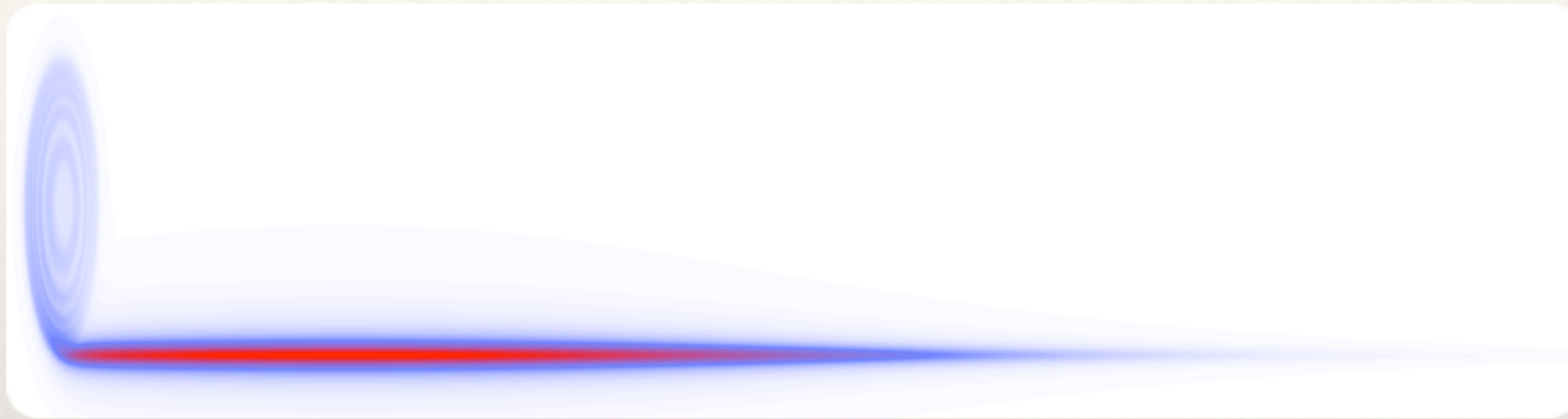


Weyl semimetals — from chiral anomaly to fractional chiral metal

Jens Hjörleifur Bärðarson

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KTH Royal Institute of Technology, Stockholm



J. Behrends, A. G. Grushin, T. Ojanen, JHB *Phys. Rev. B* **93**, 075114 (2016)
F. Arnold, ...A. G. Grushin, JHB, ... C. Felser, E. Hassinger, B. Yan *Nat. Comm.* **7**, 11615 (2016)
T. Meng, A. G. Grushin, K. Shtengel JHB arXiv:1602.08856 (to appear in PRB)

Weyl Fermions

Weyl fermions



Hermann Weyl (1885-1955)

PhD Göttingen 1908 with Hilbert

1908 - 1913 — Göttingen

1913 - 1930 — ETH Zürich

1930 - 1933 — Göttingen

1930 - 1951 — Princeton

— My work always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually chose the beautiful —

Dirac equation

$$(i\gamma^\mu \partial_\mu - m) \psi = 0$$

$$\gamma^0 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

Massless Weyl fermions: $m = 0$

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} \psi_1 + \psi_2 \\ \psi_1 - \psi_2 \end{pmatrix} = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix}$$

$$(i\partial_t \pm \mathbf{p} \cdot \boldsymbol{\sigma}) \psi_{R/L} = 0$$

Accidental crossing of two bands in 3D band structure is a Weyl point



$$H = \begin{pmatrix} A & B \\ B^* & C \end{pmatrix} = a + \mathbf{b} \cdot \boldsymbol{\sigma}$$

$$H(p_x, p_y, p_z) = a(\mathbf{p}) + \mathbf{b}(\mathbf{p}) \cdot \boldsymbol{\sigma}$$

$$\mathbf{b}(\mathbf{p}_*) = 0$$

$$b_i(\mathbf{p}) = \sum_j b_{ij} (\mathbf{p} - \mathbf{p}_*)_j + \mathcal{O}((\mathbf{p} - \mathbf{p}_*)^2)$$

$$b_{ij} = \left. \frac{\partial b_i}{\partial p_j} \right|_{\mathbf{p}=\mathbf{p}_*}$$

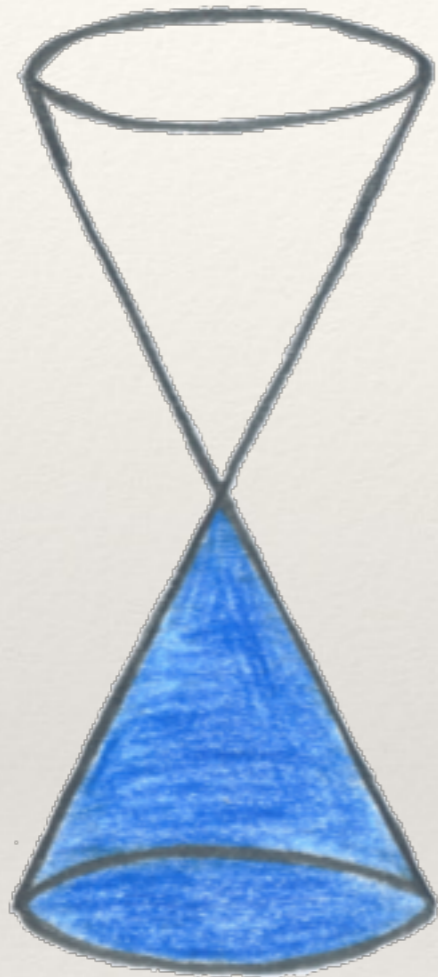
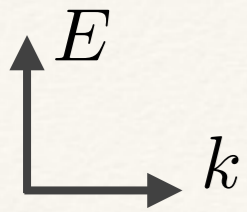
$$H' = \sum_{i,j} \sigma_i b_{ij} (\mathbf{p} - \mathbf{p}_*)_j \quad \rightarrow \quad \mathbf{p} \cdot \boldsymbol{\sigma}$$

Note $a(p)$!

Type II Weyl semimetals

see, e.g., Soluyanov et al Nature 2015

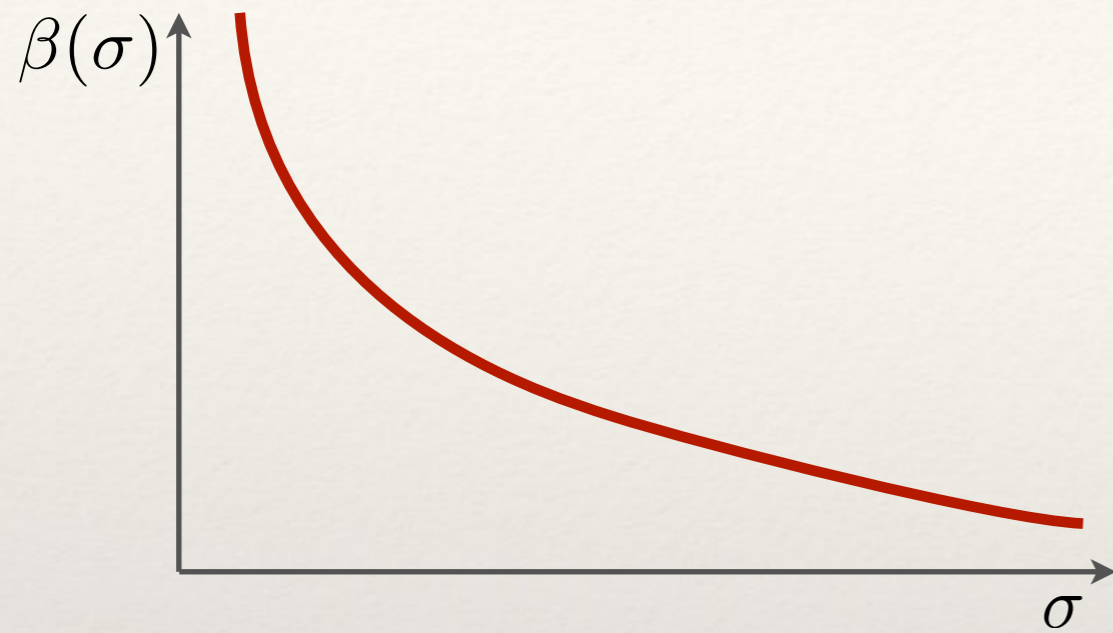
At low energy Weyl semimetals are described by Weyl fermions



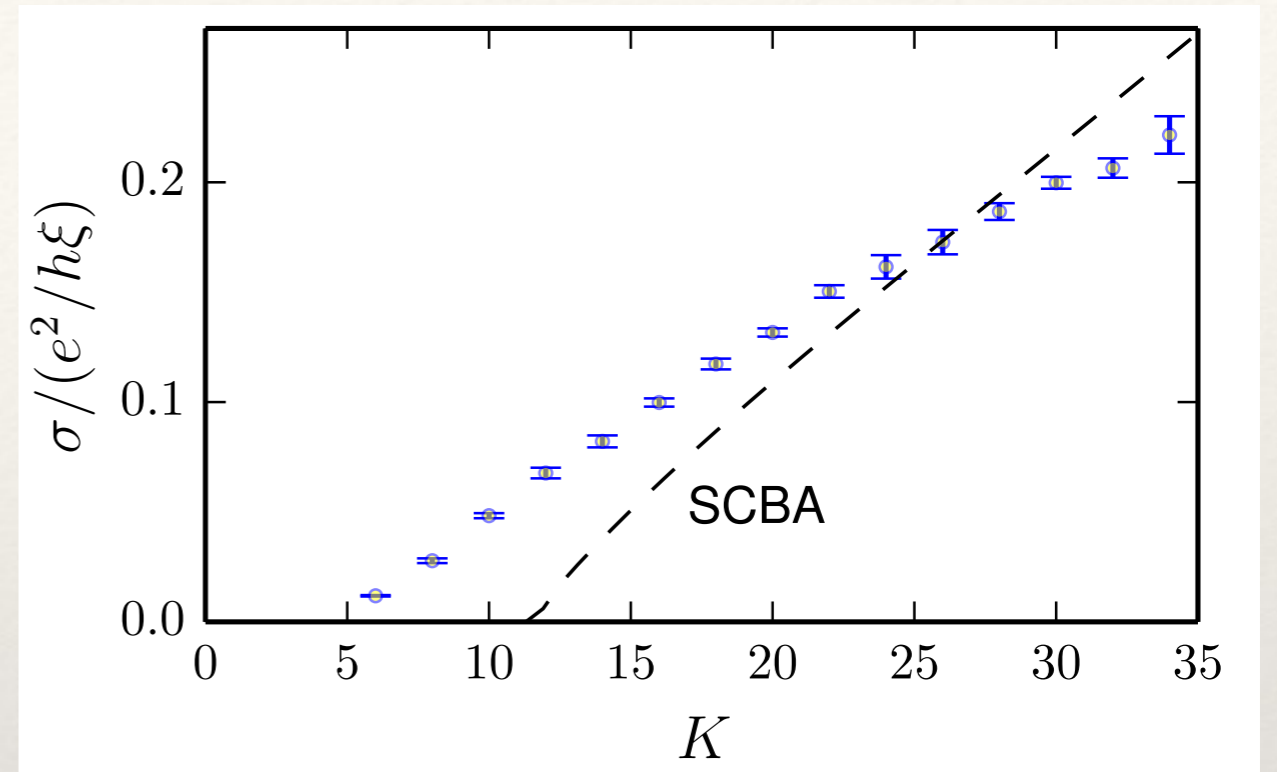
$$H = \mathbf{p} \cdot \boldsymbol{\sigma} = p_x \sigma_x + p_y \sigma_y + p_z \sigma_z$$

Scalar disorder is irrelevant for 3D Weyl nodes

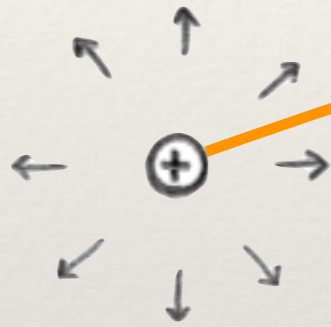
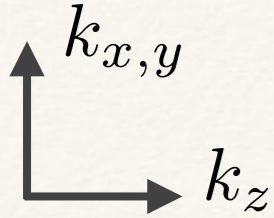
2D



3D



Weyl node is a topologically stable monopole of Berry flux



$$\mathbf{A}_{\mathbf{p}} = i \langle u_{\mathbf{p}} | \nabla u_{\mathbf{p}} \rangle$$

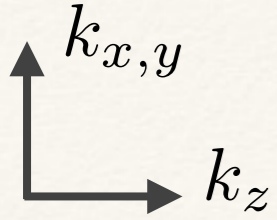
Berry phase

$$\mathbf{\Omega}_{\mathbf{p}} = \nabla_{\mathbf{p}} \times \mathbf{A}_{\mathbf{p}}$$

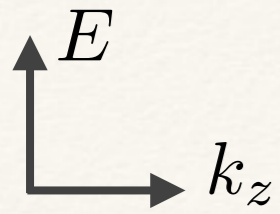
Berry curvature

$$\nu^{(i)} = \frac{1}{2\pi\hbar} \oint d\mathbf{S} \cdot \mathbf{\Omega}_{\mathbf{p}}^{(i)} \in \mathbb{Z} \quad \text{topological invariant}$$

In momentum space the monopoles always come in pairs



The minimal model of a Weyl semimetal has two Weyl nodes



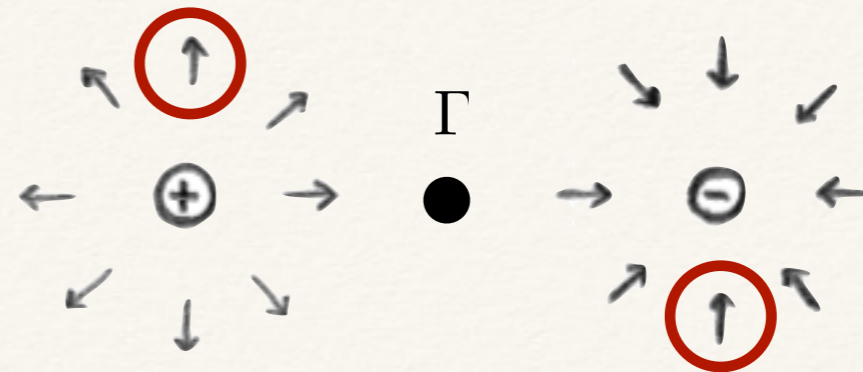
Weyl semimetals necessarily break time reversal and/or inversion symmetry

Inversion symmetry

$$\mathbf{k} \rightarrow -\mathbf{k}$$

$$\sigma \rightarrow \sigma$$

$$\varepsilon_{\uparrow}(\mathbf{k}) = \varepsilon_{\uparrow}(-\mathbf{k})$$

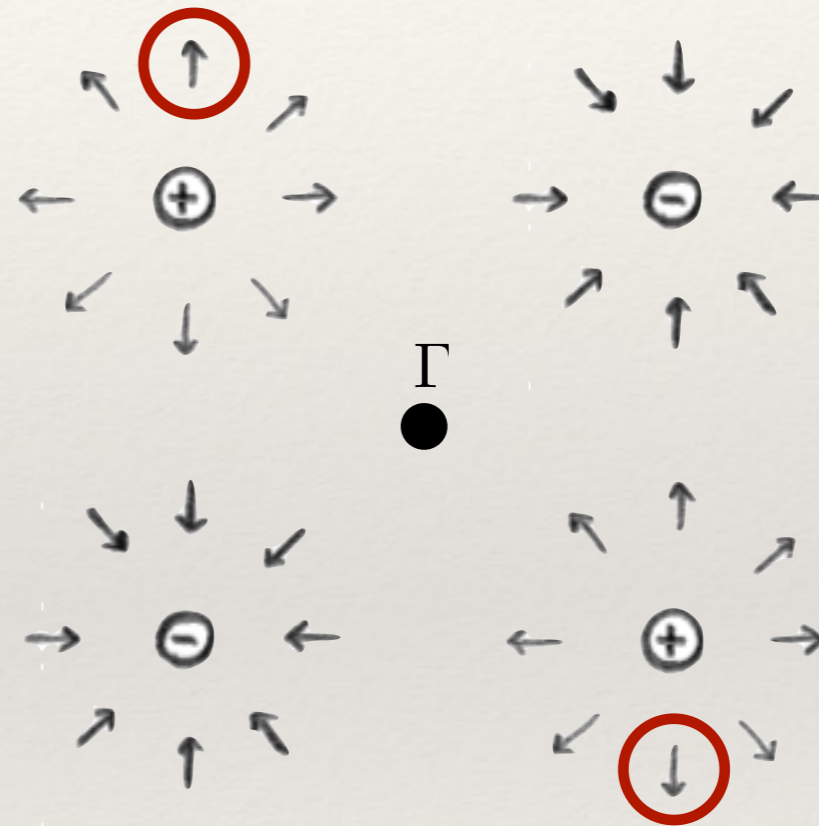


Time reversal symmetry

$$\mathbf{k} \rightarrow -\mathbf{k}$$

$$\sigma \rightarrow -\sigma$$

$$\varepsilon_{\downarrow}(\mathbf{k}) = \varepsilon_{\uparrow}(-\mathbf{k})$$



TRS + IS Dirac semimetal

$$\varepsilon_{\uparrow}(\mathbf{k}) = \varepsilon_{\uparrow}(-\mathbf{k}) = \varepsilon_{\downarrow}(\mathbf{k})$$

Weyl (and Dirac) semimetals exist!

Dirac semimetals — 2014

Na₃Bi

Xu et al. Science 2015
Liu et al. Science 2014
Kushwaha et al. APL Mat. 2015
...

Cd₃As₂

Neupane et al. Nature Comm. 2014
Borisenko et al. PRL 2014
Yi et al. Sci. Rep. 2014
Liu et al. Nature Mater. 2014
Liang et al. Nature Mater. 2014
He et al. PRL 2014
...

...

See Claudia Felser's talk!

Weyl semimetals — 2015

Weng et al. PRX 2015 (th)

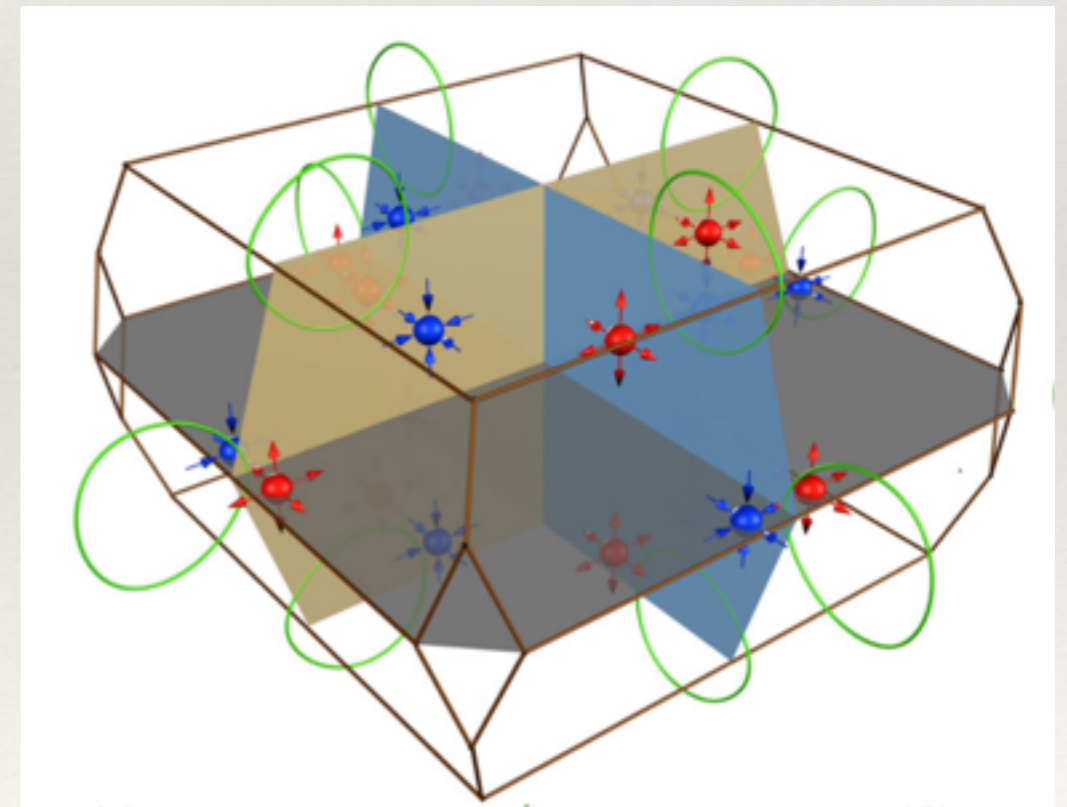
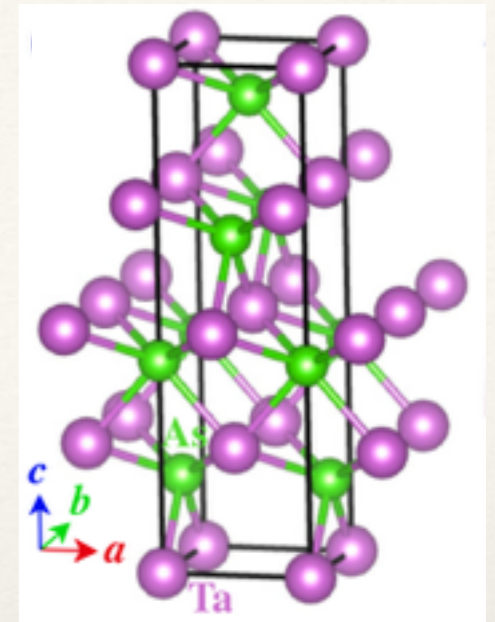
TaAs

Xu et al. Science 2015
Yang et al. Nature Physics 2015,
Lv et al. Nature Physics 2015, PRX 2015
...

NbAs

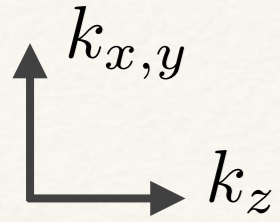
Xu et al. Nature Physics 2015
...

...

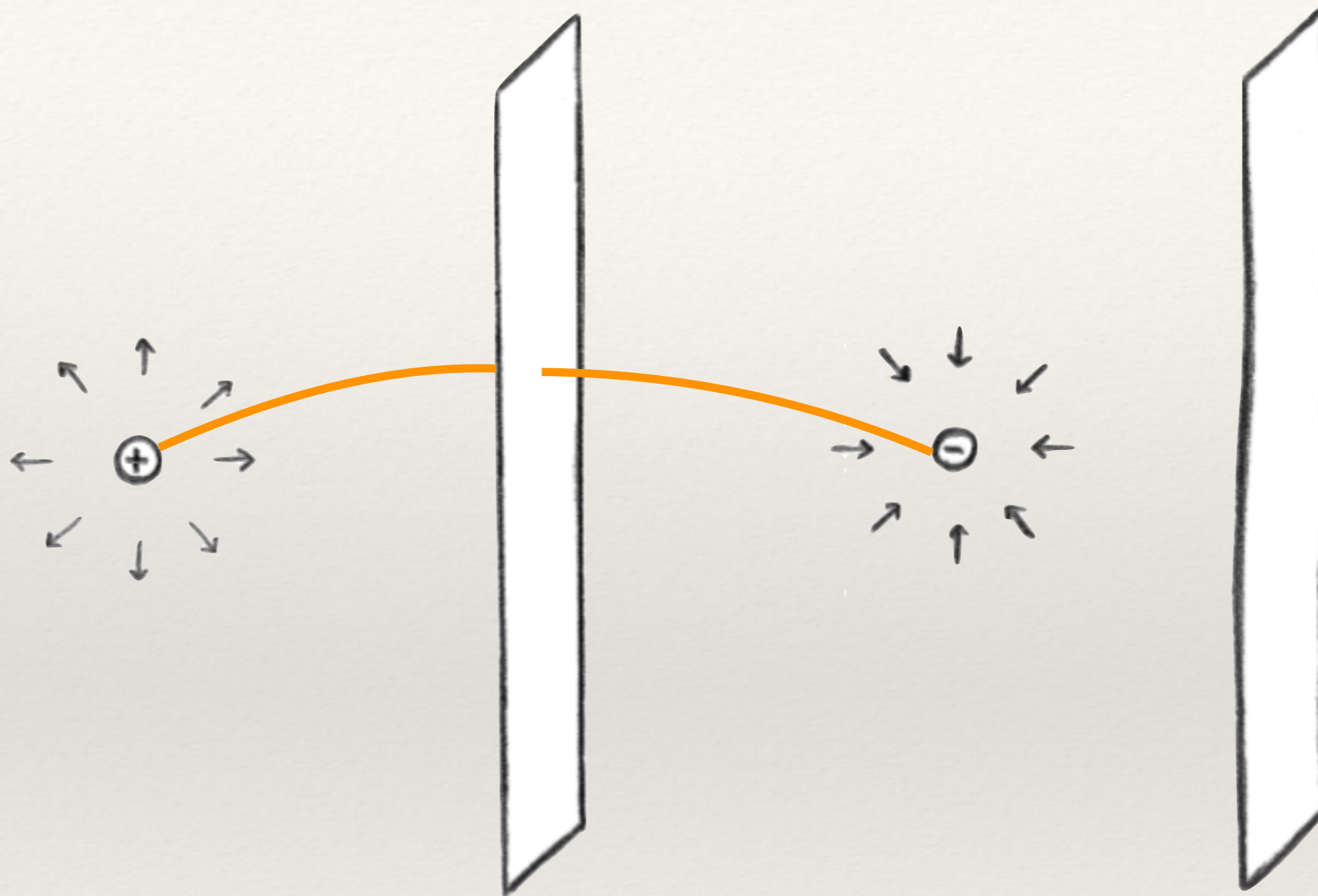
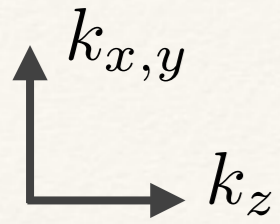


Fermi Arcs

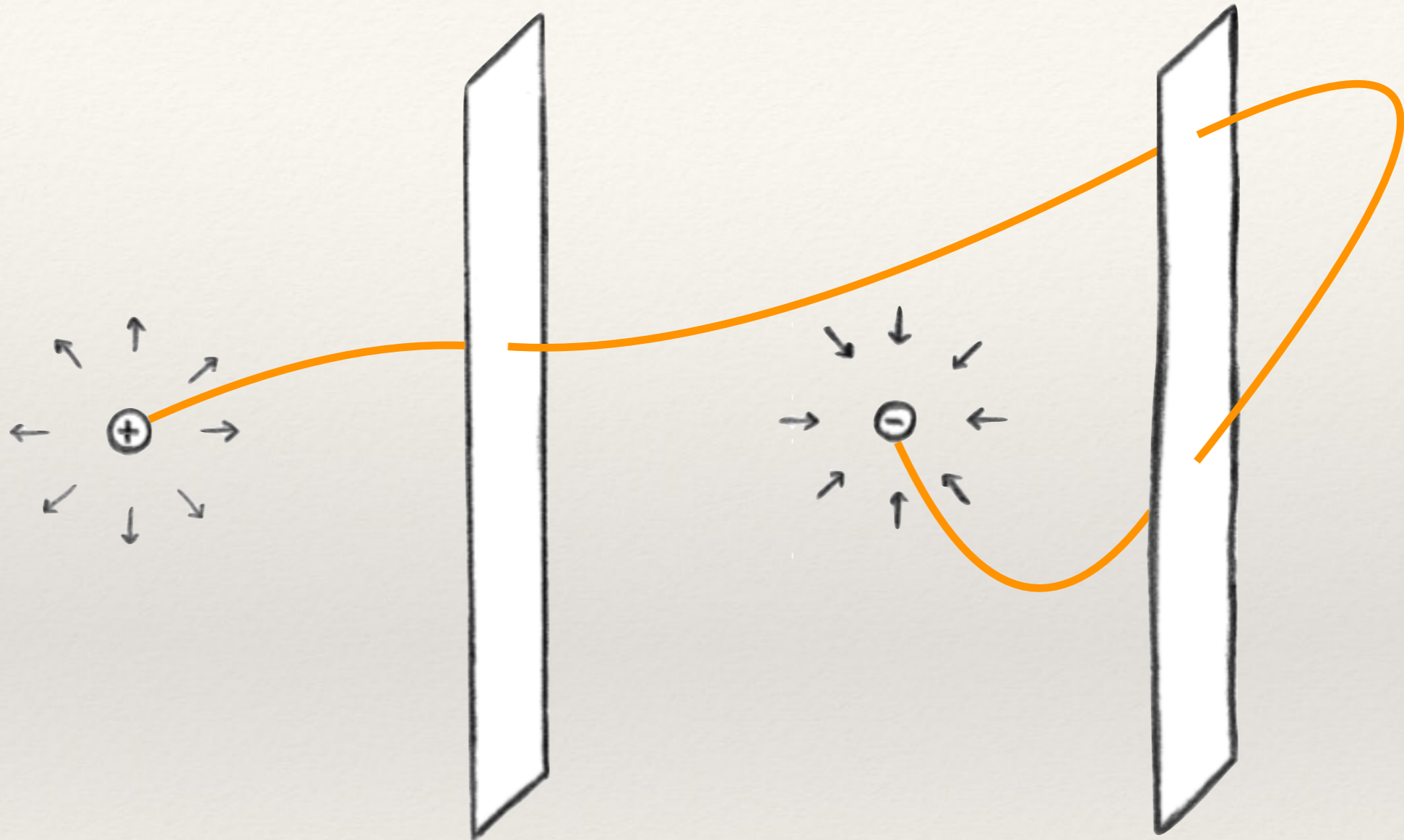
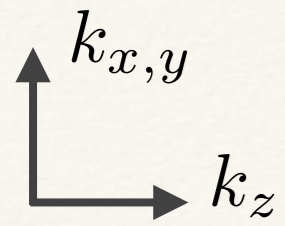
In momentum space the monopoles always come in pairs



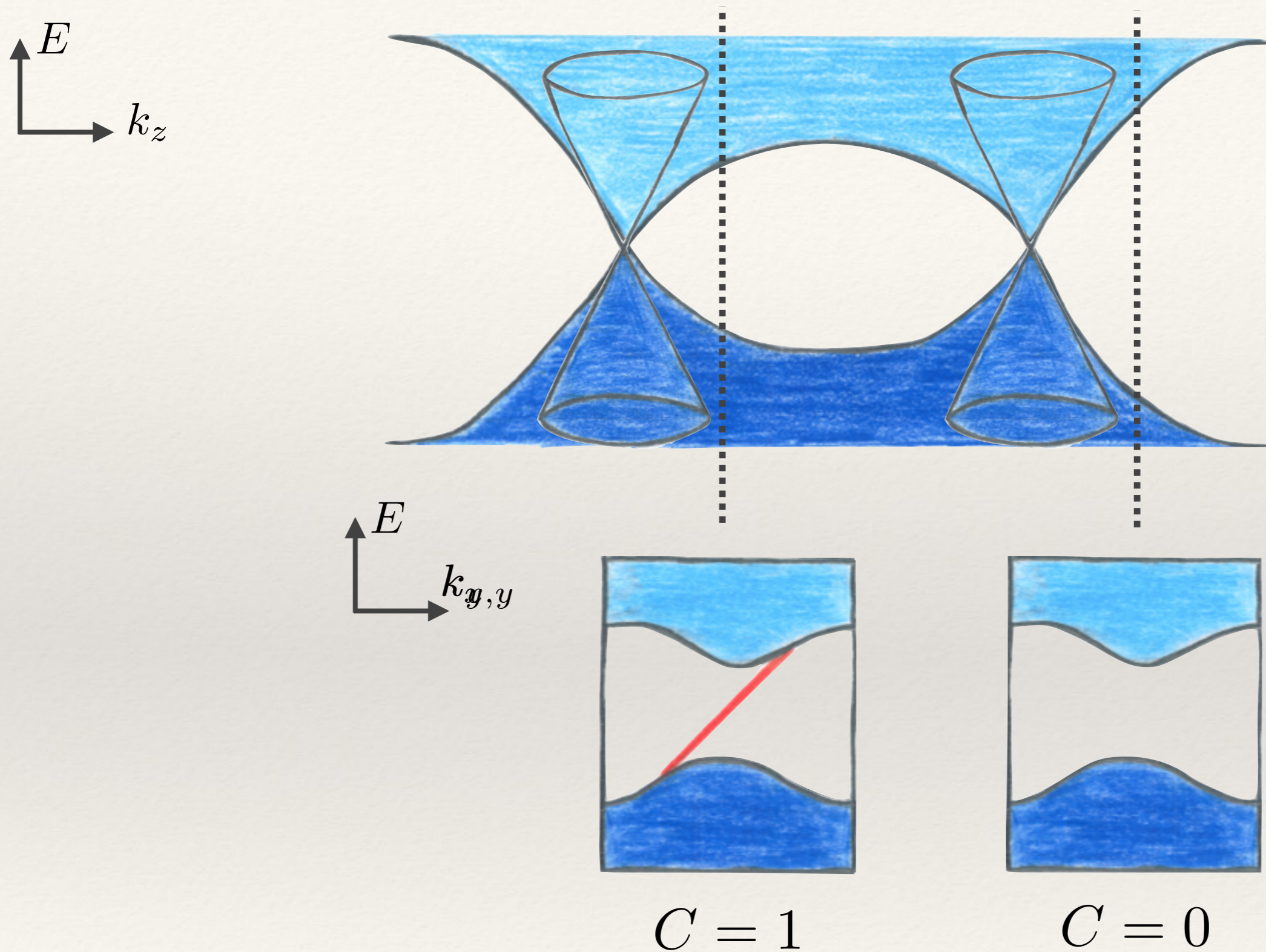
In some parts of momentum space, Dirac string cuts a plane odd times



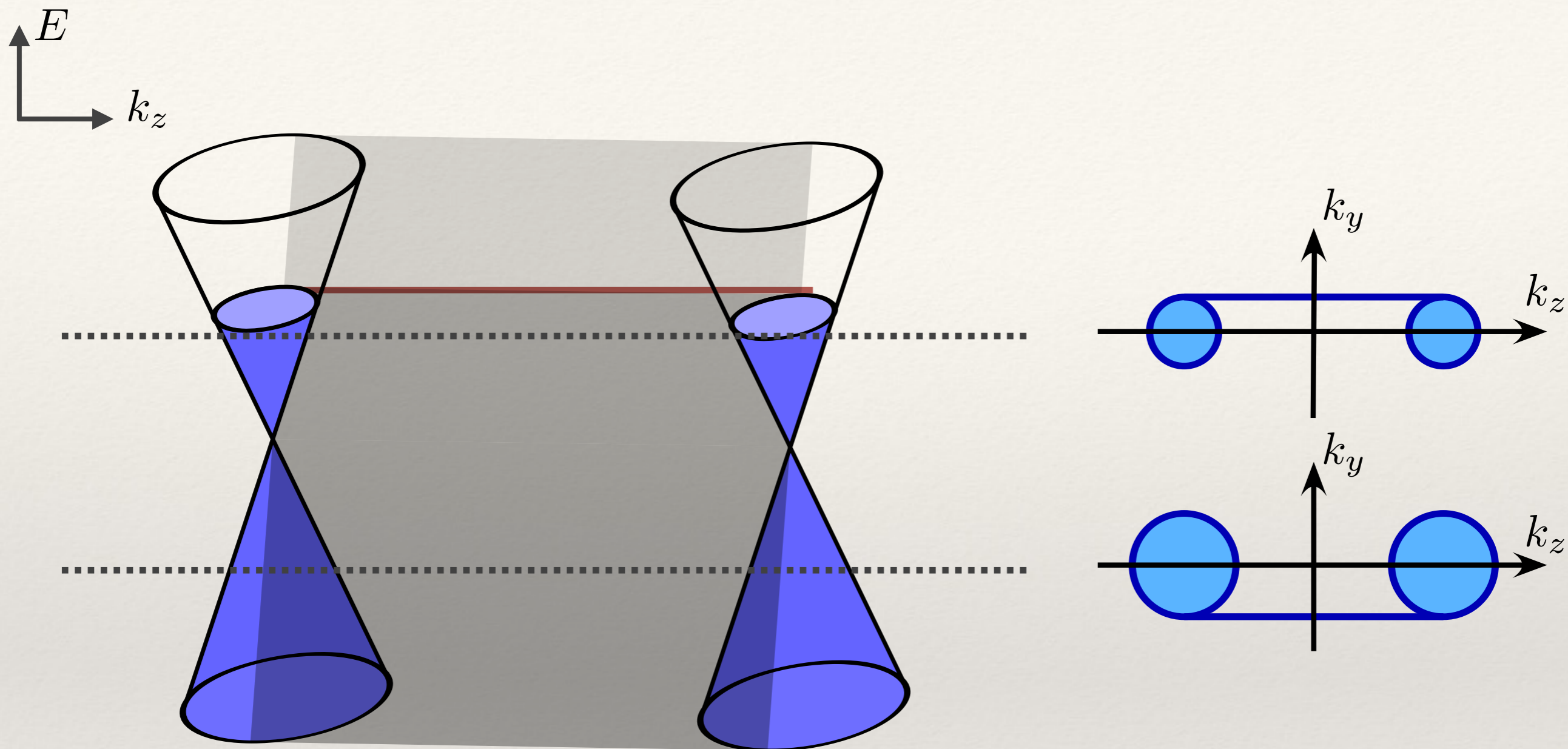
In some parts of momentum space, Dirac string cuts a plane odd times



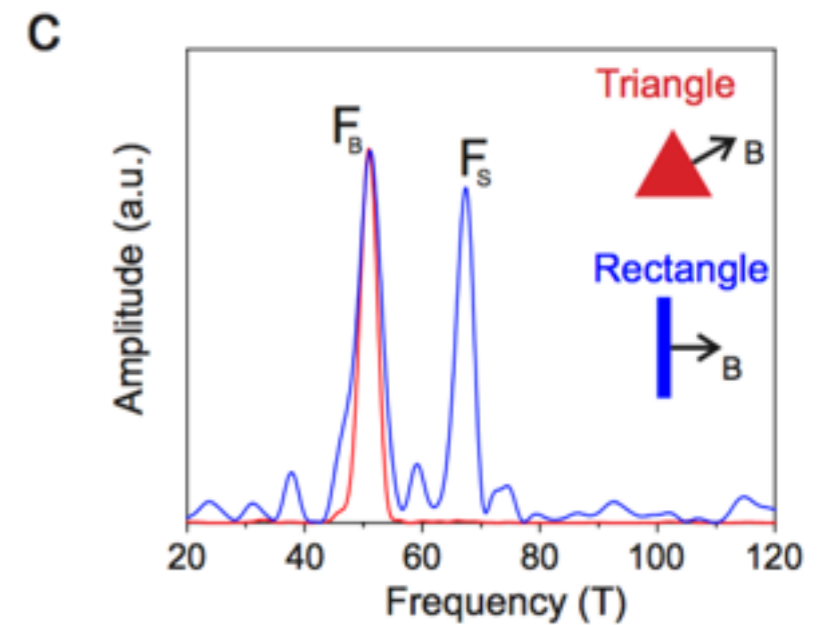
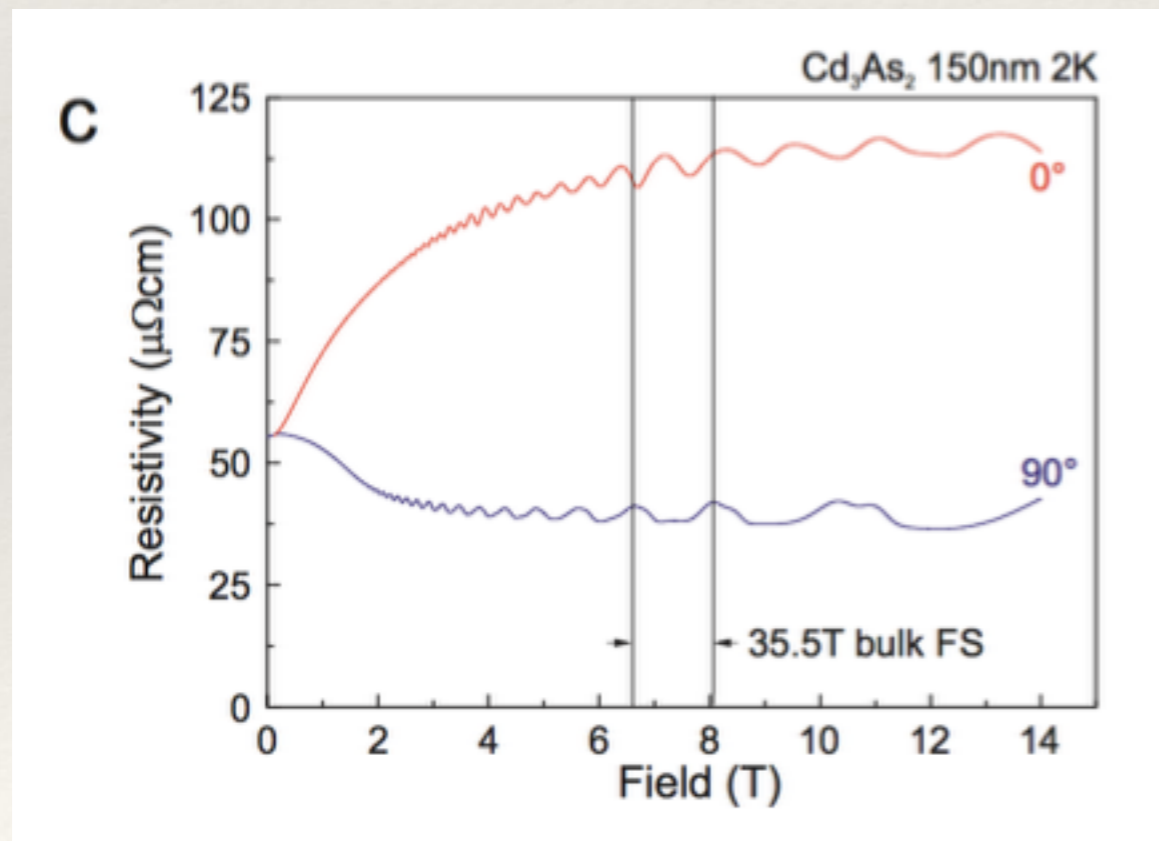
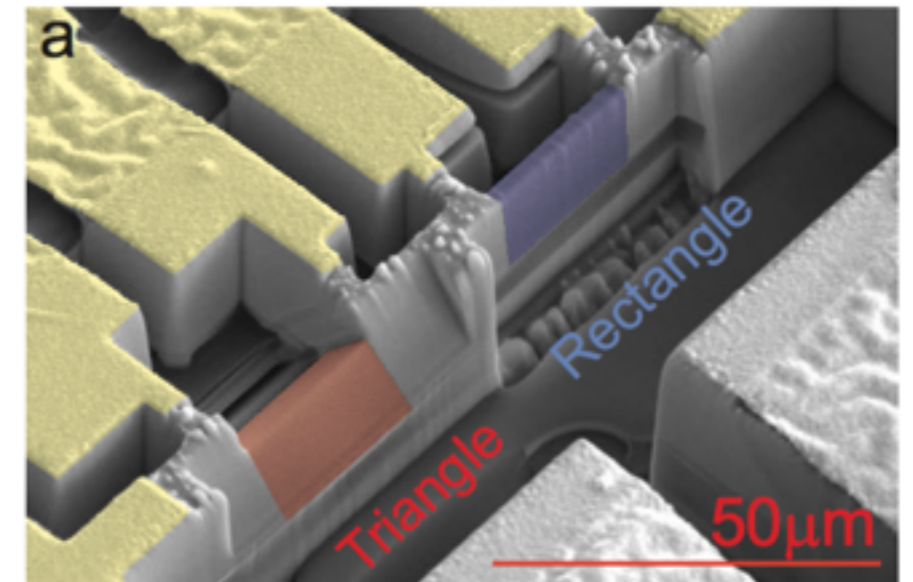
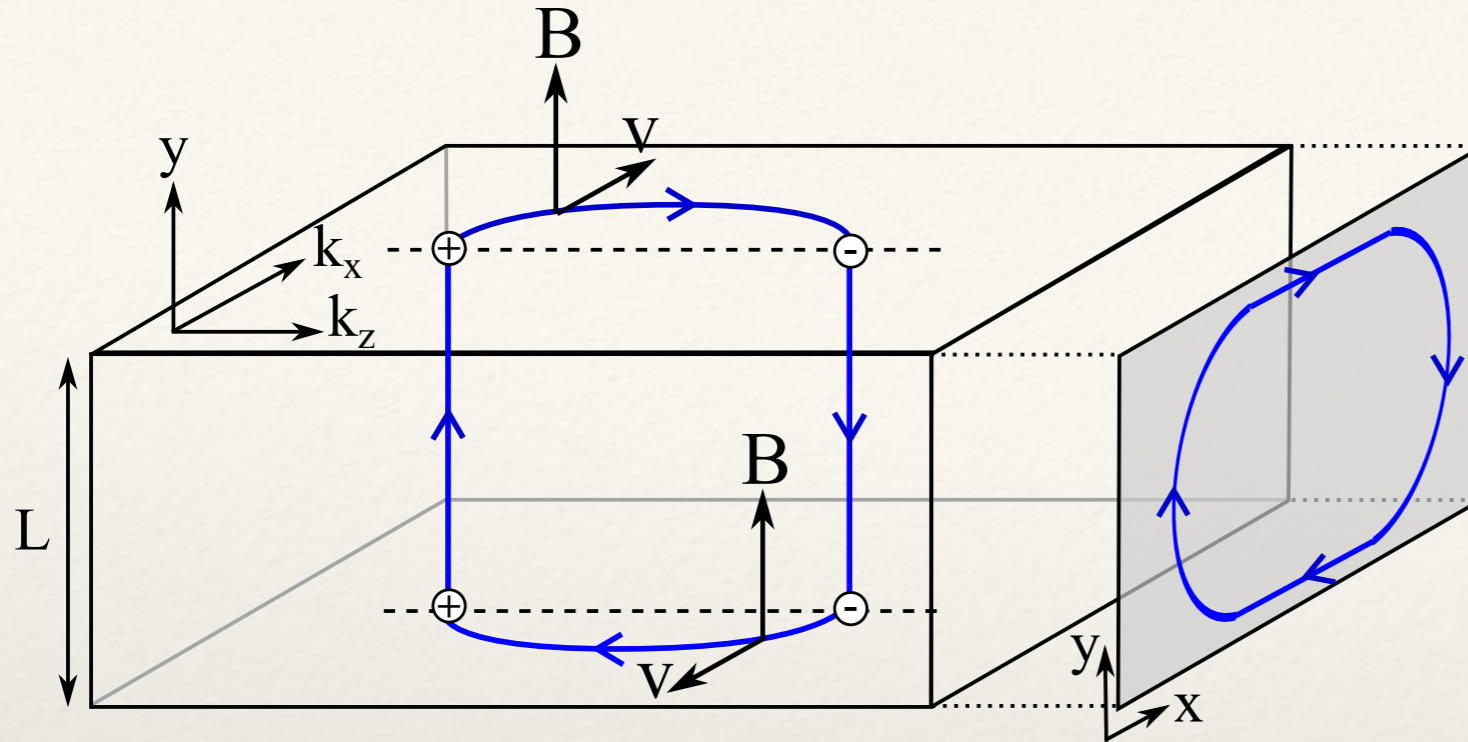
Topological surface states appear for certain momentum values



The Fermi arc separates occupied and unoccupied surface states



Quantum oscillations from Fermi arcs



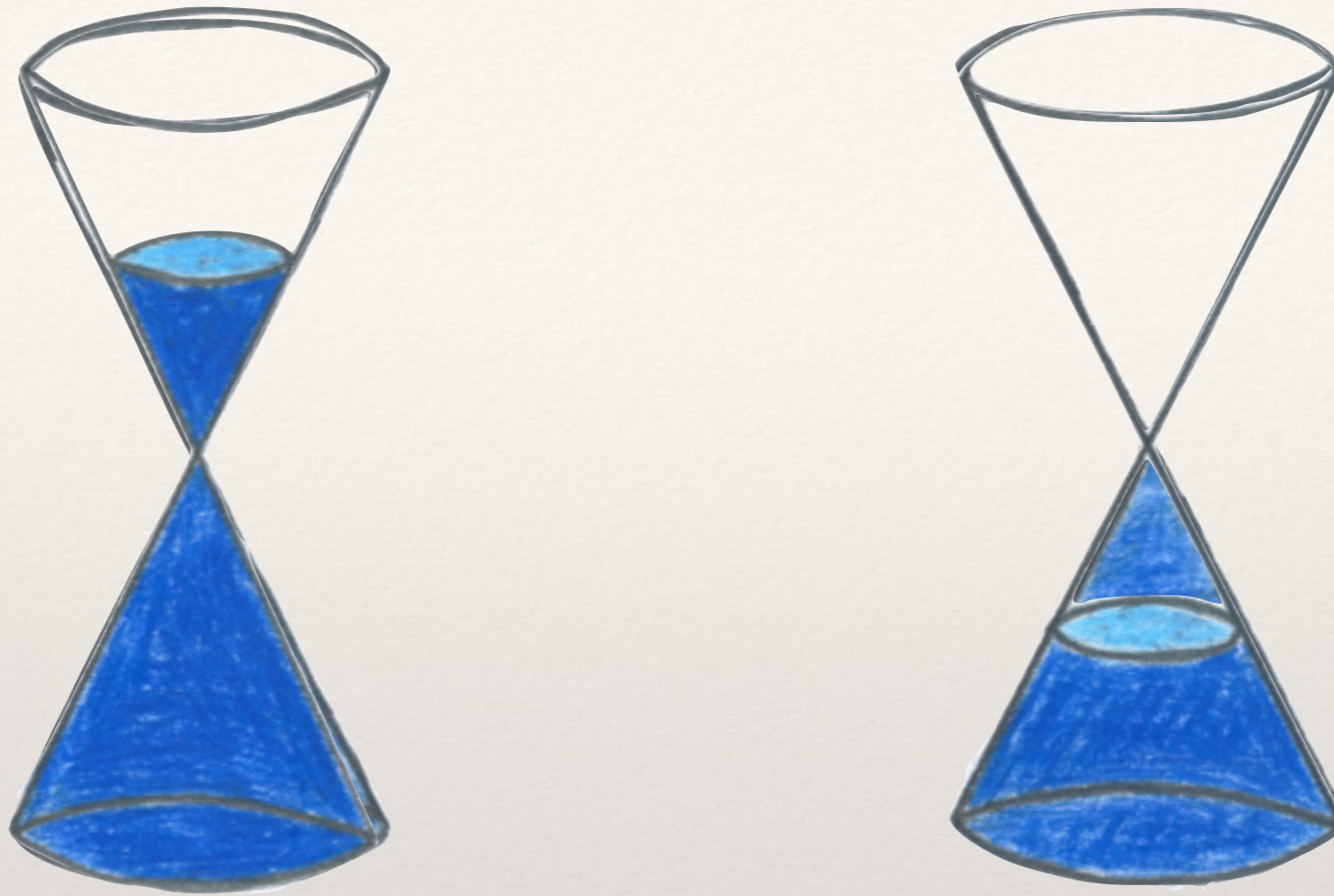
Potter, Kimchi, Vishwanath, Nature Comm. 2014

Baum, Berg, Parameswaran, Stern PRX 2015

Moll, Nair, Helm, Potter, Kimchi, Vishwanath, Analytis, Nature 2016

Chiral anomaly

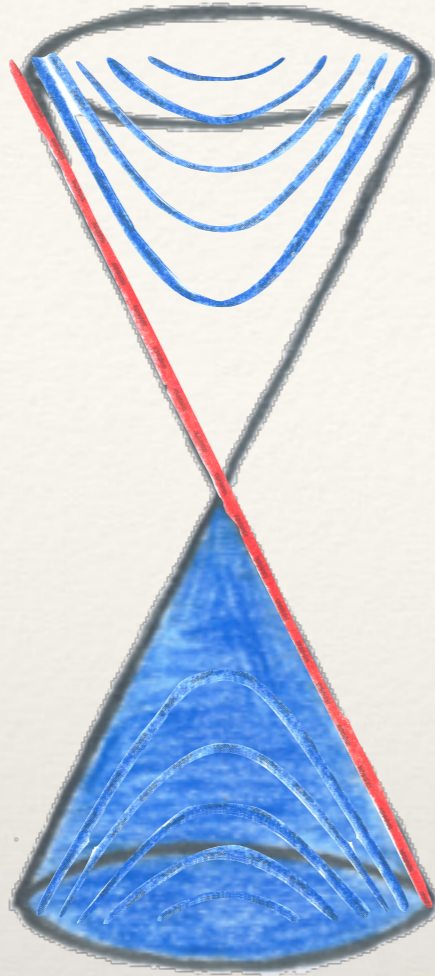
Chiral anomaly in a Weyl semimetal



$$\frac{\partial}{\partial t} (n_L + n_R) = 0$$

$$\frac{\partial}{\partial t} (n_L - n_R) = \frac{e^2}{2\pi^2 \hbar^2} \mathbf{E} \cdot \mathbf{B} - \frac{n_L - n_R}{\tau_v}$$

Zero Landau level is chiral



$$H = \chi(\mathbf{p} - e\mathbf{A}) \cdot \boldsymbol{\sigma} \quad \chi = \pm \quad \mathbf{B} = B\mathbf{e}_3$$

$$a_{p_2} = \frac{1}{\sqrt{2}} \left(\frac{x_1 - p_2 \ell_B^2}{\ell_B} + ip_1 \ell_B \right)$$

$$[a_{p_2}, a_{p_2}^\dagger] = 1$$

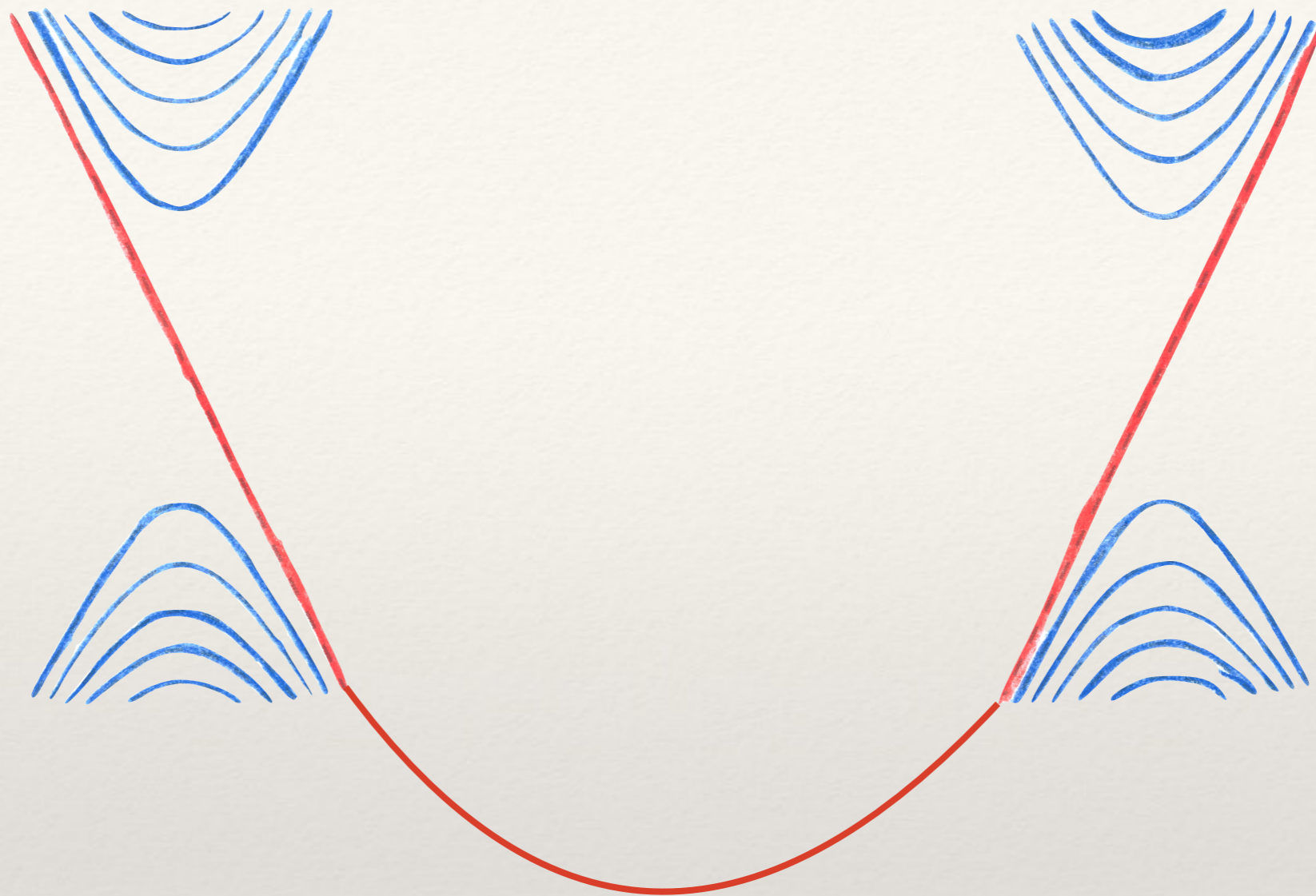
$$H = \chi \begin{pmatrix} p_3 & i\sqrt{2}a_{p_2}^\dagger/\ell_B \\ -i\sqrt{2}a_{p_2}/\ell_B & -p_3 \end{pmatrix}$$

$$\psi_n = \begin{pmatrix} |n\rangle \\ \lambda|n-1\rangle \end{pmatrix}$$

$$E_n = \pm \chi \sqrt{p_3^2 + 2n\ell_B^2}$$

$$\psi_0 = \begin{pmatrix} |0\rangle \\ 0 \end{pmatrix} \quad E_0 = \chi p_3$$

Zero Landau level is chiral



$$\frac{\partial p_3}{\partial t} = eE$$

$$\frac{\partial n}{\partial t} = \frac{eE}{2\pi\hbar} \frac{eB}{2\pi\hbar} = \frac{e^2}{4\pi^2\hbar^2} \mathbf{E} \cdot \mathbf{B}$$

Semiclassical Boltzmann theory gives the same answer

$$\frac{\partial n_{\mathbf{p}}}{\partial t} + \dot{\mathbf{r}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{r}} + \dot{\mathbf{p}} \cdot \frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}} = I_{\text{coll}}\{n_{\mathbf{p}}\}$$

$$\dot{\mathbf{r}} = \frac{\partial \epsilon_{\mathbf{p}}}{\partial \mathbf{p}} + \dot{\mathbf{p}} \times \boldsymbol{\Omega}_{\mathbf{p}} \quad \boldsymbol{\Omega}_{\mathbf{p}} = \nabla_{\mathbf{p}} \times \mathbf{A}_{\mathbf{p}} \quad \mathbf{A}_{\mathbf{p}} = i \langle u_{\mathbf{p}} | \nabla u_{\mathbf{p}} \rangle$$

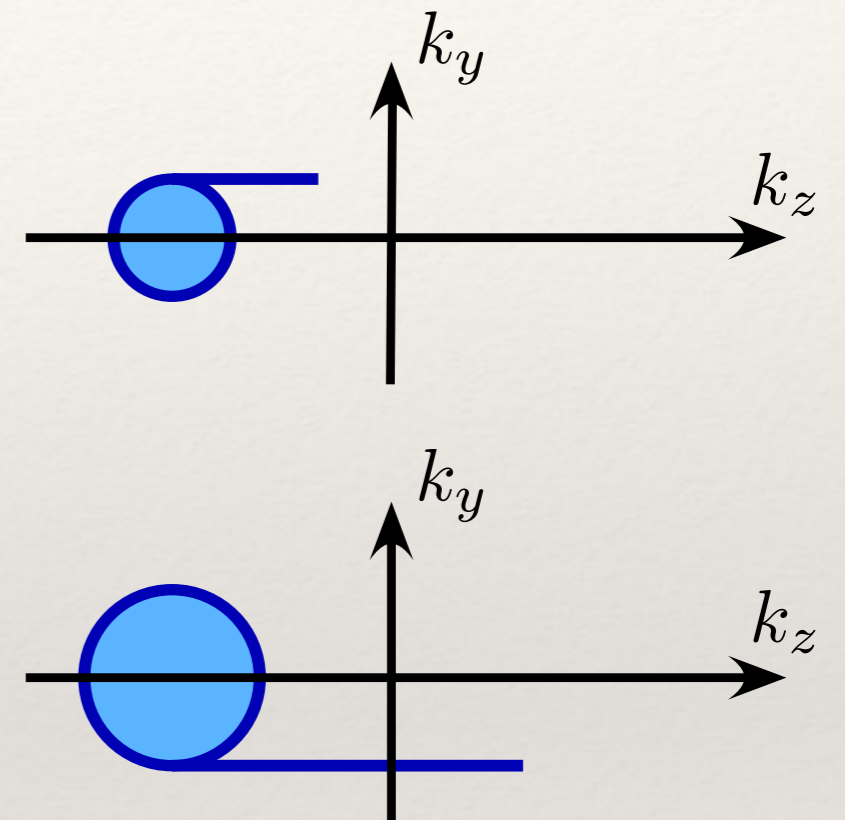
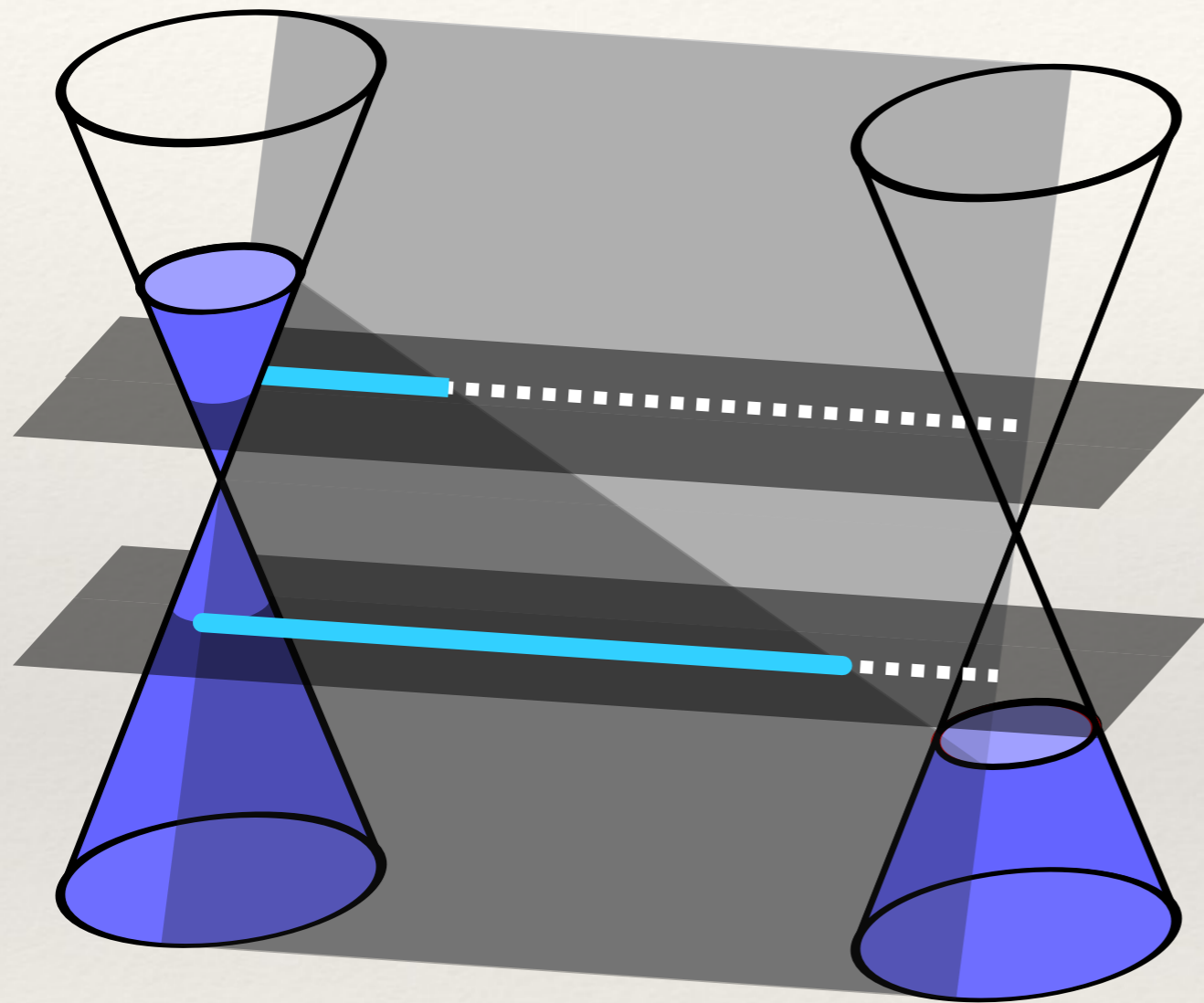
$$\dot{\mathbf{p}} = e\mathbf{E} + \frac{e}{c} \dot{\mathbf{r}} \times \mathbf{B}$$

$$\nu^{(i)} = \frac{1}{2\pi\hbar} \oint d\mathbf{S} \cdot \boldsymbol{\Omega}_{\mathbf{p}}^{(i)} \in \mathbb{Z}$$

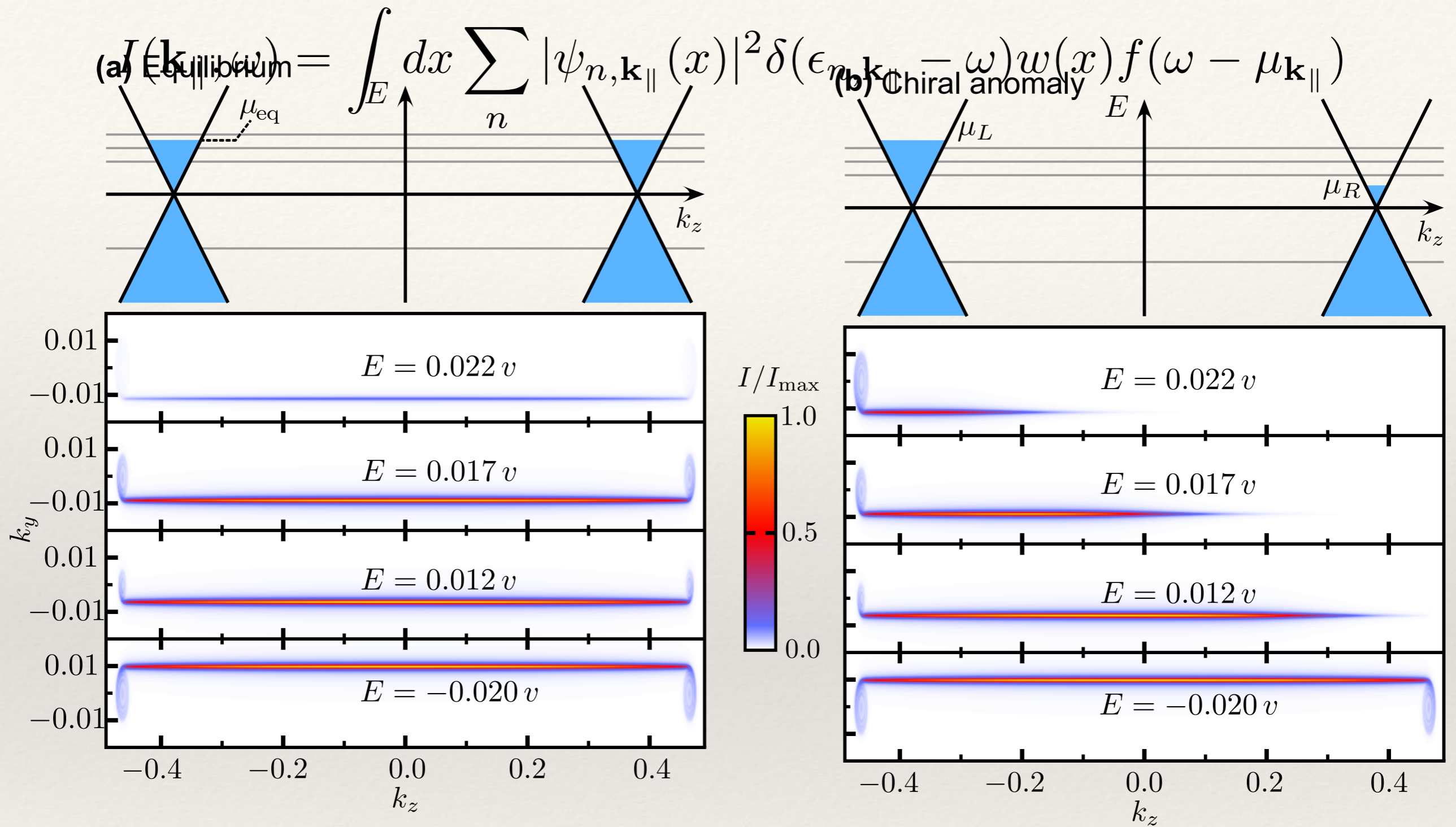
...

$$\frac{\partial n^{(i)}}{\partial t} + \nabla \cdot \mathbf{j}^{(i)} = \nu^{(i)} \frac{e^2}{4\pi^2 \hbar^2} \mathbf{E} \cdot \mathbf{B} - \frac{\delta n^{(i)}}{\tau}$$

The chiral anomaly induced chiral chemical potential tilts the Fermi arc



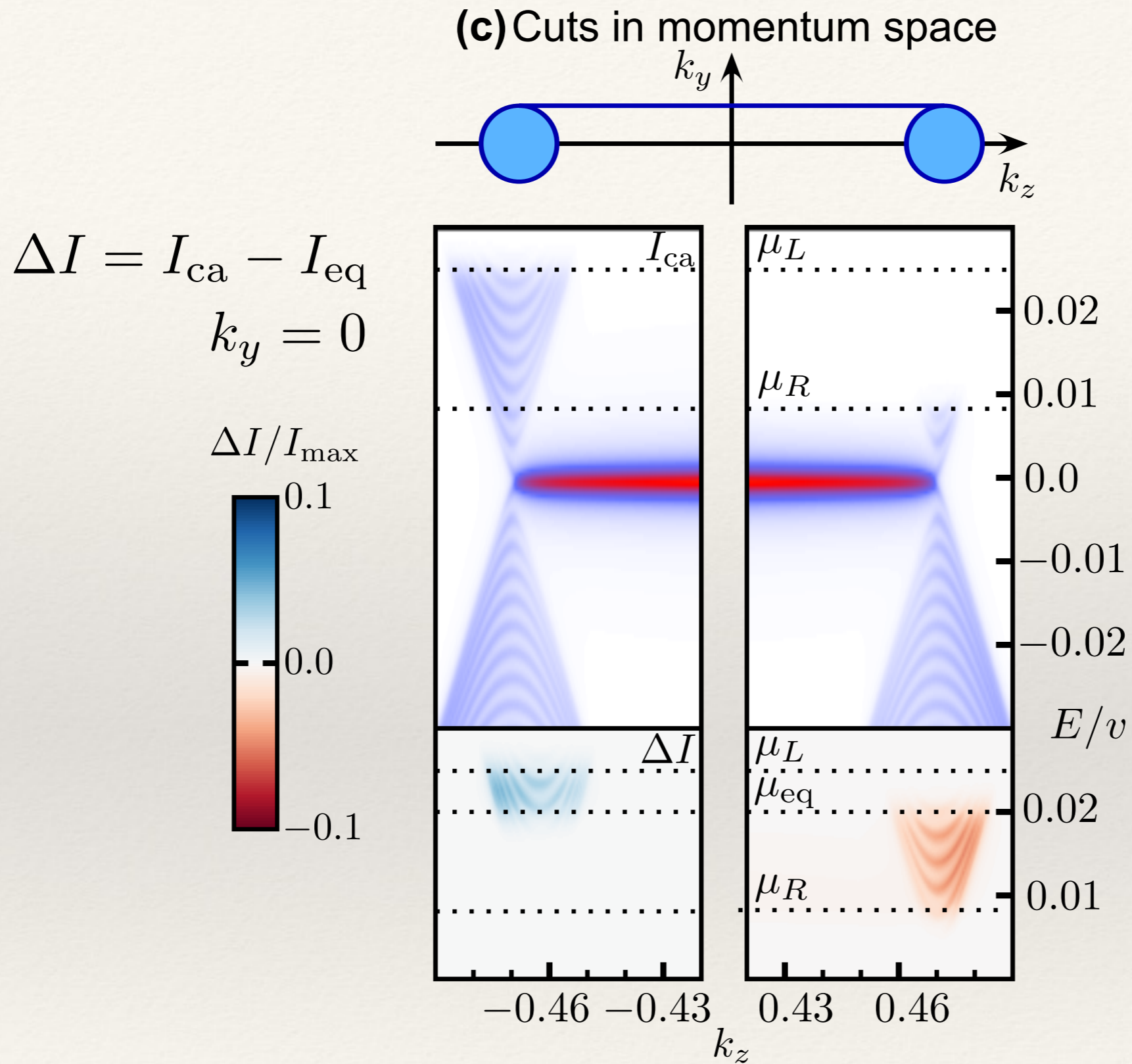
ARPES — Weyl semimetal



use tight binding model from Vazifeh and Franz PRL 2013

Behrends, Grushin, Ojanen, JHB PRB 2016

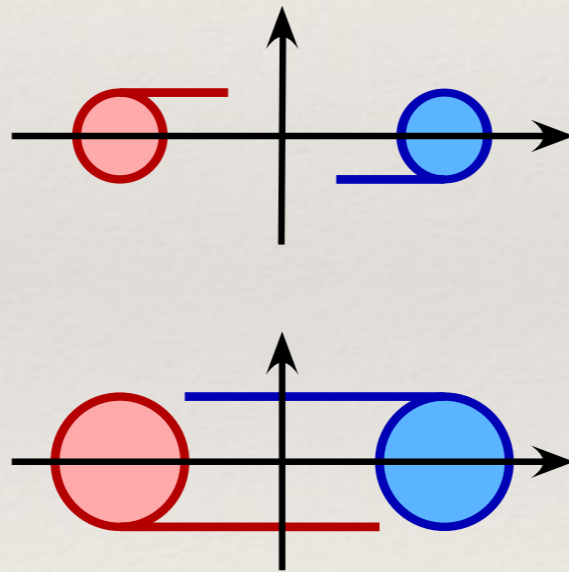
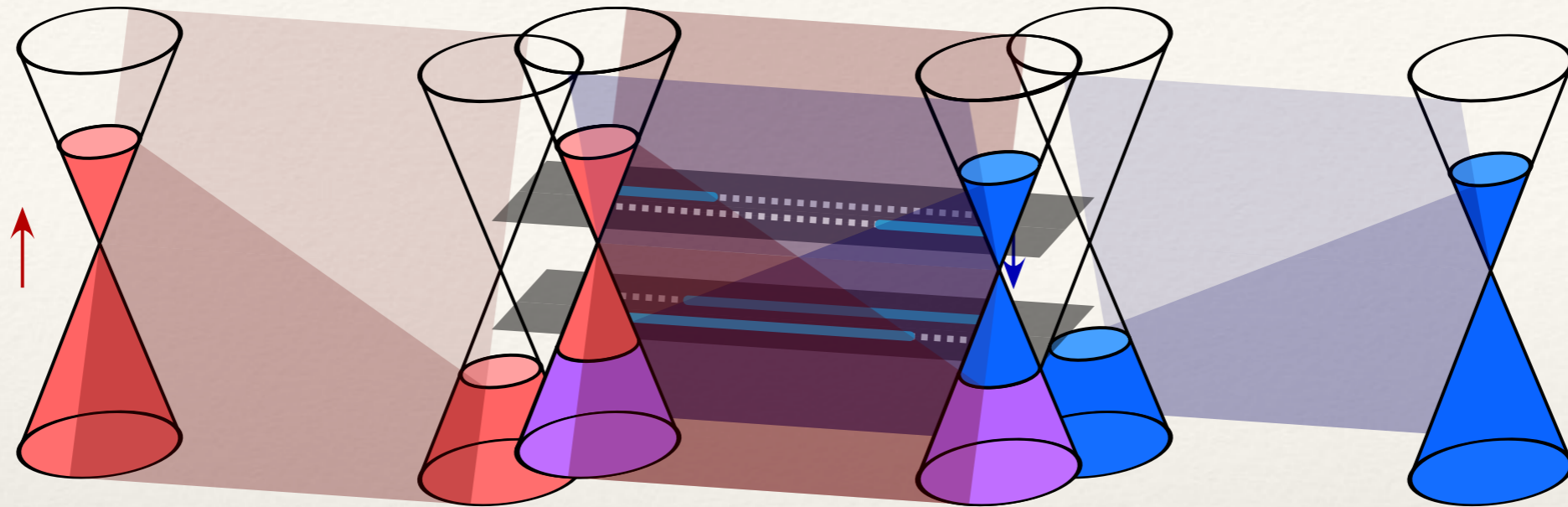
ARPES — Weyl semimetal



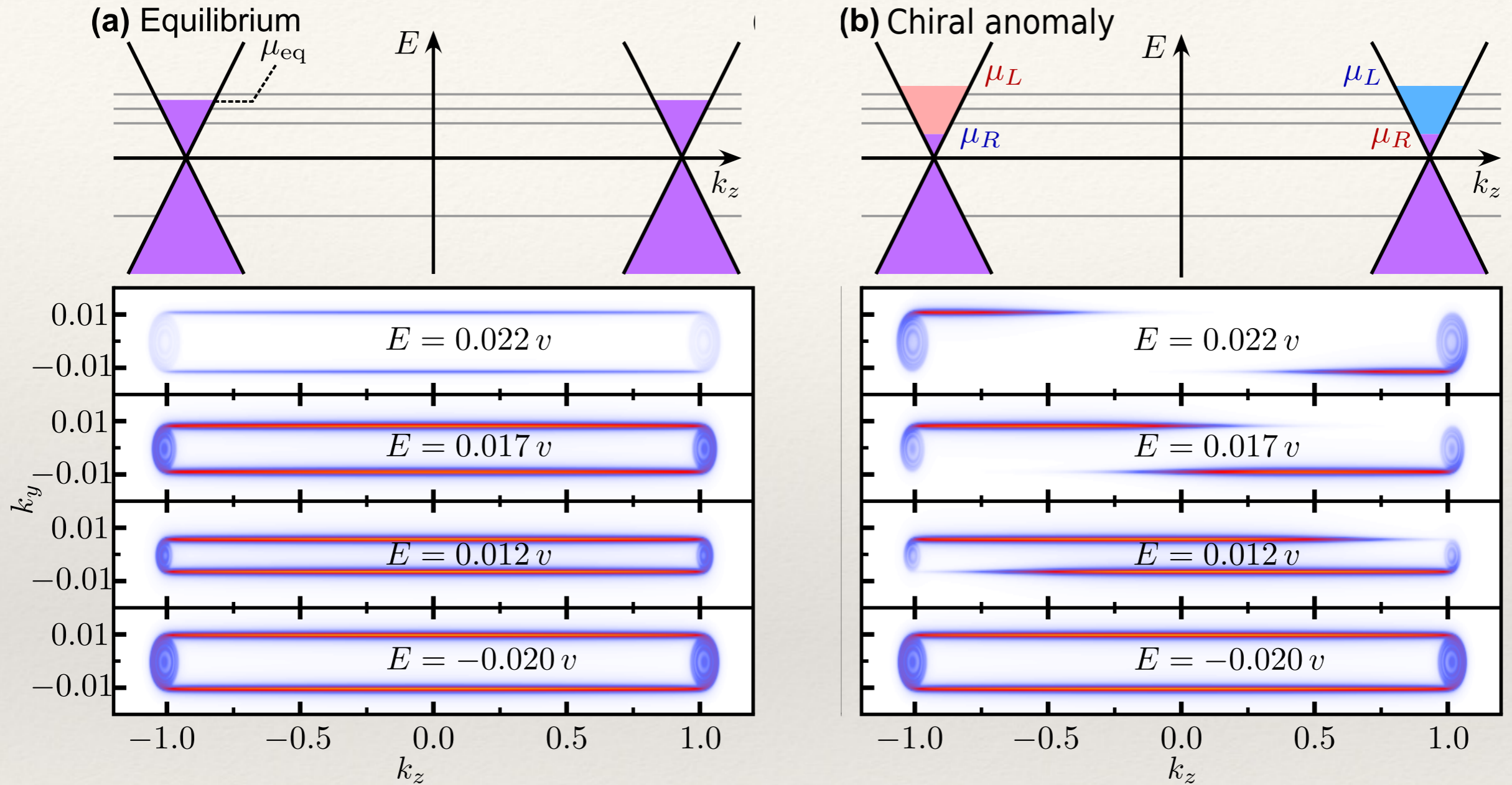
use tight binding model from Vazifeh and Franz PRL 2013

Behrends, Grushin, Ojanen, JHB PRB 2016

Dirac semimetal as two copies of a Weyl semimetal



ARPES — Dirac semimetal



use tight binding models from Wang et al. PRB 2012; PRB 2013

Behrends, Grushin, Ojanen, JHB PRB 2016

Experimental estimates

Na₃Bi

$$E = 10^4 \text{ V m}^{-1}$$

$$B = 1 \text{ mT}$$

$$\tau_v/\tau_c = 10^4$$

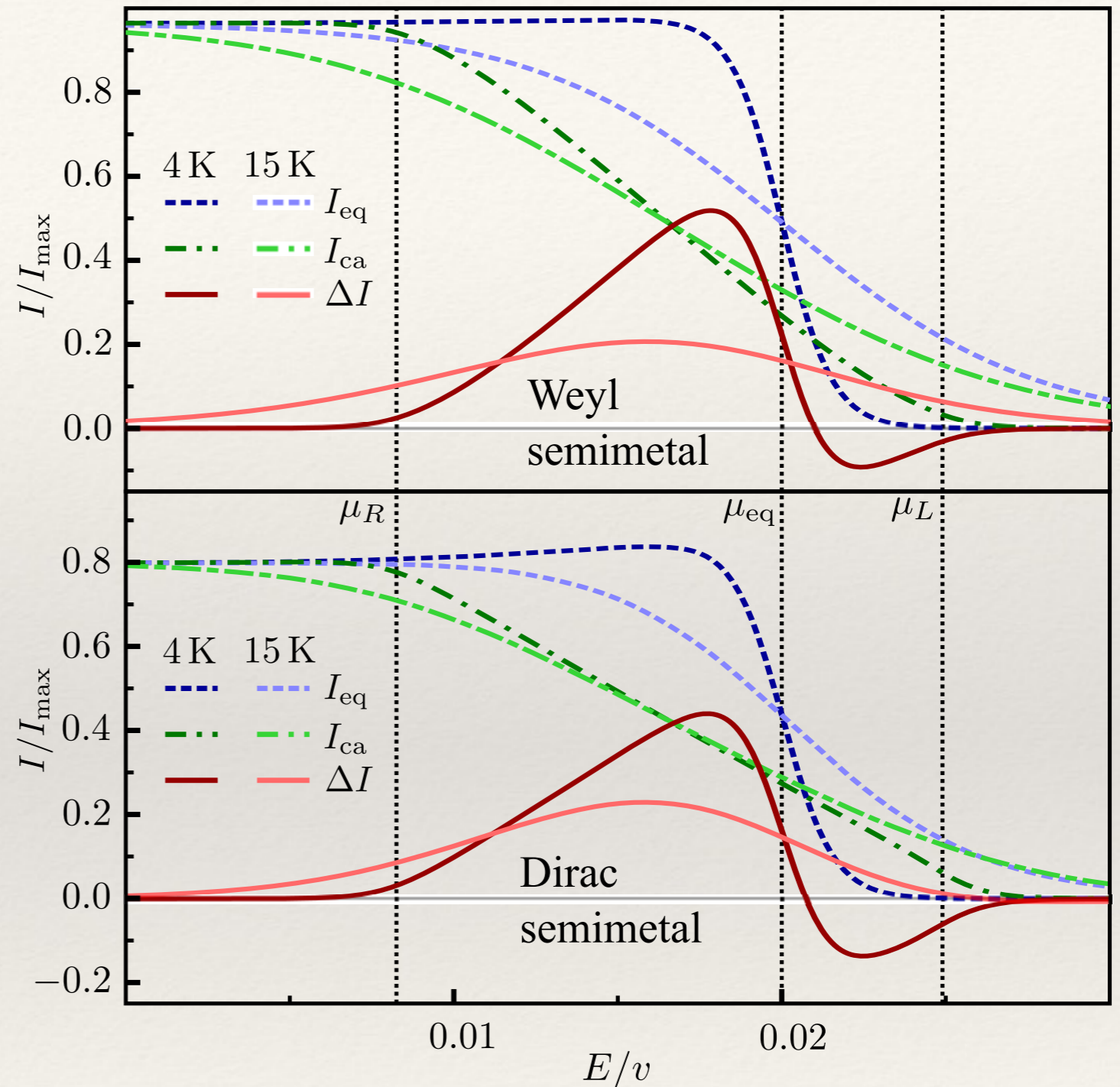
$$\delta\mu = \mu_L - \mu_R \approx 10 \text{ meV}$$

But, ARPES doesn't like B!

Magnetic substrate

Pump-probe

...



discussion of scattering times: Parameswaran et al. PRX 2014

Behrends, Grushin, Ojanen, JHB PRB 2016

Transport: negative magnetoresistance

Semiclassical Boltzmann theory and conductance

$$\frac{\partial n^{(i)}}{\partial t} + \nabla \cdot \mathbf{j}^{(i)} = \nu^{(i)} \frac{e^2}{4\pi^2 \hbar^2} \mathbf{E} \cdot \mathbf{B} - \frac{\delta n^{(i)}}{\tau}$$

Chiral magnetic effect:

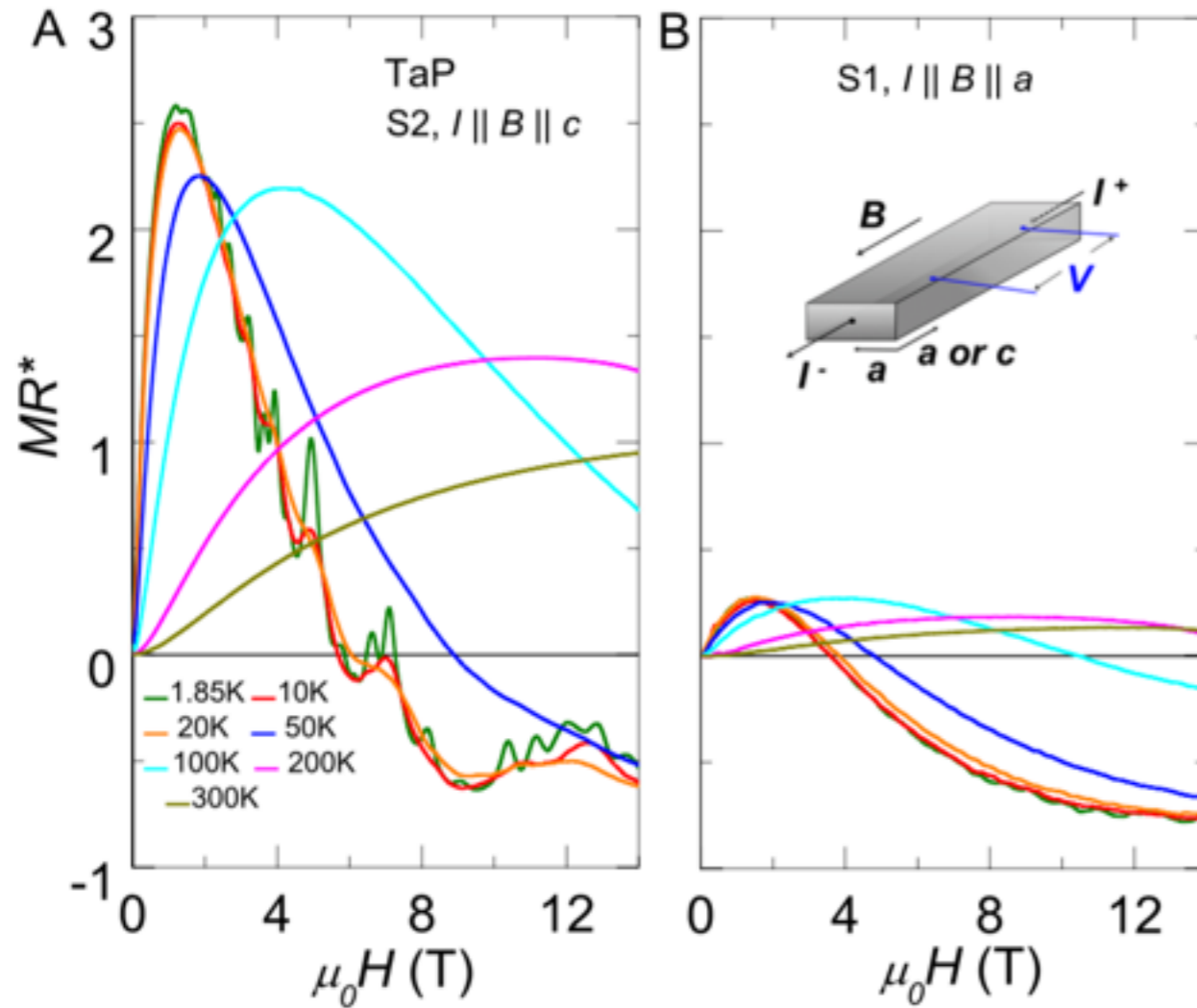
$$\mathbf{j} = \frac{e^2}{4\pi^2 \hbar^2 c} \mathbf{B} \sum_i k^{(i)} \mu^{(i)}$$

Negative magnetoresistance

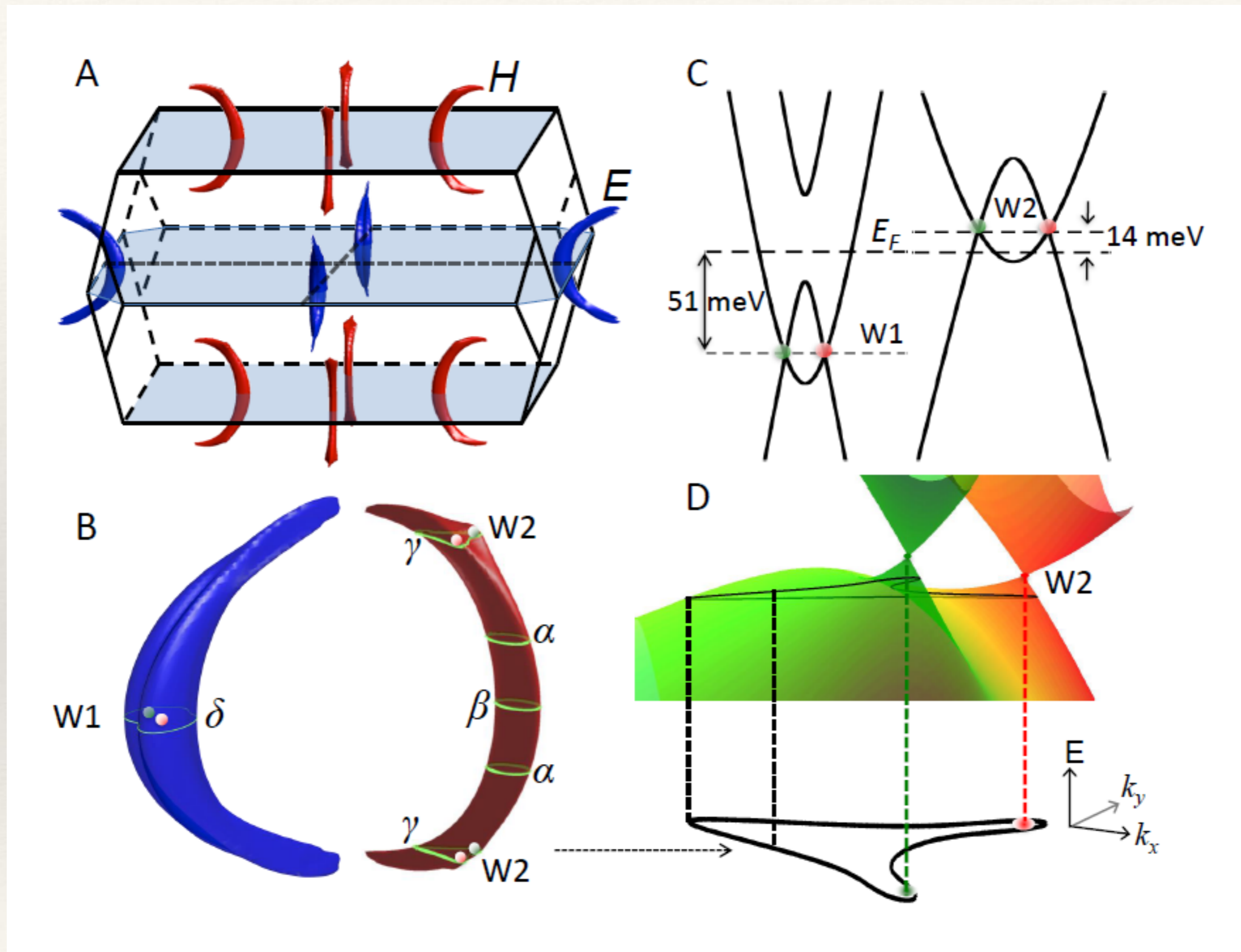
$$\sigma_{zz} = \frac{e^2}{4\pi^2 \hbar} \frac{(eB)^2 v^3}{\mu^2} \tau \quad \tau \gg \tau_{\text{tr}}$$

Negative magnetoresistance in TaP

$$\sigma = \sigma_0 + \frac{e^4 B^s \tau_a}{4\pi^4 g(E_F)}$$



TaP has several electron and hole pockets and no well defined chirality



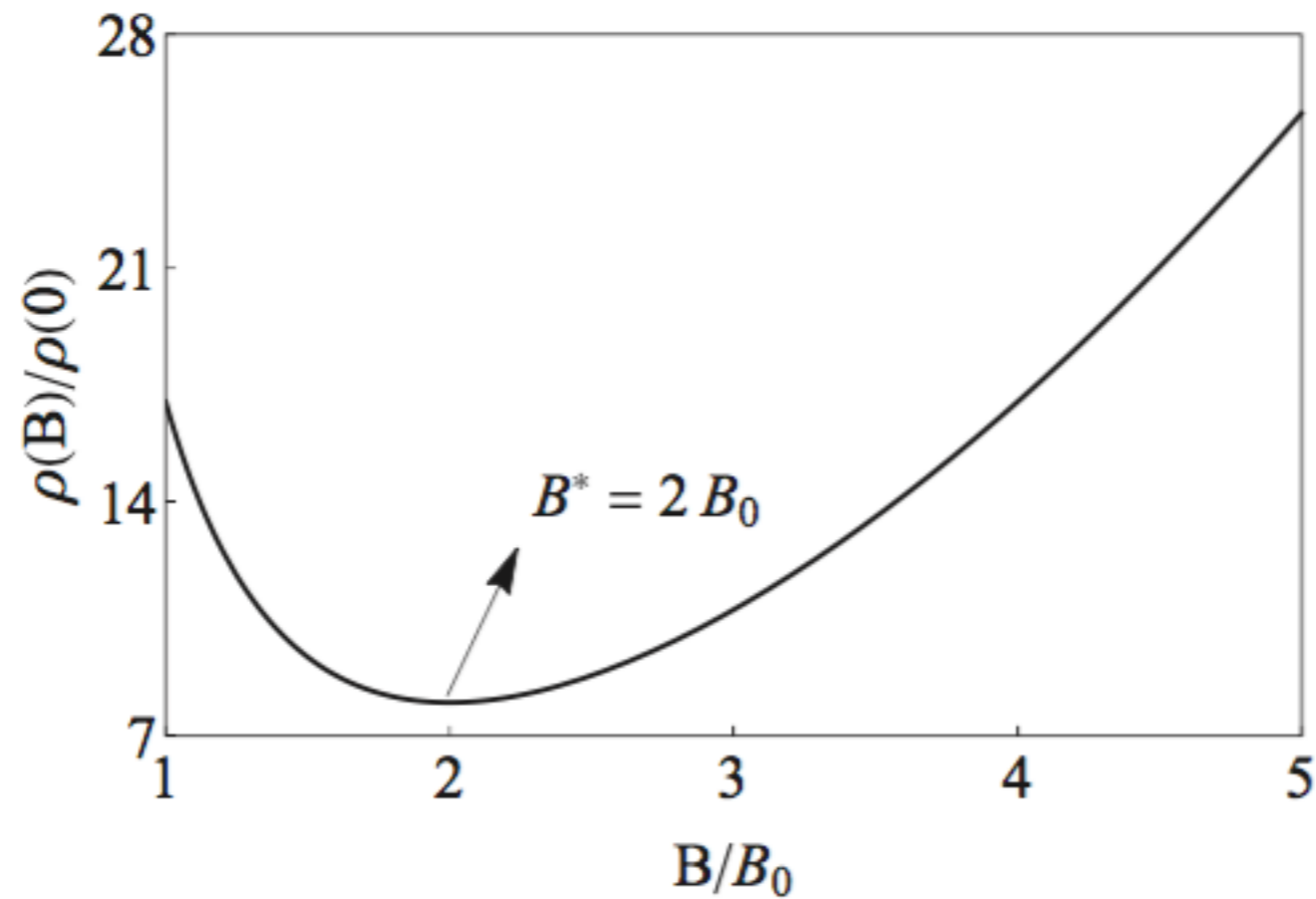
Negative magnetoresistance also for electron pockets in quantum limit

PHYSICAL REVIEW B **92**, 075205 (2015)



Axial anomaly and longitudinal magnetoresistance of a generic three-dimensional metal

Pallab Goswami, J. H. Pixley, and S. Das Sarma



Gyrotropic magnetic effect

$$j_i = \alpha_{ij}^{\text{GME}} B_j$$

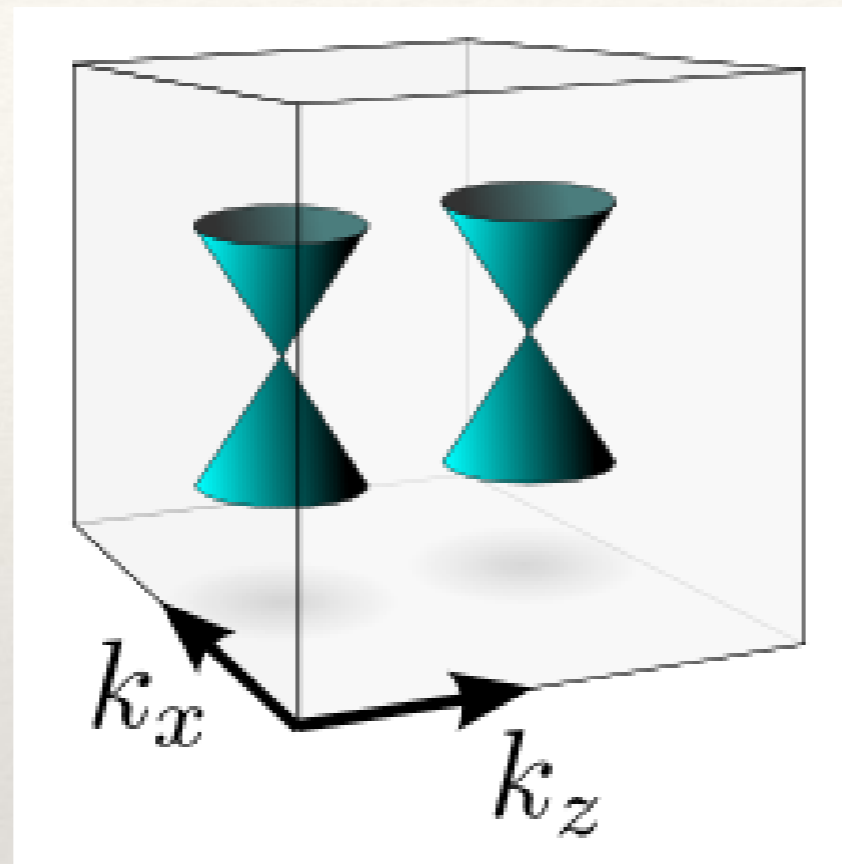
$$\alpha_{ij}^{\text{GME}} = \frac{i\omega\tau}{i\omega\tau - 1} \frac{e}{(2\pi)^2 \hbar} \sum_{n,a} \int_{S_{n,a}} dS \hat{v}_{F,i} m_{\mathbf{k}n,j}$$

$$\mathbf{m}_{\mathbf{k}n} = -(eg_s/2m_e) \mathbf{S}_{\mathbf{k}n} + \mathbf{m}_{\mathbf{k}n}^{\text{orb}}$$

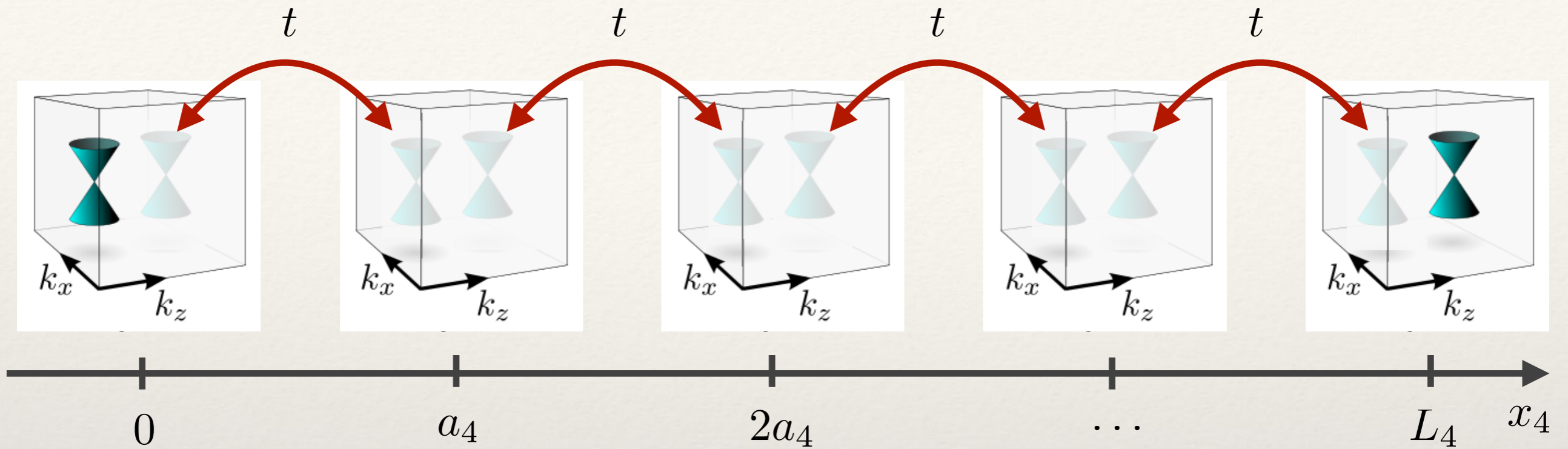
$$\mathbf{m}_{\mathbf{k}n}^{\text{orb}} = \frac{e}{2\hbar} \text{Im} \langle \partial_{\mathbf{k}} u_{\mathbf{k}n} | \times (H_{\mathbf{k}} - \epsilon_{\mathbf{k}n}) | \partial_{\mathbf{k}} u_{\mathbf{k}n} \rangle$$

Fractional chiral metal

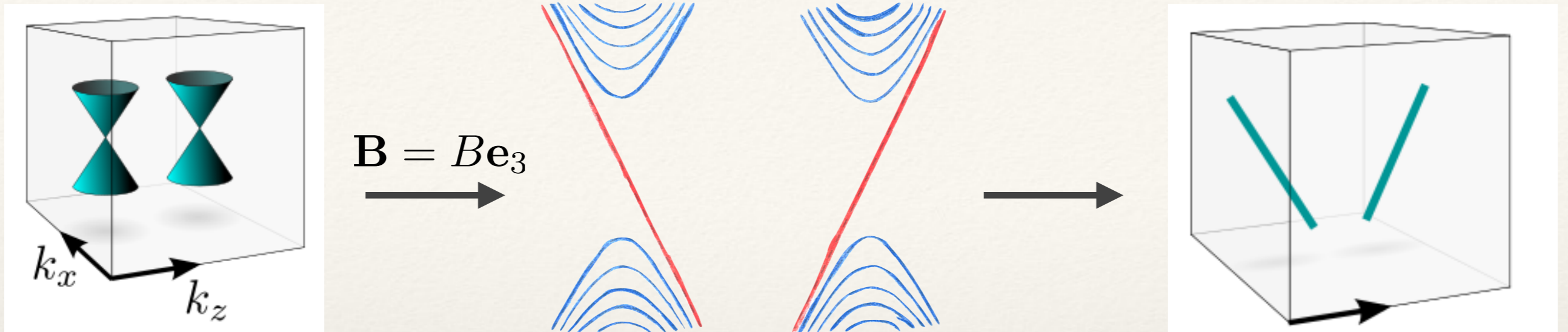
A 4D construction of a Weyl semimetal



A 4D construction of a Weyl semimetal



In presence of interactions first go to quantum limit and bosonize



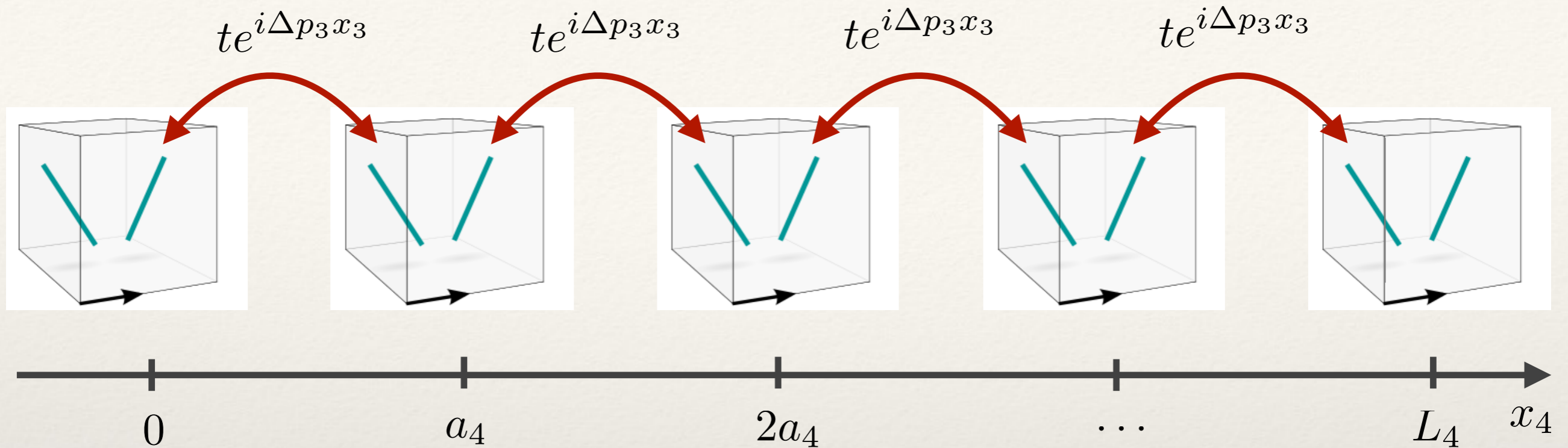
$$H_0 = \sum_{x_4, p_2} \int_0^b dx_3 \left(R_{p_2} \sum_{r, q, q', p_2} \int_{x_3}^{x_4} dx_3' \left(\partial_3 \Phi_{rp_2}(x_3, x_4) \right)^T V_{p_2, q, q'} \left(\partial_3 \Phi_{r'p_2}(x_3', x_4') \right) \right) (x_3, x_4)$$

$$V_{p_2, q, q'} = \frac{v_F}{4\pi} \delta_{q, q'} \mathbb{1} + \tilde{U}_{p_2, q, q'}$$

$$c_{p_2}(x_3, x_4) = \frac{1}{\sqrt{2\pi\alpha}} \left(e^{-i x_3 b/2} E_{p_2} \left(\frac{x_3 - x_4}{\alpha} \right) + e^{i x_3 b/2} R_{p_2} \left(\frac{x_3 - x_4}{\alpha} \right) \right)$$

$$[\Phi_{rp_2}(x_3, x_4), \Phi_{r'p_2}(x_3', x_4')] = \delta_{rr'} \delta_{p_2 p_2'} \delta_{x_4 x_4'} i\pi \hat{r} \text{sgn}(x_3 - x_3')$$

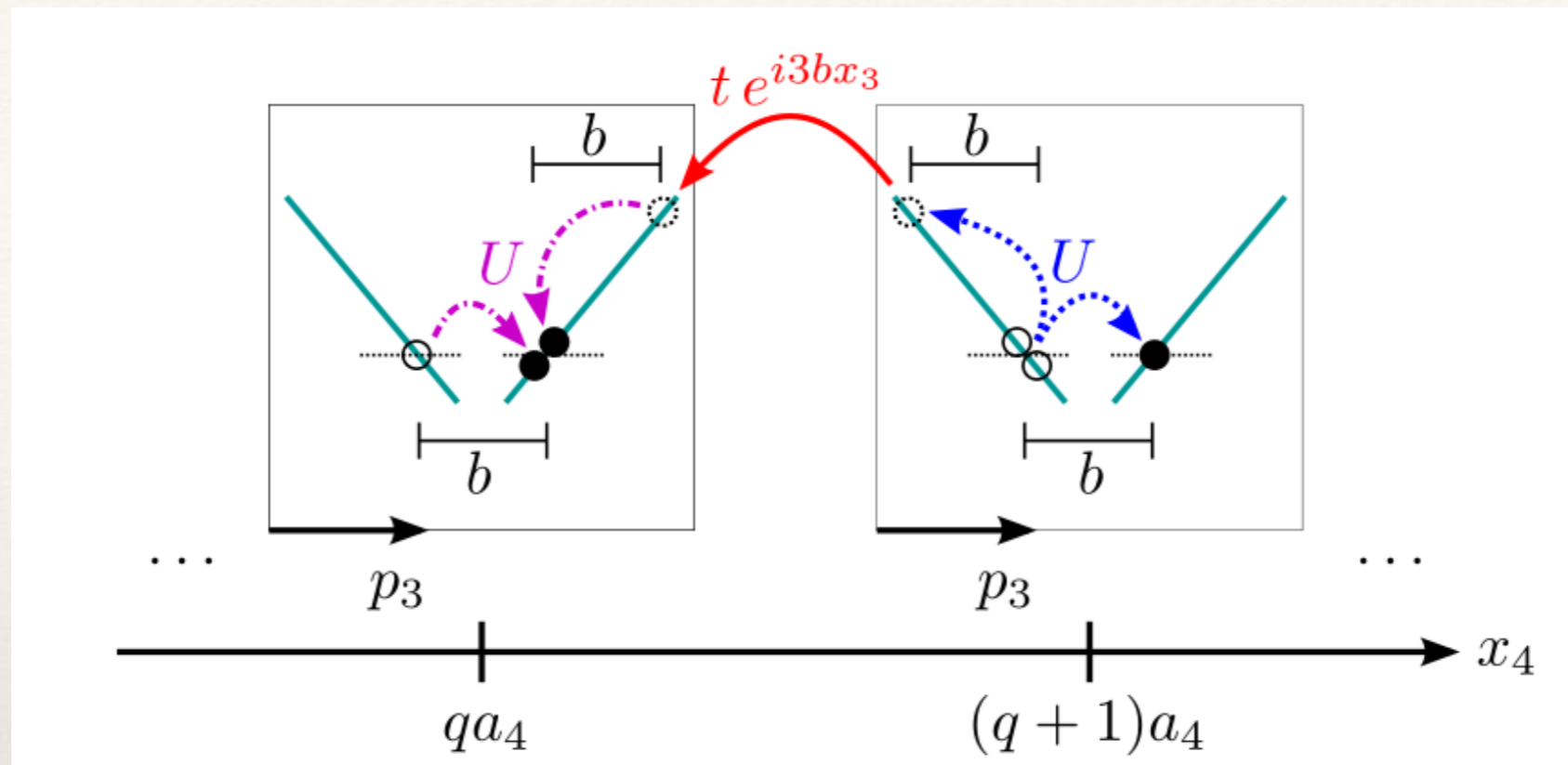
Coupled wires construction



$$H_{\text{tun}} = \sum_{q, p_2, p_3} t c_{p_2, p_3 + \Delta p_3}^\dagger ((q + 1)a_4) c_{p_2, p_3}(q a_4) + \text{h.c.}$$

$$H_{\text{int}} = \int dx_3 \sum_{p_2, x_4} U c_{p_2}^\dagger(x_3, x_4) (\partial_3 c_{p_2}^\dagger(x_3, x_4)) (\partial_3 c_{p_2}(x_3, x_4)) c_{p_2}(x_3, x_4)$$

Fractional quantum Hall state obtained for correlated hopping



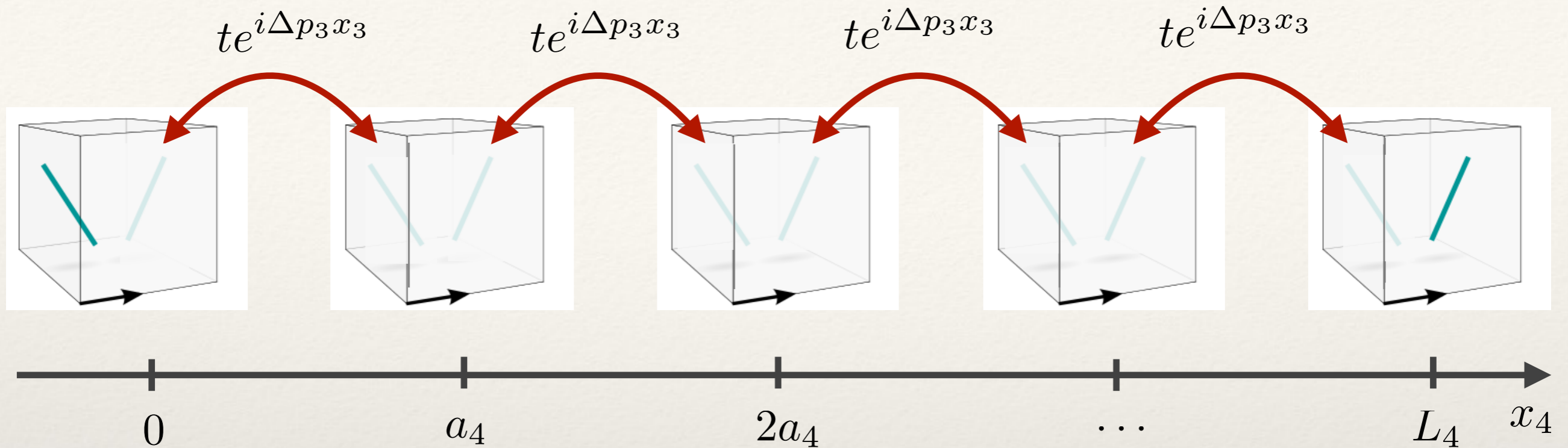
$$\tilde{\Phi}_{Lp_2}(x_3, x_4) = (m+1)\Phi_{Lp_2}(x_3, x_4) - m\Phi_{Rp_2}(x_3, x_4)$$

$$\tilde{\Phi}_{Rp_2}(x_3, x_4) = (m+1)\Phi_{Rp_2}(x_3, x_4) - m\Phi_{Lp_2}(x_3, x_4)$$

$$[\tilde{\Phi}_{rp_2}(x_3, x_4), \tilde{\Phi}_{r'p'_2}(x'_3, x'_4)] = \delta_{rr'}\delta_{p_2p'_2}\delta_{x_4x'_4}(2m+1)i\pi\hat{r}\text{sgn}(x_3 - x'_3)$$

$$H_{\text{tun}}^{(2m)} \sim tU^{2m} \sum_{q,p_2} \int dx_3 \cos \left(\tilde{\Phi}_{Lp_2}(x_3, (q+1)a_4) - \tilde{\Phi}_{Rp_2}(x_3, qa_4) \right)$$

The bulk is gapped out by the correlated hopping



$$H_{\text{tun}}^{(2m)} \sim tU^{2m} \sum_{q,p_2} \int dx_3 \cos \left(\tilde{\Phi}_{Lp_2}(x_3, (q+1)a_4) - \tilde{\Phi}_{Rp_2}(x_3, qa_4) \right)$$

Electromagnetic response:
$$j^4 = -\frac{e^3}{4\pi^2(2m+1)} \mathbf{E} \cdot \mathbf{B}$$

Field theory from minimal coupling and gauge invariance

$$\mathcal{S} = \mathcal{S}_0[\Phi] + \mathcal{S}_1[\Phi, A_4] + \mathcal{S}_2[\Phi, A_\alpha]$$

$$\mathcal{S}_1[\Phi, A_4] \sim tU^{2m} \sum_{q,p_2} \int dx_0 dx_3 \cos \left(\tilde{\Phi}_{Lp_2}((q+1)a_4) - \tilde{\Phi}_{Rp_2}(qa_4) + ea_4 A_4(qA_4) \right)$$

$$\begin{aligned} \mathcal{S}_2[\Phi, A_\alpha] &= e \sum_{x_4} \int dx_0 dx_3 j^\alpha A_\alpha \quad \alpha = 0, 3 \\ &= \sum_{p_2, x_4} \frac{e}{2\pi(2m+1)} \int dx_0 dx_3 \epsilon^{\alpha\beta} \partial_\alpha (\tilde{\Phi}_{Rp_2} - \tilde{\Phi}_{Lp_2}) A_\beta \end{aligned}$$

$$\partial_\alpha j^\alpha = 0$$

$$\rho = \sum_{p_2} \frac{1}{2\pi} \partial_3 (\Phi_{Lp_2} - \Phi_{Rp_2}) = \sum_{p_2} \frac{1}{2\pi(2m+1)} \partial_3 (\tilde{\Phi}_{Lp_2} - \tilde{\Phi}_{Rp_2})$$

Strong coupling and electromagnetic response: fractional chiral anomaly

$$\tilde{\Phi}_{Rp_2}(qa_4) - \tilde{\Phi}_{Lp_2}((q+1)a_4) = ea_4 A_4(qa_4)$$

$$\mathcal{S}_2[A_4, A_\alpha] = \sum_{p_2} \frac{-e^2}{2\pi(2m+1)} \int dx_0 dx_3 dx_4 \epsilon^{\alpha\beta} A_4 \partial_\alpha A_\beta$$

$$\sum_{p_2} \rightarrow N_{LL} = eB_3 L_1 L_2 / 2\pi$$

$$j^4 = \frac{\delta \mathcal{S}}{\delta A_4} = -\frac{e^3}{4\pi^2(2m+1)} B_3 E_3$$

Action is an anisotropic part of the isotropic 4+1D Chern-Simons action

$$N_{\text{LL}} = \frac{B_3 L_1 L_2}{2\pi/e} = \int dx_1 dx_2 \frac{1}{2\pi/e} \epsilon^{\gamma\delta} \partial_\gamma A_\delta$$

$$\mathcal{S}_2[A_\mu] = \frac{-e^3}{(2\pi)^2(2m+1)} \int d^5x \epsilon^{\alpha\beta} \epsilon^{\gamma\delta} A_4 (\partial_\alpha A_\beta) (\partial_\gamma A_\delta)$$

compare with

$$\mathcal{S}_{\text{CS}}^{(4+1)}[A_\mu] = \frac{-e^3}{6(2\pi)^2(2m+1)} \int d^5x \epsilon^{\mu\nu\rho\sigma\eta} A_\mu \partial_\nu A_\rho \partial_\sigma A_\eta$$

$$j^4 = -\frac{e^3}{4\pi^2(2m+1)} \mathbf{E} \cdot \mathbf{B}$$

Surface field theory — fractional chiral metal

3+1D surface state constructed from N_{LL} copies of the edge theory of 2+1D Chern-Simons theory, labeled by p_2

$$\mathcal{S}_{\text{surface}}[\Phi_{R,L}] = \frac{2m+1}{4\pi} N_{LL} \int_{\partial\Sigma} \left[\partial_0 \Phi_L \partial_3 \Phi_L - v_F (\partial_3 \Phi_L)^2 - \right. \\ \left. (\partial_0 \Phi_R \partial_3 \Phi_R + v_F (\partial_3 \Phi_R)^2) \right]$$

$$\partial_0 \rho_{R,L} + \hat{r} v_F \partial_3 \rho_{R,L} = 0 \quad \hat{r} = \pm 1$$

Consistent with current algebra analysis of the 4+1D Chern-Simons field theory

$$S_{\phi F} = \kappa \int_{\partial\Sigma} \partial_0 \phi d\phi \wedge F$$

K. Gupta and A. Stern, Nucl. Phys. B 1995

Collaborators



Jan Behrends



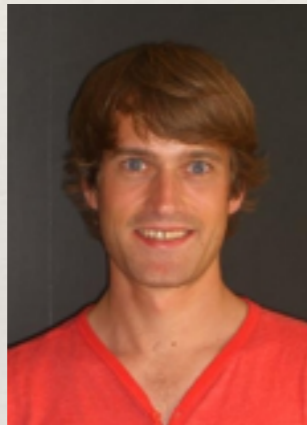
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Teemu Ojanen
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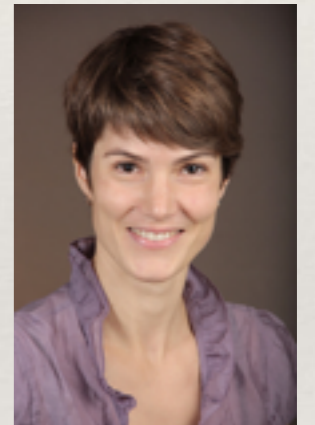
Tobias Meng



Frank Arnold

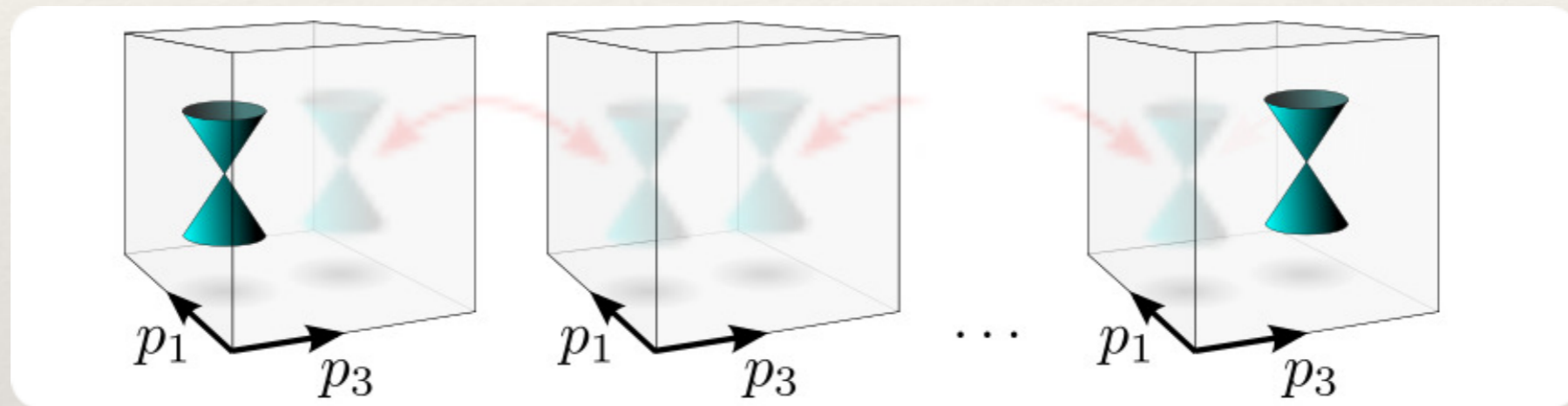
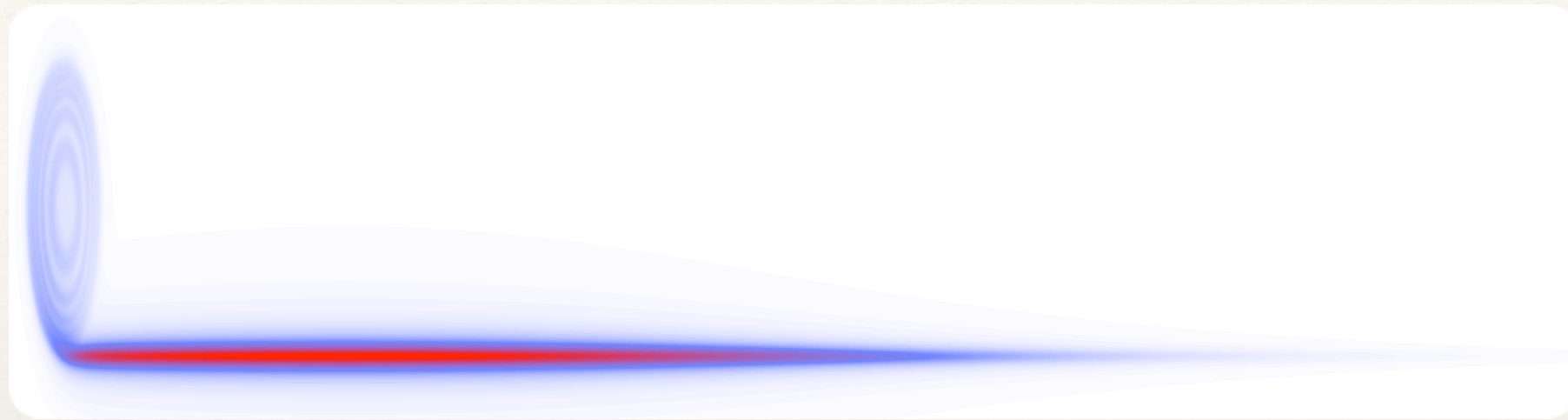


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and more

Summary: chiral anomaly visualized through the blue note and fractional chiral metals



Open PhD and Postdoc positions at KTH



J. Behrends, A. G. Grushin, T. Ojanen, JHB Phys. Rev. B **93**, 075114 (2016)
F. Arnold, ...A. G. Grushin, JHB, ... C. Felser, E. Hassinger, B. Yan Nat. Comm. **7**, 11615 (2016)
T. Meng, A. G. Grushin, K. Shtengel JHB arXiv:1602.08856 (to appear in PRB)